Ergodic Capacity Evaluation of MIMO Channel in Wireless Communication System

Nnaji, Genevieve Ada[1], Nwabueze, Christopher A.[2], Iloh, Johnpaul I. [3]

[1]Department of Electrical and Electronic Engineering, University of Agriculture and Environmental Sciences, Umuagwo, Nigeria
[2]Department of Electrical and Electronic Engineering, Chukwuemeka Odumegwu Ojukwu University, Uli, Nigeria
[3]Department of Electrical and Electronic Engineering, Chukwuemeka Odumegwu Ojukwu University, Uli, Nigeria

Abstract: *This paper considered ergodic capacity evaluation of multiple-input multiple-output (MIMO) channel in wireless communication system. Three main parameters of interest for the channels are the signal power expressed in terms of signal to noise ratio (SNR) in dB, the spacing between antennas, and the number of antennas. A MATLAB C code was developed to examined the performance of the proposed system considering channel capacity in terms of cumulative distribution function (CDF) to determine the probabilistic effect of increasing number of antennas at BS and MS on the rate of data transfer and ergodic capacity performance of the channel when different BS antennas transmits to MS different antennas. Simulation results revealed that the ergodic capacity cumulative distribution function (CDF), when SNR was fixed at 10 dB, 15 dB, 20 dB, 25 dB, and 30 dB while varying the number of antennas at base station (BS), M and mobile station (MS), N, resulted in ergodic capacity increase as the number of antennas increase with CDF (or probability) of 100%. The ergodic capacities for different multiple antenna elements configurations showed that increasing the number of antenna arrays from conventional MIMO to massive MIMO by means of antenna selection method can increase the rate (speed) at which information are transmitted over wireless channel. Besides, by making the BS antennas very much larger than the MS antennas provides much more capacity such that at M = 128 and N = 20, the capacity achieved was 197 bps/Hz.*

Keywords— Antennas, Communication system, Ergodic capacity, MIMO channel

1. INTRODUCTION

The emergence of multimedia devices has resulted in an increasing demand for data transmission over multiple devices at the same time to multiple users. For instance, in a typical residential apartment individuals may be using multimedia devices such as tablets, phones, and/or laptops and each user accesses the available network independently at the same time. Accessing the information using single-input single-output (SISO) devices can limit bandwidth (system capacity) and thereby causing prolonged downloads, timeout errors, and inability to maintain communication connection. Besides, more mobile devices that can establish communication between individuals are now readily available. However, the bandwidth and speed allotted for these devices seem to be limited by software, hardware, or handshaking potentials involving each electronic device. With antenna technology evolving into more complex system requiring the use of multiple antennas due to demand for information and communication at high rate of data transfer without loss of reliability, several studies have been carried out in this aspect to address this demand.

Consequently, as existing wireless communication systems across the world are subject to significant pressure, a dramatic increase of wireless data traffic has occurred [1]. The reduction of the size of antenna element perfectly outfits large scale MIMO requirement and makes the arrays a potential technology [2]. In addition, improved performance and better signal to interference-plus-noise ratio (SINR) is achieved when the number of antenna elements is increased [1]. While maximum of 8 antennas are used in each side of the transmitter and receiver (8 × 8 MIMO systems) respectively for conventional MIMO system, massive MIMO technology, depending on the implemented prototype, for 5G new wireless radio communication, provide supports for base station (BS) antennas of up to 256 and user equipment (UE) that will be up to 32 [3]. Significant improvement is achieved in cellular network including throughput and coverage, by increasing the number of elements of antenna array. Furthermore, using multiple antennas to combine energy in required direction helps to overcome higher path loss because of high frequencies [1].

Recent innovations in wireless system such as I-PAD, Wi-Fi router (WI-MAX), and mobile equipment using long term evaluation (LTE) standards are geared towards using MIMO systems for a reliable and faster means of data transfer. Hence, the need for information via multimedia piece of equipment at a seamlessly high speed necessitates improvement of wireless technology. MIMO is a technology that is believed to provide this requirement. This paper is designed to evaluate the ergodic capacity of MIMO channel wireless network data transmission for multimedia application.

2. LITERATURE REVIEW

In this section, some of the recent studies regarding ergodic capacity of MIMO channel, and correlated and uncorrelated channel in wireless communication system are reviewed. This is to provide literature background for the present study.

Thakur and Naithani [4] evaluated the ergodic capacity of random MIMO channel when the channel state information is not known at the transmitter. The authors also considered the

MIMIO channel capacity when there is channel gain correlation between the transmitter and the receiver. Mathematical model were presented to compute the ergodic capacity of random MIMO system. The simulation analysis was limited to four transmit antennas and four receive antennas. The simulation results revealed that channel correlation resulted in capacity reduction.

Mohamed and Abdulsattar [5] considered the performance of correlated and uncorrelated channels of MIMO system for different number of antennas and different signal to noise ratio (SNR) over Rayleigh fading channel. The authors considered both CSI technique and water filling power allocation scheme at the transmitter. The capacity of MIMO system was considered for different MIMO system arrangement by varying number of antennas. Simulation analysis conducted in MATLAB environment revealed that with known CSI at the transmitter with the application of water filling technique, the system capacity was improved. Also, by varying the number of transmit antennas, a significant gain in system capacity was achieved with number of receive antenna kept constant. The study further showed that the capacity of the MIMO channel will drop when correlated factor increases.

Belaoura et al. [6] examined downlink MU-MIMO systems ergodic capacity. It was showed by the study that the use of massive MIMO in next generation wireless communication systems would be a significant technology. The objective of the study was to use two correlative models (Kronecker and Weichselberger) over Rayleigh fading to demonstrate the ergodic capacity of MU-MIMO. The results of the simulations revealed that number of transmit antennas was more significant than the number of users. In addition, results indicated that the loss of capacity was more or less not consistent even with correlation coefficient between transmit antennas, reaching as high as 0.6.

Omer et al. [7] evaluated the performance of 5G mobile system based on ergodic capacity parameters. The study stated that several metrics significantly affect the performance of wireless communication system, but focused on three namely, number of antennas, the source of power, and distance between transmitter and receiver. The performance of the system was analyzed considering user equipment (UEs) with equal and non-equal powers besides employing various antenna techniques to show distinction between massive MIMO, MIMO, and SISO. The BS was designed to communicate with two mobile stations at different distances from it, which then showed how the use of massive MIMO would offer better performance than conventional MIMO and SISO systems over Rayleigh fading channel. The analysis of the ergodic capacity of wireless system was carried out in MATLAB simulation environment and the result of the simulation indicated that increasing source power and BS antennas improved ergodic channel capacity, with massive MIMO offering the finest performance.

Jiancun [8] examined ergodic capacity for downlink multiuser MIMO (MU-MIMO) systems. The main objective of the study was to demonstrate the ergodic capacity of MU-MIMO downlink network employing two correlative models, namely Kronecker and Weichselberger over Rayleigh fading channels. The simulation analysis revealed that the number of transmit antennas was more vital than number of users. In addition, the analysis indicated that capacity loss was almost not consistent even with a correlation coefficient between transmit antennas of nearly as high as 0.6.

Widad [9] studied ergodic capacity of downlink massive MU-MIMO system with spatially correlated Rayleigh fading channels. The study was designed to improve the performance of 5G mobile systems that employs small cell scheme. It proposed different power settings for different users. The effect of these settings on the performance of the system was demonstrated using ergodic capacity metric. The performance of the system was validated by comparison with several transmitting antennas greater than the number of users. The simulation analysis revealed that the power loss was almost negligible.

Shiney et al. [10] presented a scheme for efficient antenna selection for a distributed large scale MIMO system. The system employed random antenna units (RAUs) scattered in the cell and maximum ratio transmission (MRT) technique for downlink transmission. The study was designed to employ the developed algorithm to achieve ergodic achievable rate to any extent possible by selection of transmit antenna in a constrained capacity limited system. In order to select the optimal subset of transmit antenna elements from a set of antennas, the selection of antenna set was initially performed in terms of channel amplitude and correlation and after which iterative method was employed. Simulation results showed that antenna selection scheme provided improved sum rate compared with the select all schemes with no much system performance degradation. A near optimal performance with least throughput loss was reported to be achieved by the system. However, the developed scheme particularly suitable for a distributed massive MIMO that has capacity constraint.

3. SYSTEM MODEL

Given a massive multiple input multiple output (MIMO) system with a base station (BS) having M number of antennas serving active receive N number of mobile terminals (MTs) as shown in Fig. 1. The following assumptions are made:

- All BS antennas transmit the same signal simultaneously.
- Spacing between antennas is based on wavelength distance
- Among the N number of MTs, the same time and frequency resources are shared.
- Power is equally spread over all transmit antennas.
- Narrow-band time invariant channel.

- The receivers have perfect channel state information (CSI).

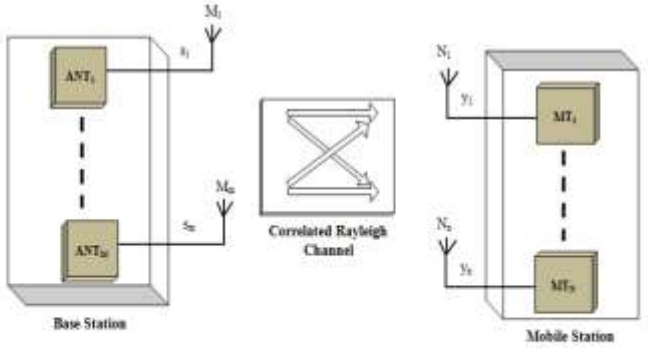


Fig. 1 Massive MIMO system model

Generally, for a MIMO technique, the received signal (vector symbol) is related to the transmitted signal (vector symbol) given by [11]:

$$y = Hs + \delta \qquad (1)$$

where s denotes the M×1 matrix of the transmitted vector symbols, y is the N×1 matrix of the received vector symbol by mobile terminals, δ is N×1 mobile terminal additive white Gaussian noise (AGWN) vector, and H represents the channel vector. The matrix of the channel vector H is given by:

$$H = \begin{bmatrix} h_{1,1} & h_{1,2} & \cdots & h_{1,N} \\ h_{2,1} & h_{2,2} & \cdots & h_{2,N} \\ \vdots & \vdots & \cdots & \vdots \\ h_{M,1} & h_{M,2} & \cdots & h_{M,N} \end{bmatrix}$$

(2)

Considering a Rayleigh channel matrix, each input to Rayleigh channel matrix may be expressed as given by [11]:

$$h_{i,j} = \alpha + j\beta$$
$$h_{i,j} = \left|h_{i,j}\right| e^{j\phi_{i,j}} \qquad (3)$$

where α and β represent randomly distributed variables and $\left|h_{i,j}\right|$ denotes a Rayleigh distributed random variables, $h_{i,j}$ is the element of the channel matrix for ith transmit antenna and jth receive antenna with $i = \{1, 2, 3, 5, 4, \cdots, M\}$ and $j = \{1, 2, 3, 4, 5, \cdots, N\}$. The block diagram representing the system model according to Eq. (1) is shown in Fig. 2.

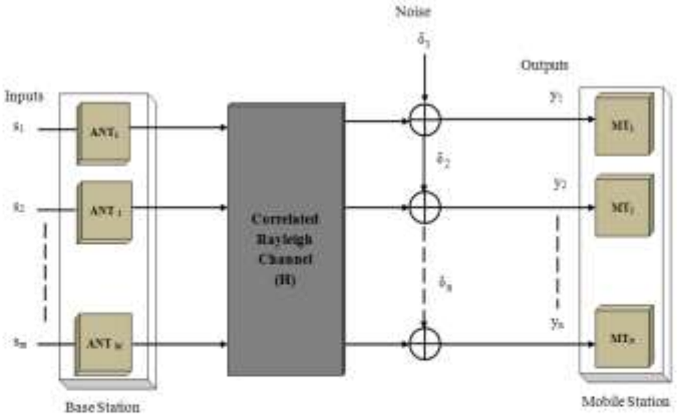


Fig. 2 Block diagram of proposed system model with channel matrix and noise

3.1 CHANNEL CAPACITY AND ERGODIC CAPACITY

Assuming the MIMO system with multiple M antennas at the BS and N receive antennas such that M > 1 and N >1 such that in a transmission time interval (TTI), only a single active MT is scheduled to receive a transmitted signal vector from mth transmit antenna. Therefore Eq. (1) can be rewritten as:

$$y_n = \sqrt{\rho} H_{m,n} s_m + \delta_n$$

(4)

where y_n is the received signal vector of nth receive antenna for Nth mobile terminal, s_m is the transmitted signal vector, ρ is a scalar that denotes the normalized transmitted (which is summing the total power of the transmitted signal to unity, $E\{\|s_m\|\}^2 = 1$), $H_{m,n}$ is the channel with deterministic and constant vector matrix, and δ_n is the nth receive signal channel noise. The capacity of MIMO system with M transmit antennas and N receive antennas in bit per second per hertz (bps/Hz) is given by [12]:

$$C = \log_2\left[\det\left(I_M + \frac{1}{\sigma_n^2} H R_T H^*\right)\right]$$

(5)

where I_M is an identity matrix, H is the M×N transmission matrix of the channel, H^* is the transpose-conjugate of H, σ_n^2 is the noise power, R_T is the transmitted covariance that shows the power allocated to the transmit signal and it is determined by:

$$R_T = \frac{P_T}{M} \times I_M$$

(6)

where P_T is the transmit signal power. Substituting Eq. (6) into Eq. (5), the channel capacity can be expressed as given by:

$$C = \log_2\left[\det\left(I_M + \frac{P_T}{M\sigma_n^2} \times I_M \times HH^*\right)\right]$$

(7)

Assuming the number of transmit antennas is very much larger than the number of receive antennas (that is M >> N) for the case of massive MIMO, then $HH^* = M \times I_M$ and substituting this expression into Eq. (7) gives the channel capacity as:

$$C = \log\left[\det\left(I_M + \frac{P_T}{\sigma_n^2} I_M\right)\right]$$

(8)

Now, from Omer et al. (2020):

$$\left(I_M + \frac{P_T}{\sigma_n^2} I_M\right) = \left(1 + \frac{P_T}{\sigma_n^2}\right)^N$$

(9)

Substituting Eq. (9) into Eq. (10), the MIMO capacity can be expressed as:

$$C = N\log_2\left(1 + \frac{P_T}{\sigma_n^2}\right)$$

(10)

Eq. (10) can be written for M >> N as given by [7]:

$$C = \min(M, N)\log_2\left(1 + \frac{P_T}{\sigma_n^2}\right)$$

(11)

Equation (11) shows that the capacity of MIMO channel increases directly as the number of antennas at both transmit and receive sides, but no growing transmit power and channel bandwidth [7].

Now, the channel ergodic capacity is considered, and is the statistical average of the mutual information taking place. Since, MIMO channels are random and as such the channel capacity is also random. Taking the total average of the information rate over dispersion of the channel matrix (H) elements, the ergodic capacity of MIMO channel can be defined by [13], [7]:

$$C = E\left\{\sum_{i=1}^{m} \log_2\left(1 + \frac{P_T}{M\sigma_n^2}\lambda_i\right)\right\}$$

(12)

where λ_i are the eigenvalues of HH^*. Equation (12) is the channel ergodic capacity when the total power is allocated equally to all transmit antennas. In this case the channel state information (CSI) is not available to the transmit side. Since the channel capacity is not improved at all by the availability of CSI [14], it has been assumed in this study that all transmit antennas are allocated with equal total transmit power.

3.2 Simulation Parameters

The parameters used for the simulations conducted in this paper are defined in this section. These parameters are for micro strip antennas designed and implemented for MIMO system. Number of BS antennas (M) is $8 \le M \le 128$, Number of mobile station antennas (or receive antennas) (N) is ≤ 20, signal to noise ratio (SNR) is 30, distance between antennas (spacing) is 0.5λ, modulation scheme PSK, azimuth spread (at transmitter) is 10 degrees, angle of arrival (at transmitter and receiver) is 20 degrees, frequency 1×10^9 Hz, and wavelength is 3×10^8/frequency.

4. SIMULATION RESULTS

This section presents the various simulation results for the evaluation of performance of channel capacity in wireless network data transmission for multimedia application in terms of ergodic capacity and correlation of channel. Basically, there are three parameters of concern namely, the signal-to-noise ratio (SNR), and number of antennas at base station (transmitter) and the receiver antennas. Performance results are presented for channel capacity in terms of ergodic capacity considering cumulative distribution function (CDF), and ergodic capacity against SNR.

4.1 Ergodic Capacity Considering CDF

In this scenario, simulations have carried out considering the channel coefficients to have a Gaussian distribution of zero mean and variance of 0.5. Thus, with the channel coefficient random, the capacity (rate of data transfer) is also considered a random variable with a certain distribution. Hence, for such channel, cumulative distribution function (CDF) is an important performance metric to quantify its capacity and basically offers the probability that the capacity of the multiple antenna system is above certain threshold [15]. The simulation results for CDF of ergodic capacities for multiple antenna system with varying base station (BS) antennas and mobile station (MS) antennas are presented in this section as shown in Fig. 3. Also, Fig. 4 is CDFs of various ergodic capacities with the number of antennas at BS (M) and MS (N) constant.

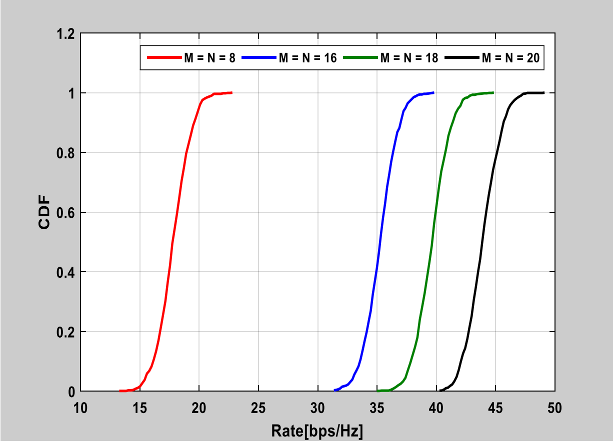


Fig. 3a CDF of capacity (SNR =10)

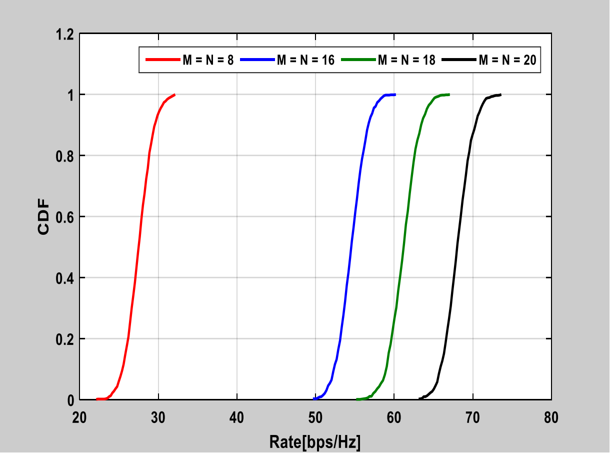


Fig. 3b CDF of capacity (SNR =15)

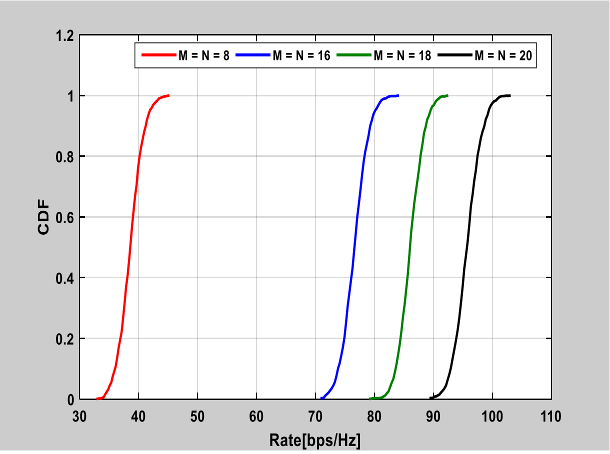
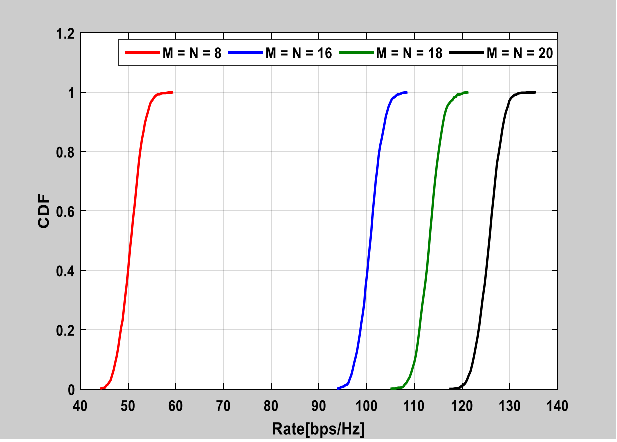


Fig. 3c CDF of Capacity (SNR =20)



Fig, 3d CDFof Capacity (SNR =25)

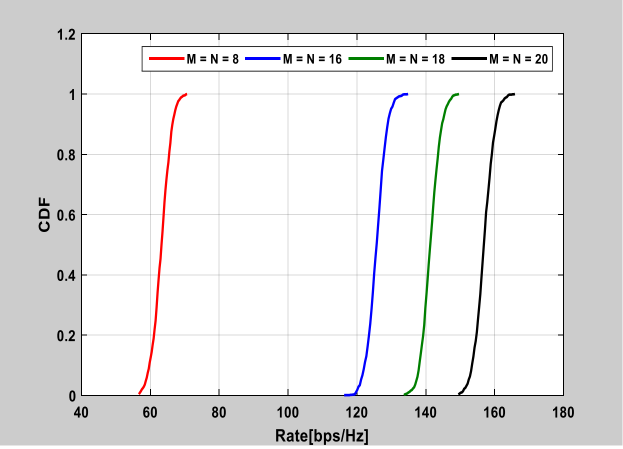


Fig. 3e CDF of Capacity (SNR = 30)

The system performance is evaluated by varying the SNR metric of the system from 10 to 30 dB. The analysis is conducted for perfect channel state information (CSI). The numerical evaluations of the simulation curves in Fig. 3 are shown in Table 1.

Table 1 Ergodic capacity CDF performance

M / N	Capacity (bps/Hz)				
	SNR 10 dB	SNR 15 dB	SNR 20 dB	SNR 25 dB	SNR 30 dB
8	22.8	31.74	45.21	59.46	70.67
16	39.82	60.21	84.14	108.6	134.9
18	44.85	67.07	92.49	121.3	149.6
20	48.94	73.62	103.1	135.5	165.9

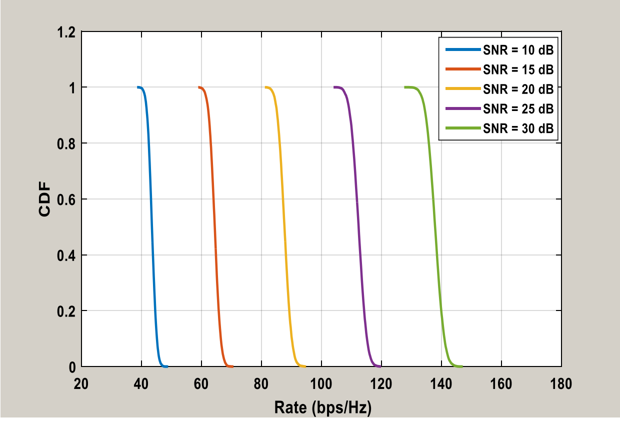


Fig. 4 CDF of capacity (for M = N = 16)

Figure 3 is the simulation result for a scenario whereby the number of base station antennas and mobile station antennas are equal but are varied with respect to constant SNR. CDF is the probability term to describe the capacity of the system. As shown in Fig. 3a to Fig. 3e, and looking at Table 1, it is obvious that the probability that the capacity obtained for the channels of multiple antenna system is significantly improved as the number of BS antennas and MS antennas increases. For example, at SNR of 30 dB, the conventional multiple antenna system (8 × 8 MIMO system) provided channel capacity (or rate of data transfer) of 70.67 bps/Hz for multimedia wireless transmission with a 100% probability (i.e. 1 CDF), whereas with the number of BS antennas and MS antennas increased to form massive multiple antenna system (for example 16 × 16 massive MIMO), the channel capacity increased to 134.9 bps/Hz for wireless transmission of data with a 100% probability. Generally, the simulation results show that there is 100% probability that capacity of the system increases as the number of multiple antenna elements increases.

In Fig. 4, it was observed that when the number of antennas (M) and (N) at the BS and MS are constant such that the performance metric is evaluated by varying the SNR from 10 dB to 30 dB, the capacities are reduced at stricter threshold. For instance, at SNR of 10 dB there is 90% probability that the capacity is greater than 41.92 bps/Hz but at stricter threshold (or probability) of 99%, the capacity is reduced to 38.58 bps/Hz. This holds for all cases where at SNR of 15, 20, 25, 30 dB, there are 90% probability the capacities are greater than 62.4 bps/Hz, 85.33 bps/Hz, 109.5 bps/Hz, and 134.7 bps/Hz, but at stricter probability of 99%, the capacities are reduced to 58.95 bps/Hz, 81. 18 bps/Hz, 104.1 bps/Hz, and 127.6 bps/Hz respectively. Generally, the simulation curves as shown in Fig. 4 revealed that the capacities increase as the CDF (probability or threshold) becomes less strict. Therefore, it can be said that the probability that increasing SNR (or signal power) without corresponding increase in multiple antenna elements will provide improved channel capacities is low.

4.2 Ergodic Capacity

The simulation results in this section show the ergodic capacities for various antenna elements with MIMO channel. The SNR was 30 dB. Simulation was initially conducted considering equal number of antennas at the transmitter and the receiver sides as shown in Fig. 5. Next simulation was performed for varying number of transmitter antennas and receiver antennas Fig. 6. Furthermore, simulation results for varying number of transmitter antennas and fixed number of receiver antennas is presented in Fig. 7, while Fig. 8 showed the simulation plots for constant number of transmitter antennas and varying number of receiver antennas. The numerical performance analysis of the various plots is presented in Table 2.

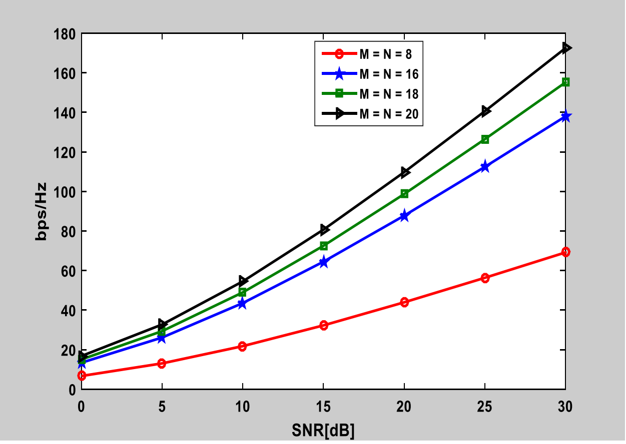


Fig. 5 Ergodic capacity for the equal transmitter and receiver antennas

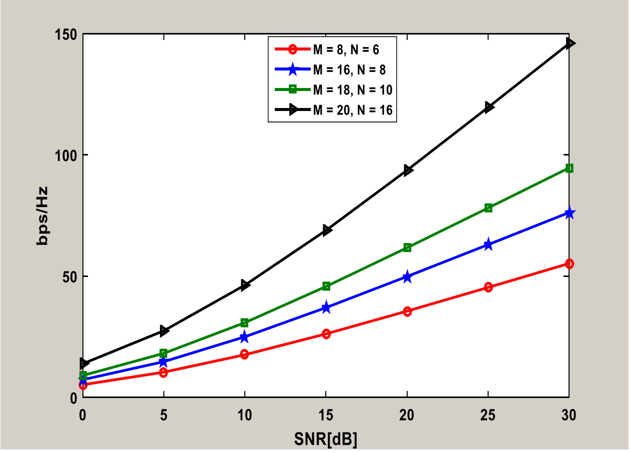


Fig. 6 Ergodic capacity for varying transmitter and receiver antennas

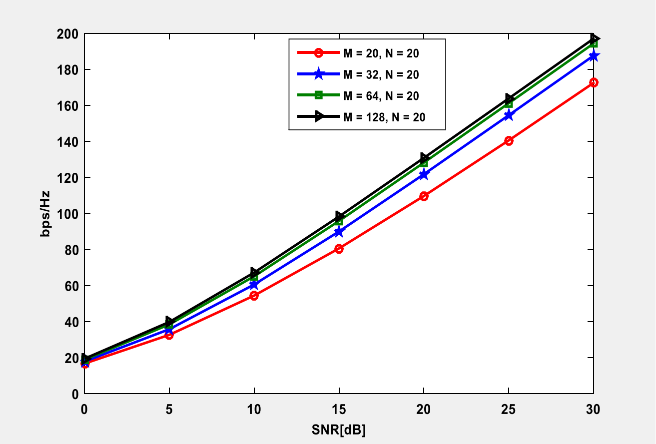


Fig. 7 Ergodic capacity for varying transmitter antennas and fixed receiver antennas

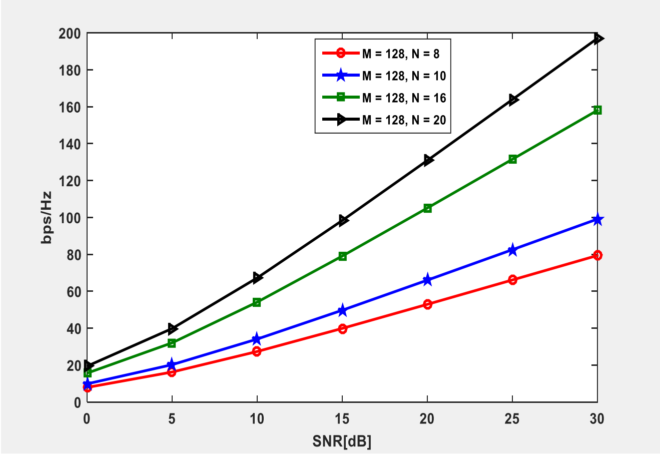


Fig. 8 Ergodic capacity for constant transmitter antenna and varying receiver antennas

Table 2 Ergodic capacity for MIMO channel

Number of BS antennas (M)	Number of MS antennas (N)	Fig. 5: Ergodic capacity (bps/Hz)
8	8	69.31
16	16	138.1
18	18	155.1
20	20	172.4
Number of BS antennas (M)	Number of MS antennas (N)	Fig. 6: Ergodic capacity (bps/Hz)
8	6	55.23
16	16	76.23
18	18	94.64
20	10	145.9
Number of BS antennas (M)	Number of MS antennas (N)	Fig. 7: Ergodic capacity (bps/Hz)
20	20	172.4
32	20	187.5
64	20	194.3
128	20	196.9
Number of BS antennas (M)	Number of MS antennas (N)	Fig. 8: Ergodic capacity (bps/Hz)
128	8	79.37
128	10	99.09
128	16	158
128	20	197

From Table 2, it can be seen that the ergodic channel capacity of wireless communication system can be increased by either increasing the number of transmitter antennas or increasing the number of receiver antennas or even both. Thus for number of transmitter (M) equal to 128 the capacity of the system was maximized, which conforms to the fact that a near optimal capacity can be achieved in MIMO channel system with large scale transmitter (or base station) antenna (Singh and Kedia, 2019). From Fig. 5 to 8, it can be deduced looking Table 2 that the capacity of large scale multiple antenna system (or massive MIMO) is higher than conventional MIMO. Besides, it is obvious that the number of antenna elements significantly contributes in the performance improvement of channel capacity in facilitating rate of data transfer for improved wireless communication in multimedia application including increasing signal power in terms of SNR dB.

5. CONCLUSION

The ergodic capacities for different multiple antenna elements configurations have shown that increasing the number of antenna arrays from conventional MIMO to massive MIMO by means of antenna selection method can increase the rate (speed) at which information are transmitted over wireless channel. Besides, by making the BS antennas very much larger than the MS antennas provides much more capacity. For instance in Table 2 at M = 128 and N = 20, the capacity achieved is 197 bps/Hz. In terms of ergodic capacity cumulative distribution function (CDF), when SNR was fixed at 10 dB, 15 dB, 20 dB, 25 dB, and 30 dB while varying the number of antennas at base station (BS), M and mobile station (MS), N, the resulted ergodic capacity increases as the number of antennas increases with CDF (or probability) of 100%. Hence, the probability that the capacity obtained for the channels of multiple antenna system is significantly improved as the number of BS antennas and MS antennas increases.

REFERENCES

[1] Khwandah, S. A., Cosmas, J. P., Lazaridis, P. I., Zaharis, Z., & Chochilouros, I. P. (2021). Massive MIMO systems for 5G communications. Wireless Personal Communications, vol. 120, pp. 2101 – 2115.

[2] Lin, Z., Du, X., Chen, H., Ai, B., Chen, Z., & Wu, D. (2019). Millimeter-wave propagation modeling and measurements for 5g mobile networks. IEEE Wireless Communications, vol. 26, No. 1, pp. 72–77.

[3] Samsung (2016). Mismatched FD-MIMO configuration fields between RAN1 and RRC," 3GPP TDocs TSG RAN WG1, St Julian's, Malta, Tech. Rep. R1–160520

[4] Thakur, S. & Naithani, A. (2016). A complete mathematical modeling and simulation of ergodic channel capacity of wireless random MIMO system. International Journal of Future Generation Communication and Networking, vol. 9, No. 11, pp. 287-298.

[5] Mohamed, E. & Abdulsattar, A. M. (2015). Evaluation of MIMO system capacity over Rayleigh fading channel. International Journal of Computer Science & Engineering Survey, vol. 6, No. 3, pp. 47-58

[6] Belaoura, W., Ghanem, K., & Bousbia-Salah, H. (2017). Effect of spatial correlation on the ergodic capacity for downlink massive MU-MIMO systems. Proceedings of the 6th International Conference on Systems and Control, University of Batna 2, Batna, Algeria, pp. 183 – 186.

[7] Omer, D. S., Hussein, M. A., and Mina, L. M. (2020). Ergodic capacity for evaluation of mobile system performance. Journal of Engineering, vol. 26, No.10, pp. 135 – 148.

[8] Jiancun, F. (2017). Performance analysis and optimization for downlink distributed MIMO systems. IEEE Wireless Personal Multimedia Communications (WPMC2017), pp.481-486.

[9] Widad, B., Khalida, G., & Hisham, B. (2017). Effect of Spatial Correlation on the Ergodic Capacity for Downlink Massive MU-MIMO systems, No. 2, pp. 183–186.

[10] Shiney, J. R. J., Indumathi, G., & Asis, A. A. C. (2020). Efficient antenna selection strategy for a massive MIMO downlink system. Progress in Electromagnetics Research M, vol. 26, pp. 203 – 211.

[11] Fakunle, M. M., Adedeji, K. B., & Olasoji, Y. O. (2021). Improving BER performance of massive MIMO scheme over Rayleigh fading channel. European Journal of Electrical Engineering and Computer Science, vol. 5, No. 6, pp. 58 – 63.

[12] Dalveer, K., Neejar, K. (2019). Enhance the capacity of MIMO wireless communication channel using SVD and optimal power allocation algorithm. Electronic and Telecommunication, vol. 65, No. 1, pp. 71–78.

[13] Yazen, S. and Ghassan, A. (2019). Capacity analysis of multiple-input-multiple-output system over Rayleigh and Rician fading channel. Cihan University-Erbil Science Journal, vol. 3, No. 2, pp. 70–74.

[14] Pottkotter, A. (2015). Maximizing Channel Capacity based on Antenna and MIMO Channel Characteristics and its Application to Multimedia Data Transmission. Master Thesis, University of Toledo.

[15] RAYMAP (2011). MIMO capacity in a fading environment. [Online]. https://www.raymaps.com/index.php/mimo-capacity-fading-environment/