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Evaluation of Teaching the Concept of Absolute Value in the Context of Middle School Teachers' Perspectives

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***Abstract**: Its importance in mathematics, especially in the teaching of limits in calculus courses and its potential to cause difficulties for students from middle school to higher education, turn the absolute value concept into a very important in mathematics education. The perspectives of teachers on a mathematical concept lead their classroom practices. Therefore, defining those on a mathematical concept can provide important clues in understanding the nature of teaching and learning of this concept. The aim of this study was to investigate the teaching of the absolute value concept through middle school teachers' perspectives. As a data collection tool, a semi-structured interview was conducted. The research group consisted of 15 middle school mathematics teachers working in a city located in the north of Turkey. The participants' responses were qualitatively analyzed to characterize patterns and categorize answers. The results of the study indicated that the teachers use wrong statements and examples that may cause some misconceptions while defining the absolute value concept. Although these do not cause much trouble in the arithmetic domain, it is thought that they will cause important problems with the transition to the algebraic domain in the following years.*

Keywords— absolute value concept; teacher perspectives; mathematics education; teaching and learning

1. INTRODUCTION

The absolute value concept is an important concept in mathematics and mathematics education. It is difficult to find situations where it is used directly in solving a problem, but since it is included in many mathematical concepts, its a good understanding is needed in terms of avoiding learning difficulties. Indeed, studies on absolute value reveal that the concept can be a problem for student at all levels from middle school to high school, even university (Aziz et al., 2019; Baştürk, 2000; Elia et al., 2016; Marcantonio, 2007).

In the literature, it is possible to come across many studies that reflect this situation of the absolute value. For instance, Allendoerfer (1963) puts forward some difficulties of students with the definition of the limit of functions, and mentions that these are difficulties, which usually arise from not being able to fully utilize absolute value, such as the inability of students to fully understand inequality and absolute value representations in the epsilon-delta technique (as cited in Yassin, 1991). In some mathematical proofs, absolute value function is an important tool and has a wide use in advanced mathematics. Bratina (1984) focused on the acquisition of the preliminary skills necessary to understand the concept of limit in the majority of her/his study on students' understanding the limit of the series and found that the participating students' weakest concepts were those related to the absolute value concept. Absolute value in itself poses a challenge for students. When a simple equation that can be easily solved by students is turned into that with absolute value, it may become a nightmare for the same those (Baştürk, 2004; Duroux, 1983a). In Baştürk's study (2004), the general success rate of the question consisting of a first order equation with one unknown variable consisting absolute value goes down to 27% (Solve in |R, $|x+5/3|=14$) while the success rate is about 97% in the question consisting of a simple equation without absolute value (If $2a+5=17$, what is a=?). Based on this, Duroux (1983) states that absolute value can be used by teachers to make an exercise more difficult.

As stated before, the use of the absolute value is very important in mathematics, especially in understanding the concept of limit in calculus and pre-calculus lessons. It is used to express the distance between two real numbers and define the concept of limit, of the continuity for the real functions and for the multivariate functions. We also use it to define the spaces such as the linear space for the mathematical analysis and for the metrics of the vectors at the analytic geometry (Almog & Ilany, 2012; Gagatsis & Panaoura, 2014). However, the absolute value perceptions of many students learning it, are quite simple and limited to the arithmetic and numerical domain understandings, and they have problems in using the absolute value concept at the algebraic domain. Indeed, some students at tertiary level cannot interpret the expression of $|x|$ by considering its different possible cases, and they limit themselves to the absolute value definition first taught in middle and high school mathematics lessons (Marcantonio, 2007).

1.1 Importance of Teacher Perspectives on Mathematics Teaching

Scholarly knowledge goes through many changes and transformations until it becomes knowledge that is learnt. Chevallard (1985) calls all these changes and transformations didactic transformation. Scholarly knowledge produced by scientists is not taught in schools as they are, and not every scholarly knowledge is included in the knowledge to be taught. Social needs, the social environment of education and many variables play a role in the selection of knowledge to be taught. Chevallard expresses these with the concept of the noosphere. The knowledge chosen among scholarly knowledge by the curriculum development experts working in the Ministry of National Education becomes the knowledge to be taught as a result of the changes and transformations made by the textbook authors, then the taught knowledge by the changes and transformations made by the teachers, and finally the learnt knowledge by changes and transformations carried out by the student after the didactic relationship between the student and the teacher. Therefore, teachers are one of the important elements of the didactic transformation process and, accordingly, the teaching and learning process in the context of containing the taught knowledge in this process and being a source for the teaching done in the classroom.

There are important resources in mathematics education research to obtain information about the nature of the teaching of a concept. Curriculum, textbooks, teacher's representations and practices, classroom observations, assessment, and evaluation practices, etc. are some of them. Undoubtedly, classroom practices from one teacher to another show some differences, even though the content is the same and the classes are similar. As revealed by some studies on teaching practices Roditi (2005), there are dramatic differences between teacher practices as well as many general and common points in the subjects taught in the classroom and methods or techniques chosen to teach them.

Teaching is a concept that includes all activities that a teacher will do to increase the learning of his/her students. Mathematics teaching includes all mathematical activities such as preparing lesson plans, selecting appropriate materials for the subject, showing students how to solve problems, answering students' questions, evaluating, and scoring students' work (Ball et al., 2008). On the teacher's side, teacher classroom practices underlie almost all his interventions. The process of these practices is under the responsibility of the teacher and appears in various forms on different occasions in the class life. For instance, during the assessment phases, in what the teacher solicits, resumes or neglects student interventions, in the comments s/he makes and explanations s/he gives, what s/he asks to note in the notebook; in the reminders s/he makes himself or that s/he solicits students; in the choice of activities s/he offers, in the selection of questions in exams (Glorian-Perrin, 1996).

Teachers' beliefs play an important role in planning and realizing a lesson. Beliefs that they have affect what they will teach, how they will teach and, accordingly, what will be learned in the classroom (Andrews & Hatch, 2000). Beliefs are elements that shape how teachers think and feel about mathematics and its teaching and learning. Therefore, knowing and recognizing beliefs is one of the best places to start if changes in teachers' practices are desired. Because it is thought that changes in beliefs will bring about changes in practices (Lerman, 2002). As underlined Lerman (2002) the relationship between beliefs and practices is a very strong and two-way. That is, changing practices depends on changing beliefs, and changing beliefs depends on changing practices.

Our perspectives on a concept are the way we think about that concept, which is particularly influenced by our beliefs and experiences. Teachers' perspectives have a direct or indirect effect on their students' learning in class or out-of-class. Social psychologists (Jodelet, 1997; Moscovici, 1986) call perspectives (representations) to each of the images, thoughts, and beliefs that we associate with a concept. When we say perspectives here, we understand every belief, thought and image of each of the teachers for the absolute value concept and its teaching. The reason we attach importance to these perspectives is that we have adopted, with many researchers, the hypothesis that teachers' perspectives condition their practices in the classroom. These perspectives are metacognitive since they introduce the elements on a concept (here for us the absolute value concept), its teaching and learning. Perspectives are much more stable, even very stable in adults (Robert et al., 1999). Since our perspectives condition our behavior (and vice versa) to a large extent, we believe that the identification of those of teachers and it may contribute to get some important clues about the nature of absolute value concept teaching and learning.

1.2 Place of Absolute Value Concept in Curriculum

In this part, where we investigated the place and function of the concept of absolute value in Turkish curricula, if we express it in Chevallard's terminology (1992), we will briefly make an ecological analysis (Chevallard, 1992, 2002). In Turkey, students encounter the absolute value concept at only 6th grade (students 11-12 aged) throughout middle school education. In curriculum, there is only one gain and it is located in whole number sub-learning area of numbers and operations learning area. In this gain, student is expected to determine and make sense of the absolute value of an integer. Furthermore, in the statement accompanying the gain, teacher is expected to emphasize the meaning of the absolute value on the number line and in real life by using the examples such as elevator, thermometer, etc.

The student's encounter with absolute value again takes place in 9th grade (aged 15-16) of high school. In the curriculum of this grade, almost all algebraic properties of absolute value are taught, and it is desired to find the solution sets of first-degree equations and inequations with absolute value. The use of equations and inequations with more than two absolute values are not desired.

1.3 Purpose and Significance of the Study

The aim of this study was to investigate the teaching of absolute value concept through middle school teachers' perspectives. As they transform the knowledge in the curriculum and textbooks into the knowledge taught in the course by integrating their personal knowledge and experience, the teachers are an important factor in the didactical relationship. Although absolute value has a limited place in the middle school curriculum, we attach great importance that for the first time both students meet with it and teachers teach it. We think that the answers to be given of the questions such as, how do teachers introduce this concept? what definitions and examples do they use? how do they interpret the place of absolute value in curriculum and textbooks? and etc. are significant for the improvement and development of the teaching of this concept that is small (considering its definition of a few lines) but poses big problems students at all level.

2. METHOD

The research design used for the study was a descriptive survey method. Thus, the teachers' representations of the teaching of absolute value were identified without influencing them in any way. Descriptive research, which is widely used in many fields such as behavioural sciences, social sciences, education, nutrition, etc., is a study of status. The fact that it refers to the acceptance that problems can be solved, and practices developed through observation, analysis and description makes it valuable.

The most well-known descriptive research method is survey. Personal interviews, questionnaire consisting of open-ended or closed-ended questions, phone surveys and normative questionnaires are among the most common of data collection tool of such research. These studies, which aim to describe the existing as it is, produce qualitative or quantitative data describing the state of nature in a certain period of time (Koh & Owen, 2000). Unlike in the experimental research, in the survey research designs, the fact that conditions are manipulated experimentally by the researchers is out of the question and they are not expected to explain the cause and effect like experimental researchers (Creswell, 2012).

2.1 Research Group

The research group consisted of middle school mathematics teachers working in a small-sized city (with a population of about 93.753 people) located in the north of Turkey. 15 teachers participated in the research as volunteers.

Most teachers have more than 5 years' experience. Their experience mean was 13 years. The most experienced one has been teaching for 21 years and the most inexperienced one for 1 year. All teachers, except for one teacher who taught only in the 8th grade, taught at each class level of middle school. All this shows that the teachers selected for the research group were suitable for the research subject in terms of both the experience and the grade levels they taught.

2.2 Data Collection Tool

In this study, a semi-structured interview form that aims to investigate the teaching of absolute value according to the middle school mathematics teachers developed by the researcher was conducted as data collection tool. In the literature, it is seen that interview is a frequently used research method in qualitative research. Interview technique, which is based on the act of speaking as the most common form of communication, is a powerful research method in terms of revealing individuals' data, views, experiences and emotions (Yıldırım & Şimşek, 2018). In this respect, it can eliminate or complement the deficiencies that may arise in writing-based data collection tools. The interview is basically about asking questions and getting answers (Punch, 2005). The interview form consisting of 10 open-ended questions was examined by 2 experts with a doctorate in mathematics education, and then revised in the light of their feedbacks.

In the questions of the semi-structured interview form, the teachers were asked to indicate the importance of absolute value according to them, what kind of activities they start with absolute value teaching, how they define it, what they do to concretize it, examples from daily life that they give, situations where students have the most difficulty in learning and mistakes they make the most, what they do to eliminate them, their thoughts on the place of absolute value in the curriculum and textbooks, and examples of three questions about absolute value that they ask in the written examination.

2.3 Data Collection and Analysis

The interviews and were recorded with a voice recorder in accordance with the permission of the participant teachers. Each interview was conducted in a quiet classroom with the teachers and lasted approximately 15-20 minutes. The teachers were selected among the teachers who wanted to participate in the research during the researcher's visits to various middle schools. It was emphasized that the answers they will give to the questions of the interview are very important to provide a better understanding of teaching and learning the absolute value concept. In addition, they were assured that their answers to the questionnaire would be used entirely within the framework of the research, and they would not be shared with any third parties.

Teachers' answers were analysed using content analysis, one of the qualitative research methods. To characterize patterns and categorize answers, the teachers' responses to interview questions were qualitatively analysed by using open coding that is one of the processes to analyse textual contents. Data translation, data labelling and grouping, and data comparison are three steps of this analyse procedure (Khandkar, 2009). Common categories and subcategories were determined by continuously comparing the answers given with each other (Creswell, 1998). As a result of these processes, 11 main categories (almost every question was taken as a main category) and a total of 61 sub-categories depending on these main categories were determined. In order to increase the reliability of the category and coding processes, the categories determined, and the coding made were examined together with two experts working in the Department of Secondary Education Mathematics Education at the university where the research was conducted. Conflicts were resolved by discussion, and in this way, a high rate of common agreement on coding and categories was achieved (Lincoln & Guba, 1985).

3. RESULTS

In this party, the findings obtained from the analysis of the answers of the teachers to the questions included in the interview were presented.

3.1 Importance level of absolute value in the view of teachers

It is clear that determining how important a concept is for teachers will give an idea of how much effort teachers will make to teach that concept. In order to determine the importance level of absolute value according to the participant teachers, they were asked to indicate the importance level of absolute value with a number between 1 and 10 (1 for not important at all, 10 for absolutely essential). The analysis of responses shows that 8 teachers stated that the absolute value is a very important topic by choosing a number between 8 and 10. For 5 teachers the absolute value is a matter of slightly above average. 2 teachers did not answer to the question. The average of their responses was calculated as 8. The following comments illustrate very well our analysis on the teachers' responses:

> *The absolute value is the distance from the place on the number line of a number to the starting point of zero. For example, it is not said that the distance of the number of -7 to the starting point of zero is -7. Because, the distance cannot be expressed with minus. Therefore, the concept of absolute value is very important (Teacher 6).*

> *The importance level of absolute value is 8 for me. There is no relationship between the absolute value and the learning objectives of middle school. Using the absolute value in four operations on integers will not give students much benefit. As the absolute value concept takes up a lot of places in the high school curriculum, it would be more appropriate to explain it in high school (T10).*

> *I say 8. As the absolute value is in relation to a lot of subjects and students have problems in learning it, I consider it to be important (T7).*

> *I want to answer this question by saying 7. The absolute value concept is important for students to learn better integers, distance, positive and negative concepts. In my opinion, absolute value is a moderately important concept and deserves a bit above-average value, not 9 or 10 (T15).*

In the first comment, the teacher states that absolute value is a very important concept because of its length meaning. S/he also explains that the absolute value cannot be negative, since the length is not negative. In the next comment, the teacher thinks that it is not appropriate to include the concept of absolute value in middle school curriculum, and the high school level with many gains in absolute value is more appropriate for the teaching of this concept. In the third comment, the teacher finds the absolute value concept as important and explains this importance with its relations to many subjects and student difficulties in learning it. The last comment states that the absolute value is important, but not much. It provides to better understand some concepts such as integers, distance, positive and negative concepts. Consequently, as it has relations with many concepts and causes difficulties in students, the participant teachers consider absolute value as an important concept.

3.2 Activities Used to Start Teaching the Absolute Value

One of the most important points to know about the teaching of a concept is, of course, how teaching first began. In the context of shaping the first impressions of the student and motivating him/her to learn the subject, the entry activities to be selected are very important. When starting to teach the absolute value, the teachers give some examples from daily life such as thermometer, length, fish and plane, elevator, number line, washing machine, dishwasher, and profit-loss samples etc. (7 teachers). Some teachers prefer to base their introduction activities on the use of number line (6 teachers). Using materials (1 teacher), lecturing (1 teacher), and asking students the definition of absolute value are other entry activities used by the teachers. The following extracts are typical of such comments:

> *When defining the absolute value, I use through the dirty and clean laundry example. In other words, I say to students that more or less dirty laundries go out from the washing machine as clean. If I do not give examples of daily life in this way, the student does not understand (T2).*

> *When teaching the absolute value, I explain the concept of distance. Talking about the distance between objects, I try to provide students perceiving the absence of the concept of direction. Philosophically I underline negative, positive thinking or not thinking anything. I talk about contributions of positive thinking to person's life (T10).*

> *First, I try to make understand to students the relationship between the concept of distance and the absolute value by using the example of sea level from daily life. Then, I start to teach with showing the numbers on the number line (T15).*

> *When I started teaching the absolute value, I first asked its definition to the students. Of*

> *course, interesting answers came from many of them. I always start teaching by asking questions even though I know they cannot answer (T3).*

The first comment expresses the washing machine example from daily life in order to define the absolute value concept. The fact that dirty laundries leave the washing machine clean is associated with the function of absolute value. In her/his opinion, students' understanding depends on this type of examples. The second comment highlights the relationship between the distance concept and the absolute value. An example of daily life associated with the absolute value is stated by the teacher such as positive thought, negative thought or neutral thought. Like its predecessors, the third comment also refers to the distance meaning of absolute value concept by basing on a daily life example such as the sea level. The number line is another teaching instrument used by the teacher. In the last comment, the teacher indicates that whatever the concept, s/he starts the lesson by asking students its definition. Her/his aim is to make students think rather than to get the right answer. As a result, while starting their teaching, the participant teachers try to give examples that appear to refer to the function of absolute value from daily life such as washing machine, negative and positive thought, and sea level etc.

3.3 Definition of Absolute Value of the Teachers

There are different definitions of absolute value. Of these, the algebraic definition, and the definition in terms of distance, are among the most commonly used. By considering which definition teachers prefer, we can predict what effects will have on students' conceptual understanding. Most teachers define the absolute value concept by basing on its definition in terms of distance (14). Some give the classic definition (i.e., algebraic definition) in the textbook (2 teachers), while others define it by giving examples from daily life (2 teachers). Only one teacher indicates that s/he defines the concept by playing educational games with students. We think that the next comments allow the reader to understand better, how the teachers define the absolute value concept:

> *When defining the absolute value, I put myself at a point in the classroom and I get one student on my right side and the other on my left side. I ask them standing the one on 2 and the other on -2. Then, I ask them their distance from me. In this way, they understand that their distance from me is 2 (T2).*

> *For example, I take the numbers -5 and 5. I say that the first is a debt and the second is a credit. If we remove the signs, the rest is 5 as debt or credit. Thus, I underline the number 5. Finally, I give the definition of absolute value in terms of distance (T1).*

> *I define the absolute value to be an operation that always transforms negative or positive numbers written in absolute value bars into positive numbers (T4).*

> *(…) After the explanations above, I pass the classic definition. I define the absolute value of a number as being the distance, on the number line, of that number from zero (T10).*

In the first comment, it is seen that the absolute value is defined with a drama activity realized by the teacher and two students. S/he tries to make students feel the definition in terms of distance. In a way, the number line is animated or modelled in the classroom. The next comment explains the absolute value with a credit and debt example. -5 means debt and 5 means credit. In this example, the goal of the teacher is to achieve number 5, regardless of whether it is a debt or a credit. In the third comment, the teacher considers the absolute value concept as an operation that transforms all numbers in absolute value bars into positive. In the last comment, the absolute value concept is defined by basing on its number-line-based definition that explains absolute value as a term used in mathematics to indicate the distance of a point or number from the origin (zero point) of a number line. As a result, teachers often define the absolute value concept by referring to the number-line-based definition or examples that evoke this definition. In addition, it is seen that some teachers tend to define the concept as an operation transforming negative numbers into positive.

3.4 Concretization of the Teaching of Absolute Value

The concept of absolute value, which is abstract in nature, is related to many subjects in mathematics. Abstract concepts are difficult to understand by students. Therefore, it is important to reveal how the concept of absolute value is embodied by the teachers. Although there are many ways, one of the most well-known ways of concretization is to give individuals examples from their own environment and daily life. In this section, we presented the results of the analysis of the question that asked the teachers to talk about how to concretize the teaching of absolute value concept. The teachers mostly profit from the number line to embody their absolute value teaching (7 teachers) and some of them use examples from daily life modelling the number line such as sea level, elevator, symmetry mirror, and debt-credit etc. (6 teachers). While 2 teachers use materials for concretization, 2 teachers do not anything. Only one teacher states that s/he uses examples highlighting the transformation property of the absolute value, such as the washing machine. This was supported by the interview comments, as illustrated below:

> *To concretize the absolute value concept, I use examples such as debt-credit, elevator, and direction concept (T1).*

> *I ask students to interpret the distance to sea level of an airplane or that of a submarine (T6).*
>
> *In teaching of the absolute value, I focus more on the concept of distance. I associate the washing machine with the absolute value. If we put a clean laundry in the washing machine, it will be clean. In the same way, if we put a dirty laundry, it will be clean (T10).*
>
> *I construct a number line with the kids, and I ask the one going to 6 and the other one going to -6, and I pass to the starting point of zero. I show that my distance to both is equal (T12).*

The first comment expresses three examples from daily life in order to concretize the absolute value concept. There are debt-credit, elevator, and direction concept. In the second comment, there is an example based on the number line. The teacher uses the sea level example in the case of an airplane and a submarine. In the next comment, the teacher highlights the number-line-based definition. At the same time, s/he also uses examples referring to transformative side of absolute value. Leaving clean or dirty laundry, as clean, out of the washing machine is associated by the teacher with the function of absolute value on numbers. The last comment reveals a number line-based drama activity exhibited by the teacher in the classroom with the students. The teacher stands at the starting point zero and places students at some different symmetric points such as 6 and -6. Consequently, the teachers often use examples of daily life based on the number line or the number line to concretize the absolute value. In addition, there are examples that show the transformative side of absolute value.

3.5 Examples from Daily Life

When the teachers were asked whether they gave examples in daily life when teaching the absolute value, only one of them responded negatively to this question. The analysis of responses reveals the distribution of the answers of the teachers benefiting from daily life. Sea-level example and washing machine or dishwasher example are popular and used by the same number of teacher (4 teachers for both). The thermometer example and examples referring to the number line are also a popular example and expressed by three teachers. In order to exemplify the absolute value concept, some teachers give elevator example (2 teachers), and examples revealing non-negativity of length (2 teachers). On the other hand, even if their number is limited, there are also some very interesting examples expressed by the teachers such as prisoner and prison, goodness and evil, and profit and loss (1 teacher for each). This is supported by the interview comments, as illustrated below:

> *Yeah. When we put dirty plates in the dishwasher, they go out clean. When we put clean plates, they also go out clean. Here the dirty represents negative and the clean represents positive (T9).*
>
> *From daily life, I give the examples that I mentioned above. On the other hand, I focus opposite situations such as good and evil. Moreover, I also use the thermometer and elevator examples (T10).*
>
> *Yes, I determine some lengths by using my steps in the classroom and in this way, I try to emphasize that a length cannot be negative. Then I talk about examples of everyday life that can take a negative value such as temperature and depth (T15).*
>
> *I give students an example of a plane and a fish that are at the same distance to the sea level. I want them to see that the distance between these two objects is equal (T5).*

The first comment expresses dishwasher example in which dirty plates mean negative numbers and clean plates mean positive numbers. Just as all dirty and clean plates come out of dishwasher as clean, all numbers come out of absolute value bars as positive. In the second comment, the teacher indicates that s/he uses thermometer and elevator examples as well as goodness and bad example. The next comment highlights that a length cannot be negative. The teacher measures some distances in the classroom by step at aimed to demonstrate that a length could not be negative. In addition, s/he also uses thermometer and depth samples. In the last comment, the teacher takes sea level example containing a plane and a fish. S/he aims to make students comprehend that the distance between these two symmetrical objects, to the sea level, is equal. As a result, it is seen that the examples from daily life that teachers chose to make students comprehend the absolute value are many and varied. Most of these examples are often intended to illustrate the number-line-based definition of absolute value. On the other hand, some teachers give examples illustrating the transformative side of absolute value such as washing machine, and dishwasher.

3.6 Most Encountered Student Misconceptions and Difficulties

An effective teacher is asked to be aware of mathematical concepts or topics that students consider difficult to learn, and common misconceptions committed by them. This skill takes place inside the most important teaching skills of a teacher. Student misconceptions and difficulties help, teachers who know to take advantage of them, in forming their lesson effectively. When we asked the participant teachers to state misconceptions or difficulties related to the absolute value concept, most frequently observed by them in students. Therefore, students have difficulties in solving exercises about absolute value (7 teachers). They extract negative expression from the absolute value bars as it is (5 teachers), more generalize the property of absolute value concept in numbers, make positive all in the absolute value bars (4 teachers), turn negative number into positive and vice-versa (3 teachers), and think that the unknown algebraic expression should be only positive (1 teacher). There is only one teacher, who responds to this question by saying no difficulty. All this is reflected in the following comments:

> *If students do not pay attention to zero, they can write + in front of zero. Students who have recognized the absolute value can take out negative numbers as positive from the absolute value bars. However, in test questions some students can take out numbers in the bars without looking at their sign, and for example, they can write the absolute value of -5 as -5 (T2).*

> *If a value of x is less than zero, its absolute value is -x. Some students generalize this situation to numbers and so, they can write a minus sign in front of the absolute value of a positive number when determining its absolute value (T11).*

> *As the absolute value is very abstract, when there is no activity, students make mistakes. Sometimes they can think that negative sign turns into positive sign or vice versa. In other words, they always associate the absolute value with the opposite sign (T5).*

> *Students may think that the unknown expression in the absolute value bars should only be positive. They mostly confuse absolute value bars with parentheses (T14).*

In the first comment, the teacher states that students sometimes take it out from two vertical bars by putting a plus sign even to zero. S/he also says that students simply extract number enclosed in absolute value bars without doing anything to it. Of course, this creates problems in negative numbers. The next comment deals with the problem that the minus sign in front of the variable, one of the most common mistakes, is converted by the student into a plus sign, regardless of whether its value is positive or negative. The teacher indicates that some students extend this situation to numbers by writing a minus sign in front of the absolute value of a positive number when determining its absolute value. In the third comment, the teacher highlights that the absolute value concept is associated by students with writing the opposite sign of expression enclosed in bars. In his/her opinion, as the absolute value concept is abstract, its teaching should be based on activities to minimize students' mistakes. The last comment states that students make the mistake of thinking that unknown expression in the absolute value bars should only be positive, and some of them take the absolute value bars as parentheses. As a result, it is possible to say that students have difficulties in solving exercises about absolute value and they tend to make every expression they see within absolute value bars positive. Students convert minus sign of a variable in absolute value bars into plus sign regardless of whether its value is positive or negative.

3.7 Strategies Developed for Overcoming Student Difficulties and Mistakes

Misconceptions, mistakes and student difficulties are among the most important research subjects of mathematics education, and they are the most important student characteristics to be known. Misconceptions, in particular, can be permanent in students' mind and it is often difficult that they are removed by traditional teaching methods. Teachers need to develop appropriate and effective strategies in order to overcome them. Regarding what kind of strategies, the participant teachers develop against student difficulties and misconceptions, solving many exercises and repeating the subject are most frequently preferred strategies by them (10 teachers). Some of the teachers prefer to give examples from everyday life (3 teachers), while 2 teachers draw students' attention to the precedence of absolute value in order of operations. On the other hand, giving concrete examples (2 teachers) and using materials (1 teacher) are considered by the teachers in order to overcome difficulties and mistakes. All this is reflected in the following comments:

> *In general, we solve many questions. We underline the precedence of absolute value in order of operations. We try to tell students that they should first solve the absolute value (T4).*

> *I do repetitions and I solve similar examples. In this way, I completely try to provide that students understand that they should solve the absolute value before other operations (T15).*

> *I re-explain the subject and solve many questions (T10).*

> *I use the metaphor of negative or positive people by including the absolute value. I say that the absolute value of negative people is positive, that of positive is also positive and that of neutral people is zero (T7).*

In the first comment, the teacher expresses two strategies against mistakes and difficulties: The first one is to solve many questions and the second one is to underline the precedence of absolute value in order of operations. The next comment includes three strategies such as repetition, solving similar examples, and precedence of absolute value. The third comment also underlines repetition of subjects and solving many questions. In the last comment, the teacher indicates that s/he uses a negative and positive people metaphor that links the absolute value concept to human psychology. As a result, it can be said that the most common strategy used by the participant teachers against misconceptions and mistakes is to solve many questions and to repeat the subject. In addition, strategies such as warning students against mistakes and giving concrete examples from daily life are also encountered.

3.8 Characteristics of a Student Who Understood the Absolute Value

One of the issues we wanted to learn within the scope of the research was how the teachers judge student behaviours to determine whether they understood the subject. We looked for an answer to the question of what a student can do, if s/he has understood the absolute value concept according to the teachers. It was thought that the answers given would be important in terms of showing the satisfaction levels of teachers regarding absolute value. The analysis indicates that most teachers think that if students realize the distance is not negative, they have understood the absolute value concept (11 teachers). Finding the absolute value of a number (2 teacher), giving the definition of the concept (1 teacher), solving relevant examples (1 teacher), solving the absolute value of an algebraic expression (1 teacher) and showing the place of numbers on the number line (1 teacher) are among learning skills asked by the teachers. This is supported by the interview comments, as illustrated below:

> *For example, when solving the exercise |-5|+2, if the student finds the result as 7 by determining that the absolute value of -5 is 5, then it can be considered that s/he understood the absolute value concept, otherwise s/he did not (T4).*
>
> *If the student recognized that the distance concept is always equal to zero or greater than zero, it is said that s/he understood the absolute value concept (T14).*
>
> *To recognize the absolute value, it is needed that the student should understand that it means the distance. For instance, s/he has to recognize that the place of a fish under the sea level is expressed to be negative, but its distance to the sea level is expressed with the absolute value concept (T11).*

The first comment associates student's understanding of absolute value with her/his skill of accurately determining the absolute value of a negative number in operations. In the second comment, the teacher underlines the relationship between the distance concept and the absolute value concept such as a distance is never negative. If the student understood this, then s/he understood the absolute value. The next comment is parallel to the previous. The only difference between the two comments is that the teacher articulates his/her opinion with a sea level example from daily life. Consequently, according to the participant teachers, the most important indicator that the student understands the absolute value concept is recognizing the interpretation of absolute value on the number line. Therefore, the teachers try to place in their students the idea that "distance is never negative, so the absolute value of a number will never be negative".

3.9 Place of Absolute Value in the Curriculum

The relationship of teachers with the curriculum contains important clues about the teaching of a concept. A teacher is asked to know objectives, concepts, and materials, related to his/her lesson, given in the curriculum. As the teacher is the most important practitioner of the program, it is clear that s/he is one of the best ones who knows the functioning and non-functioning aspects of the program. We see that most of the teachers think the place of absolute value in the curriculum is appropriate to class and age level (6 teachers). Some teachers find ordering of learning objectives as good (5 teachers), while the curriculum is evaluated to be sufficient by 4 teachers. For one teacher, time devoted to the absolute value in the curriculum is sufficient. On the other hand, two teachers criticize the curriculum from some points. The first one thinks the place of the subject in the curriculum is not meaningful and the other one finds learning objectives to be not clear. The following comments allow the reader to understand better, how the teachers consider the place of absolute value in the curriculum:

> *In my opinion, the place of absolute in the curriculum value is appropriate to class and age level (T1).*
>
> *I believe that the place allocated to the absolute value in the curriculum and students' understanding of distance concept is sufficient (T8).*
>
> *The topic of absolute value is now taught in grades 6. Because of the curriculum changes, 6. grades do not see addition and subtraction with integers. At least, it would be more meaningful it the absolute value is taught with integers (T10).*
>
> *The topic of absolute value comes after the numbers. I think that there is no problem, because it is where it should be. The fact that the absolute value is taught in 6. Grade is also appropriate. It doesn't get heavy for students (T3).*

In the two first comments, the teachers find the place of absolute value in the curriculum as appropriate and sufficient to class and age level. The second comment expects that addition and subtraction with integers are not taught to students at the sixth grade because of the curriculum changes, but the absolute value concept is taught at this level. According to the teacher, the absolute value should be taught with integers. The last comment, unlike the previous, does not consider as a problem the fact that the absolute value is taught at sixth grade and after the numbers. As a result, the vast majority of the teachers consider that the place of absolute value in the curriculum is appropriate for the class and age level, the ranking of the gains is good, and the curriculum is sufficient for the middle school.

3.10 Place of Absolute Value in Textbooks

Textbooks, as an instructional material have many purposes and characteristics. They can be listed as explaining the basis of the knowledge to be taught, being effective tools for teaching and learning, summarizing school mathematics and defining them in accordance with examinations, being necessary for effective learning in developing countries, and functioning as controls with examinations and assessments. Therefore, the teachers' evaluations of textbooks in the context of teaching a concept will provide important information about the teaching of this concept. The participant teachers' evaluations of the place of absolute value concept in textbooks can be summarized as follows: According to the teachers, textbooks are inadequate in terms of having sufficient examples and exercises (6 teachers). While in terms of content, textbooks are sufficient for some teachers (6 teachers), inadequate for some others (5 teachers). Textbooks are considered by only two teachers to be adequate with examples and exercises. Sample excerpts from the typical answers given by the teachers during the interview are presented below:

> *Textbooks are inadequate sources in this sense. There are too many incorrect examples and definitions. Therefore, I use my own experiences rather than textbooks (T1).*
>
> *Obviously, I don't really use textbooks, but I think they are enough about the absolute value. In my opinion, textbooks are generally insufficient (T3).*
>
> *I can say that the activities about the absolute value in textbooks are few and exercises are insufficient (T6).*

The first comment indicates that textbooks are inadequate in terms of placing enough examples and exercises. In opinion of the teacher, there are many incorrect examples and definitions in textbooks. This leads her/him to use his/her own experiences rather than textbooks. In the second comment, the teacher indicates that s/he does not use textbooks. However, he considers textbooks as sufficient in terms of the absolute value concept. His/her general idea about textbooks is that they are often inadequate. The last comment, like the first, finds textbooks to be inadequate and criticizes them for the lack of activities, examples, and exercises. Consequently, we can assert that although the teachers find textbooks adequate for content, they critically criticize them due to lack of examples and exercises.

3.11 Questions Related to Absolute Value in Written Exams

The questions in the written exams are very important in terms of showing teachers' preferences and expectations from students. What knowledge and skills the teacher asks students to demonstrate in order to decide whether or not the subject is understood can be revealed by analysing assessment tools. Since the most widely used assessment methods in our country are written exams, it is thought that analysing them will give important clues about teaching the concept of absolute value. In this context, the participant teachers were asked to give three examples of questions they asked (or will ask) in written exams about the absolute value concept.

The vast majority of questions asked by the teachers are associated with finding and/or operating the absolute value of integer numbers (14 teachers). In some exam questions, students are asked to calculate the absolute value of integer numbers and then order them (8 teachers), It is seen that it is used in written exams for questions requiring the solution of absolute valued equations with one term unknown (7 teachers). 2 teachers ask questions that require an inequality solution with an unknown term with an absolute value, while 2 ask questions related to the length meaning of the absolute value. As a result, the teachers often ask questions that require absolute values of integers in the written exam and use them in operations such as addition and subtraction or ordering from small to large or vice versa. There are also teachers who ask questions that require simple equality and inequality solutions of absolute value.

4. Conclusion and Discussion

In this study, we investigated the teaching of the absolute value concept through middle school teachers' perspectives. The participant teachers consider absolute value as an important concept. Even if the absolute value concept takes up a lot of places in the high school curriculum versus middle school, its relations with many concepts and difficulties caused by it in students at all levels increase the importance of absolute value concept in their eyes. All of this justifies researchers like (Duroux, 1983) who considers absolute value as a minor concept (taking account in its definition of a few lines), but posing major difficulties.

While starting their teaching of absolute value concept, the teachers give examples that appear to refer to the function of absolute value from daily life such as "washing machine", "negative and positive thought", and "sea level" etc. These examples are in parallel with the middle school math curriculum. The only learning gain regarding absolute value is to determine and make sense of absolute value of an integer.

In curriculum, teachers are asked to emphasis what sense of absolute value in daily life (elevator, thermometer, etc.) and number line.

We think that these examples are problematic in some ways. Negative attitude and rejection of negative numbers is a common situation in students even in mathematicians. These examples may increase this even more. Negative numbers already have a misfortune in their name. This causes the image of "something to be avoided" that they do not deserve against them (Bofferding, 2019). When looking at the history of mathematics, we see that mathematicians do not easily accept negative numbers. Maybe this negativity in their names has been inherited from mathematicians (e.g., Bishop et al., 2014; Vlassis, 2008). In addition, these examples do not refer directly to the definition of the concept, in other words they do not have a conceptual nature, they are mostly referred to the operations performed by transforming negative numbers in absolute value bars into positive.

Regarding how the teachers define the absolute value concept, we observe that they tend more to use the definition of absolute value in the meaning of length or examples referring to this definition. At this stage, without considering how this will have unpleasant consequences for students in the future, some teachers highlight the aspect of the concept that transforms negative numbers into positive. In line with our results, Ellis and Bryson (2011) underline that a distance between two real numbers in number lines is the most common accepted explanation of absolute value in mathematics classroom. Some research report that there are some algebra students who consider absolute value as positive real numbers (Ponce, 2008; Taylor & Mittag, 2015) or a number without a sign (Gagatsis & Panaoura, 2014).

On the other hand, the teachers often use examples of daily life based on the number line or the number line to concretize the absolute value. In addition, there are examples that show the transformative aspect of absolute value (negative into positive). The examples from daily life that teachers chose to make students comprehend the absolute value are many and varied. Most of these examples are often intended to illustrate the number-line-based definition of absolute value. Some teachers give examples illustrating the transformative aspect of absolute value such as "washing machine", "prison", or "dishwasher". According to its simple and common definition in the literature, analogy is a comparison between familiar and unfamiliar domains of knowledge. However, it should rapidly be stated that this is not a simple comparison, but a special kind of comparison defined by its purpose and type of information it relates (Ünver, 2009). When making an analogy, we map two knowledge domains which the one is the base (or analog concept) and the other one is the target (target concept). The relationship between basic objects is conveyed to those between target objects (Gentner, 1983). In the success of an analogy, analog and target concepts need to have common characteristics (Glynn et al., 2007). An effective analogy use should foster students' understanding while avoiding misconceptions. However, when we look at the teachers' use of analogy about absolute value, it is seen that they are problematic by both situations. If we take the analogy of washing machine or dishwasher, once an object is washed in a washing machine or dishwasher, it does not turn into another object, it remains the same. However, when this relation is moved to absolute value, for example, the number 1 turns into -1, another different number than itself. Whereas the fundamental absolute-valued relationship between 1 and -1 is that their distance from zero is the same. Therefore, here both the relationship between analog and target concept cannot be fully established and negative numbers that already have a negative image are associated with "dirt" and so their image is made even more negative.

In the opinion of the participant teachers, the most important difficulties students face to the absolute value concept are solving exercises and making every expression they see within absolute value bars positive. Students convert minus sign of a variable in absolute value bars into plus sign regardless of whether its value is positive or negative. The frequent mention of such typical mistakes in the absolute value literature is also an indication of how common they are (Almog & Ilany, 2012; Aziz et al., 2019; Baştürk, 2000; Glorian-Perrin, 1996). In this context, Johnson (1986) indicates that some problems with absolute value arise from simplified perceptions established in many students. He states that some definitions made by teachers in a hurry such as "the absolute value can be found by dropping off the sign" are confusing, often wrong and deadly.

The most common strategy used by the participant teachers against misconceptions and mistakes is to solve many questions and to repeat the subject. In addition, strategies such as warning students against mistakes and giving concrete examples from daily life are also encountered. These strategies are also widely used among pre-service teachers. Baştürk (2009) notes that even if the types of mistakes made in the question change, the suggestions made by the preservice teachers to correct the mistakes do not change much and they do not go beyond explanation, repetition, reminder, warning and increasing number of similar exercise solutions.

Looking at these strategies, it is seen that the teachers have some illusions about what student's mistakes (we use this word to include misconceptions.). Many teachers, such as our teachers here, believe that they can avoid the mistakes that occurs by explaining the subject over and over again, or believe that the only reason for misconceptions is that the student does not listen well to the lesson or the teacher's presentation is not clear and understandable (Baştürk, 2009). However, the fact that the teaching given by the teacher is clear, well prepared, and brilliant does not guarantee to bring about learning in many students at all times. The same thing can be said for the text of a definition, theorem, or rule. That is, no matter how many texts of them are decorated with shapes, colors and repeated with familiar words, there can be a resistance in some students to using them correctly (Robert et al., 1999). So, this resistance is not about the teacher saying better or the students listening better or showing more enthusiasm. As well underlined by Robert et al. (1999) with reference to the psychologists who followed Piaget (such as Vygotsky, Brunner, and Vergnaud), the achievements of some students in math class cannot be explained by listening alone, but as a situation that requires a mental construction process in which the teacher does not have the time to occur during the lecture. For some concepts and techniques, the students need to ask questions and assimilate part of the problem on their own, to understand the answers and to benefit from the teacher's presentation.

On the other hand, in accordance with the curriculum, the teachers think that the most important indicator that the student understands the absolute value concept is recognizing the interpretation of absolute value on the number line. Therefore, the teachers try to place in their students the idea that "distance is never negative, so the absolute value of a number will never be negative". The vast majority of them consider that the place of absolute value in the curriculum is appropriate for the class and age level, the ranking of the gains is good, and the curriculum is sufficient for the middle school. The teachers find textbooks adequate for content, but they criticize them for the inadequacy of examples and exercises. In written exams, the teachers often ask questions that require absolute values of integers and use them in operations such as addition and subtraction, or ordering from small to large or vice versa. Although not in the middle school curriculum, there are also teachers who ask questions that require simple equality and inequality solutions of absolute value.

5. Suggestions for Future Research

Consequently, this study allows us to consider the teaching of the absolute value concept through middle school teachers' perspectives, but this supported only on their beliefs and so what they said. We think that further research should also examine the nature of absolute value concept teaching in middle schools by considering textbooks, teacher practices in classroom, and students' productions. It seems to us that this investigation would provide in-depth and broad knowledge on the nature of teaching and learning this concept, and of course, mathematics.

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