International Journal of Academic Management Science Research (IJAMSR)
ISSN: 2643-900X
Vol. 7 Issue 3, March - 2023, Pages: 16-20

Interval-valued Fuzzy AB-ideals on AB-Algebra

Dr. Areej Tawfeeq Hameed[1], Dr. Ahmed Hamzah Abed[2], Huda Adnan Mohammed[3]

[1]Department of Mathematics, Faculty of Education for Girls, University of Kufa, Najaf, Iraq. E-mail: areej.tawfeeq@uokufa.edu.iq
[2]College of Islamic Sciences, AL Iraqia University, Baghdad, Iraq. E-mail: ahmedh.abed@uokufa.edu.iq
[3]Department of Mathematics, Faculty of Education for Girls, University of Kufa,Najaf, Iraq. E-mail: areej238@gmail.com

Abstract: *In this paper, the notion of interval-valued fuzzy AB-ideals (briefly i-v fuzzy AB-ideal) in AB-algebras is introduced. Several theorems are stated and proved. The image and inverse image of i-v fuzzy AB-ideals are defined .*

Keywords—AB-algebras, fuzzy AB-ideals, interval-valued fuzzy AB-ideals of AB-algebra.

1. Introduction

A. T. Hameed and et al ([4-7]) introduced a new algebraic structure, called AB-algebra, they have studied a few properties of these algebras, the notion of AB-ideals and fuzzy AB-ideal on AB-algebras was formulated and some of its properties are investigated. In [13,20,21], they defined the notions of α-translation, β-magnified and generalized fuzzy AB-subalgebra, generalized fuzzy AB-ideal of AB-algebra and investigate in some of their properties. The concept of a fuzzy set, was introduced by L.A. Zadeh [27]. A.T. Hameed and et al, ([19]) we introduce the notion of anti-fuzzy AB-ideals of AB-algebras and then we study the homomorphism image and pre-image of anti-fuzzy AB-ideals. S. M. Mostafa and et al. [22,24-26] were introduced a new algebraic structure which is called KUS-algebras and investigated some related properties. In [1-3,8-9,12,15-16], A.T. Hameed and et al. introduced AT-ideals on AT-algebras and introduced the notions fuzzy AT-subalgebras, fuzzy AT-ideals of AT-algebras and investigated relations among them. They introduced the notion of cubic AT-ideals of AT-algebra and they discussed some related properties of it. They also prove that some properties of anti-fuzzy AT-ideals and anti-fuzzy AT-subalgebras. A.T. Hameed and et al, ([1]). In [19], A.T. Hameed and et al., prove that the Cartesian product of anti-fuzzy AB-ideals are anti-fuzzy AB-ideals. In this paper, using the notion of interval-valued fuzzy set, we introduce the concept of an interval-valued fuzzy AB-ideals (briefly, i-v fuzzy AB-ideals) of a AB-algebra, and study some of their properties. Using an i-v level set of an i-v fuzzy set, we state a characterization of an i-v fuzzy AB-ideals. We prove that every AB-ideals of an AB-algebra X can be realized as an i-v level AB-ideals of an i-v fuzzy AB-ideals of X. In connection with the notion of homomorphism, we study how the images and inverse images of i-v fuzzy AB-ideals become i-v fuzzy AB-ideals.

"

2. Preliminaries

In this section, we give some basic definitions and preliminaries proprieties of AB-ideals and fuzzy AB-ideals in AB-algebra such that we include some elementary aspects that are necessary for this paper.

Definition 2.1([4-6]) Let X be a set with a binary operation $*$ and a constant γ. Then $(X;*,\gamma)$ is called **an AB-algebra** if the following axioms satisfied: for all $,y,z \in X$,

(i) $((x*y)*(z*y))*(x*z) = \gamma$,

(ii) $\gamma * x = \gamma$,

(iii) $x*\gamma = x$,

Example 2.2([4-6]) Let X = {γ, 1, 2, 3, 4} in which $(*)$ is defined by the following table:

$*$	γ	1	2	3	4
γ	γ	γ	γ	γ	γ
1	1	γ	1	γ	γ
2	2	2	γ	γ	γ
3	3	3	1	γ	γ
4	4	3	4	3	γ

Then $(X;*,\gamma)$ is an AB-algebra.

Remark 2.3([4-6]) Define a binary relation ≤ on AB-algebra $(X;\ *,\gamma)$ by letting $x \le y$ if and only if $x*y = \gamma$.

Proposition 2.4([4-6]) In any AB-algebra $(X;*,\gamma)$, the following properties hold: for all $x,y,z \in X$,

(1) $(x*y)*x = \gamma$.

(2) $(x*y)*z = (x*z)*y$.

(3) $(x*(x*y))*y = \gamma$.

Proposition 2.5([4-6]) Let $(X;*,\gamma)$ be an AB-algebra. X is satisfies for all $x,y,z \in X$,

(1) $x \le y$ *implies* $x*z \le y*z$.

(2) $x \le y$ *implies* $z*y \le z*x$.

Definition 2.6([4-6]). Let $(X;\ *,\gamma)$ be an AB-algebra and let S be a nonempty subset of X. S is called an **AB-subalgebra of** X if $x * y \in S$ whenever $x \in S$ and $y \in S$.

Definition 2.7([4-6]). A nonempty subset I of an AB-algebra $(X;\ *,\gamma)$ is called **an AB-ideal of *X*** if it satisfies the following conditions: for any $x, y, z \in X$,

(I_1) $\in I$,

(I_2) $(x * y) * z \in I$ and $y \in I$ imply $x * z \in I$.

Proposition 2.9 ([4-6]). Every AB-ideal of AB-algebra is an AB-subalgebra.

Proposition 2.8 ([4-6]). Let $\{I_i \mid i\in\Lambda\}$ be a family of AB-ideals of AB-algebra $(X; *, \gamma)$. The intersection of any set of AB-ideals of X is also an AB-ideal.

Definition 2.9 ([13,20,21]). Let $(X\ ;\ *,\gamma)$ and $(Y;\ *`,\gamma`)$ be nonempty sets. The mapping $f: (X;\ *,\gamma) \to (Y;\ *`,\gamma`)$ is called **a homomorphism** if it satisfies:

$f\ (x * y) = f\ (x) *` f\ (y)$, for all $x, y \in X$. The set $\{x\in X \mid f\ (x) = \gamma'\}$ is called **the kernel of** f denoted by ker f.

Theorem 2.10 ([4-6]). Let $f: (X;\ *,\gamma) \to (Y;\ *`,\gamma`)$ be a homomorphism of an AB-algebra X into an AB-algebra Y, then :

A. $f\ (\gamma) = \gamma'$.
B. f is injective if and only if ker $f = \{\gamma\}$.
C. $x \le y$ implies $f\ (x) \le f\ (y)$.

Theorem 2.11 ([4-6]). Let $f: (X;\ *,\gamma) \to (Y;\ *`,\gamma`)$ be a homomorphism of an AB-algebra X into an AB-algebra Y, then:

(F_1) If S is an AB-subalgebra of X, then f (S) is an AB-subalgebra of Y.

(F_2) If I is an AB-ideal of X, then f (I) is an AB-ideal of Y , where f is onto.

(F_3) If H is an AB-subalgebra of Y, then f^{-1} (H) is an AB-subalgebra of .

(F_4) If J is an AB-ideal of Y, then f^{-1} (J) is an AB-ideal of X .

(F_5) ker f is an AB-ideal of X.

(F_6) Im(f) is an AB-subalgebra of Y.

Definition 2.12([27]). Let $(X;\ *,\gamma)$ be a nonempty set, a fuzzy subset μ of X is a function $\mu: X \to [\gamma, 1]$.

Definition 2.13 ([27]). Let X be a nonempty set and μ be a fuzzy subset of $(X;\ *,\gamma)$, for $t \in [\gamma, 1]$, the set $L(\mu, t) = \mu_t = \{ x \in X \mid \mu(x) \ge t\}$ is called **a level subset of μ** .

Definition 2.14([7]). Let $(X;\ *,\gamma)$ be an AB-algebra, a fuzzy subset μ of X is called **a fuzzy AB-subalgebra of *X*** if for all $x, y \in X$, $\mu\ (x * y) \ge min\ \{\mu\ (x), \mu\ (y)\}$.

Definition 2.15([7]). Let $(X;\ *,\gamma)$ be an AB-algebra, a fuzzy subset μ of X is called **a fuzzy AB-ideal of *X*** if it satisfies the following conditions, for all $x, y, z \in X$,

(FAB_1) $\mu\ (\gamma) \ge \mu\ (x)$,

(FAB_2) $\mu\ (x * z) \ge min\ \{\mu\ ((x * y) * z), \mu\ (y)\}$.

Proposition 2.17([7]).

1- The intersection of any set of fuzzy AB-ideals of AB-algebra is also fuzzy AB-ideal.

2- The union of any set of fuzzy AB-ideals of AB-algebra is also fuzzy AB-ideal where is chain.

Proposition 2.18([7]). Every fuzzy AB-ideal of AB-algebra is a fuzzy AB-subalgebra.

Proposition 2.19([7]).

1- Let μ be a fuzzy subset of AB-algebra $(X;\ *,\gamma)$. If μ is a fuzzy AB-subalgebra of X if and only if for every t ∈ [γ,1], μ_t is an AB-subalgebra of X .

2- Let μ be a fuzzy AB-ideal of AB-algebra $(X; *, \gamma)$, μ is a fuzzy AB-ideal of X if and only if for every t ∈ [γ,1], μ_t is an AB-ideal of X .

Lemma 2.20([7]). Let μ be a fuzzy AB-ideal of AB-algebra X and if $\le y$, then $\mu(x) \ge \mu(y)$, for all $x, y \in X$.

Definition 2.21 ([27]). Let $f: (X;\ *,\gamma) \to (Y;\ *`,\gamma`)$ be a mapping nonempty sets X and Y respectively. If μ is a fuzzy subset of X, then the fuzzy subset β of Y defined by: $f(\mu)(y) =$

$$\begin{cases} sup\{\ \mu(x): x \in f^{-1}(y)\} & if\ \ f^{-1}(y) = \{x \in X, f(x) = y\} \neq \emptyset \\ \gamma & otherwise \end{cases}$$

is said to be **the image of μ under f.**

Similarly if β is a fuzzy subset of , then the fuzzy subset $\mu = (\beta \circ f)$ of X (i.e the fuzzy subset defined by $\mu\ (x) = \beta\ (f\ (x))$, for all $x \in X$) is called **the pre-image of β under f** .

Definition 2.22 ([27]). A fuzzy subset μ of a set X has sup property if for any subset T of X , there exist $t_0 \in T$ such that $\mu(t_\gamma) = sup\ \{\mu(t) | \ t\in T\}$.

Proposition 2.23 ([7]). Let $f: (X;\ *,\gamma) \to (Y;\ *`,\gamma`)$ be a homomorphism between AB-algebras X and Y respectively.

1- For every fuzzy AB-subalgebra β of Y, $f^{-1}\ (\beta)$ is a fuzzy AB-subalgebra of X.

2- For every fuzzy AB-subalgebra μ of X , $f\ (\mu)$ is a fuzzy AB-subalgebra of Y.

3- For every fuzzy AB-ideal β of Y, $f^{-1}\ (\beta)$ is a fuzzy AB-ideal of X.

4- For every fuzzy AB-ideal μ of X with sup property, $f\ (\mu)$ is a fuzzy AB-ideal of Y, where f is onto.”

3. Interval-valued fuzzy AB-ideals of AB-algebra

International Journal of Academic Management Science Research (IJAMSR)
ISSN: 2643-900X
Vol. 7 Issue 3, March - 2023, Pages: 16-20

In this section, we will introduce a new notion called interval-valued AB-ideals of AB-algebra and study several properties of it.

Remark 3.1([8,11,23,26]).

An interval number is $\tilde{a} = [\, a^-, a^+]$, where $\gamma \le a^- \le a^+ \le 1$. Let I be a closed unit interval, (i.e., I = [γ, 1]) . Let D[γ, 1] denote the family of all closed subintervals of I = [γ, 1] , that is, D[γ, 1] = { $\tilde{a} = [\, a^-, a^+] \mid a^- \le a^+$, for $a^-, a^+ \in$ I} .

Now, we define what is known as refined minimum (briefly, rmin) of two element in D[γ,1].

Definition 3.2([8,11,23,26]).

We also define the symbols (≽) , (≼) , (=) , rmin and rmax in case of two elements in D[γ, 1] . Consider two interval numbers (elements numbers) $\tilde{a} = [\, a^-, a^+]$, $\tilde{b} = [\, b^-, b^+]$in D[γ, 1] : Then

(1) $\tilde{a} \succcurlyeq \tilde{b}$ if and only if, $a^- \ge b^-$ and $a^+ \ge b^+$,
(2) $\tilde{a} \preccurlyeq \tilde{b}$ if and only if, $a^- \le b^-$ and $a^+ \le b^+$,
(3) $\tilde{a} = \tilde{b}$ if and only if, $a^- = b^-$ and $a^+ = b^+$,
(4) rmin $\{\tilde{a}, \tilde{b}\}$= [min $\{a^-, b^-\}$, min $\{a^+, b^+\}$],
(5) rmax $\{\tilde{a}, \tilde{b}\}$= [max $\{a^-, b^-\}$, max $\{a^+, b^+\}$],

Remark 3.3([8,11,23,26]).

It is obvious that (D[γ, 1] , ≼ , ∨ , ∧) is a complete lattice with $\tilde{\gamma}$ = [γ, γ] as its least element and $\tilde{1}$ = [1, 1] as its greatest element. Let $\tilde{a}_i \in$D[γ, 1] where i ∈Λ .

We define $r\, inf_{i\in\Lambda}\tilde{a}$= [$r\, inf_{i\in\Lambda} a^-$, $r\, inf_{i\in\Lambda} a^+$],
$r\, sup_{i\in\Lambda}\tilde{a}$= [$r\, sup_{i\in\Lambda} a^-$, $r\, sup_{i\in\Lambda} a^+$].

In what follows, let X denote an AB-algebra unless otherwise specified, we begin with the following definition.

Definition 3.4([8,11,23,26]).

An **interval-valued fuzzy subset $\tilde{\mu}_A$ on X** is defined as

$\tilde{\mu}_A$ = {< x, [μ_A^- (x) , μ_A^+ (x)]>| x∈ X} . Where μ_A^- (x) ≤ μ_A^+ (x), for all x∈ X. Then the ordinary fuzzy subsets μ_A^-: X → [γ, 1] and μ_A^+: X → [γ, 1] are called a **lower fuzzy subset and an upper fuzzy subset** of $\tilde{\mu}_A$ respectively.

Let $\tilde{\mu}_A$ (x) = [μ_A^- (x) , μ_A^+ (x)] , $\tilde{\mu}_A$:X → D[γ, 1], then A = {< x, $\tilde{\mu}_A$ (x) >| x∈ X} .

Definition 3.5. An i-v fuzzy subset A = {< x, $\tilde{\mu}_A$ (x) >| x∈ X } of AB-algebra $(X;*,\gamma)$ is called **an interval-valued fuzzy AB-ideal** (i-v fuzzy AB-ideal, in short) if it satisfies the following conditions:

(A_1) $\tilde{\mu}_A$ (γ) ≽ $\tilde{\mu}_A$ (x) ,

(A_2) $\tilde{\mu}_A$ (x ∗ z) ≽ rmin{$\tilde{\mu}_A$ $((x*y)*z)$, $\tilde{\mu}_A$ (y)}, for all $x, y, z \in X$.

Proposition 3.6. An i-v fuzzy subset A = [μ_A^- , μ_A^+] of AB-algebra $(X;*,\gamma)$ is an i-v fuzzy AB-ideal of X if and only if μ_A^- and μ_A^+ are fuzzy AB-ideals of X.

Proof. Suppose that A is an i-v fuzzy AB-ideal of X . For all $x, y, z \in X$ we have [μ_A^- $(x*z)$, μ_A^+ $(x*z)$]= $\tilde{\mu}_A$ $(x*z)$ ≽ rmin{$\tilde{\mu}_A$ $((x*y)*z)$, $\tilde{\mu}_A$ (y)}

= [min{min{ μ_A^- $((x*y)*z)$, μ_A^+ $((x*y)*z)$}, min {μ_A^- (y), μ_A^+ (y) }]

Therefore min $\{\gamma_1, \gamma_3\} > \lambda_1 = \frac{1}{2}(\delta_1 + \min\{\gamma_1, \gamma_3\}) > \delta_1$

min $\{\gamma_2, \gamma_4\} > \lambda_2 = \frac{1}{2}(\delta_2 + \min\{\gamma_2, \gamma_4\}) > \delta_2$.

Hence [min $\{\gamma_1, \gamma_3\}$, min $\{\gamma_2, \gamma_4\}$] > [λ_1 , λ_2] > [δ_1 , δ_2] = $\tilde{\mu}_A$ ($z_0 * x_0$) ,

so that, ($x_0 * z_0$) ∉ $\tilde{U}$ (A ; [λ_1, λ_2]) . which is a contradiction , since

$\tilde{\mu}_A$ (($x_0 * y_0$) ∗ z_0) = [γ_1, γ_2] ≥ [min{γ_1, γ_3} , min {γ_2, γ_4}] > [λ_1, λ_2] .

$\tilde{\mu}_A$ (y_0) =[γ_3, γ_4] ≥ [min{γ_1, γ_3} , min {γ_2, γ_4}] > [λ_1, λ_2] , imply that

(($x_0 * y_0$) ∗ z_0) ,(y_0) ∈$\tilde{U}$ (A ; [λ_1, λ_2]). Then

$\tilde{\mu}_A$ $(x*z)$ ≽ rmin{$\tilde{\mu}_A$ $((x*y)*z)$, $\tilde{\mu}_A$ (y)}, for all $x, y, z \in X$. ⌂

Theorem 3.7. Every AB-ideal of an AB-algebra $(X;*,\gamma)$ can be realized as an i-v level AB-ideal of an i-v fuzzy AB-ideal of X.

Proof. Let Y be an AB-ideal of X and let A be an i-v fuzzy subset on X defined by

$$\tilde{\mu}_A(x) = \begin{cases} [\alpha_1, \alpha_2] & if x \in Y \\ [\gamma, \gamma] & otherwise \end{cases}.$$

Where $\alpha_1, \alpha_2 \in$ [γ, 1] with $\alpha_1 < \alpha_2$. It is clear that $\tilde{U}$ (A ; [α_1 , α_2]) = Y . We show that A is an i-v fuzzy AB-ideal of X. Let $x, y, z \in X$,

If $((x*y)*z)$, y ∈ Y, then $(x*z) \in Y$, and therefore

$\tilde{\mu}_A$ $(x*z)$ = [α_1 , α_2] ≽ rmin{[α_1 , α_2],[α_1 ,α_2]} =rmin{$\tilde{\mu}_A$ $((x*y)*z)$, $\tilde{\mu}_A$ (y)}.

If $((x*y)*z)$, y ∉ Y , then $\tilde{\mu}_A$ $((x*y)*z)$ = [γ,γ] =$\tilde{\mu}_A$ (y) and so

$\tilde{\mu}_A$ $(x*z)$ ≥ [γ,γ] = rmin{[γ,γ],[γ,γ]} ≽ rmin{$\tilde{\mu}_A$ $((x*y)*z)$, $\tilde{\mu}_A$ (y)} ,

If $((x*y)*z) \in Y$ and y ∉ Y , then $\tilde{\mu}_A$ $((x*y)*z)$ =[α_1 , α_2] and $\tilde{\mu}_A$ (y)= [γ,γ] , then $\tilde{\mu}_A$ $(x*z)$ ≽ [γ,γ] =rmin{[α_1 , α_2],[γ,γ]} = rmin{$\tilde{\mu}_A$ $((x*y)*z)$, $\tilde{\mu}_A$ (y)}.

Similarly for the case $((x*y)*z) \notin Y$ and y ∈ Y we get

$\tilde{\mu}_A$ $(x*z)$ ≽ rmin{$\tilde{\mu}_A$ $((x*y)*z)$, $\tilde{\mu}_A$ (y)}.

Therefore A is an i-v fuzzy AB-ideal of X, the proof is complete. ⌂

International Journal of Academic Management Science Research (IJAMSR)
ISSN: 2643-900X
Vol. 7 Issue 3, March - 2023, Pages: 16-20

Corollary 3.8. Let $(X;*,\gamma)$be an AB-algebra, B be a fuzzy subset of X and let A be an i-v fuzzy subset on X defined by $\tilde{\mu}_A$ (x) = $\begin{cases}[\alpha_1,\alpha_2] & if x \in Y\\ [\gamma,\gamma] & otherwise\end{cases}$. Where $\alpha_1, \alpha_2 \in (\gamma, 1]$ with $\alpha_1 < \alpha_2$. If A is an i-v fuzzy AB-subalgebra of X, then B is a fuzzy AB-subalgebra of X.

Theorem 3.9. If A is an i-v fuzzy AB-ideal of AB-algebra $(X;*,\gamma)$, then the set $X_{\tilde{M}_A}$ is an AB-ideal of X.

Proof. Let $((x*y)*z)$, y $\in X_{\tilde{M}_A}$. Then $\tilde{\mu}_A$ $((x*y)*z)$ = $\tilde{\mu}_A$ (γ) = $\tilde{\mu}_A$ (y) , and so $\tilde{\mu}_A$ $(x*z) \succcurlyeq$ rmin{$\tilde{\mu}_A$ $((x*y)*z)$, $\tilde{\mu}_A$ (y)} = rmin{$\tilde{\mu}_A$ (γ), $\tilde{\mu}_A$ (γ)} = $\tilde{\mu}_A$ (γ).

Combining this with condition (1) of Definition (4.1), we get $\tilde{\mu}_A$ $(x*z)$ =$\tilde{\mu}_A$ (γ), that is $(x*z) \in X_{\tilde{M}_A}$. Hence $X_{\tilde{M}_A}$is an AB-ideal of X. ⌂

Definition 3.10 ([26]). Let f : $(X;\ *,\gamma)$ $\rightarrow (Y;$ $*',\gamma')$ be a mapping from set X into a set Y. let B be an i-v fuzzy subset of Y. Then the inverse image of B, denoted by f^{-1} (B) , is an i-v fuzzy subset of X with the membership function given by

$\mu_{f^{-1}(B)}$ (x) =$\tilde{\mu}_B$ (f (x)), for all $x \in X$.

Proposition 3.11 ([26]). Let f : $(X;\ *,\gamma)$ $\rightarrow (Y;$ $*',\gamma')$ be a mapping from set X into a set , let $\tilde{m}$ = [m^- ,m^+],
and $\tilde{n}$ = [n^- ,n^+] be i-v fuzzy subsets of X and Y respectively. Then

(1) f^{-1} ($\tilde{n}$) = [f^{-1} (n^-), f^{-1} (n^+)] ,

(2) f ($\tilde{m}$) = [f (m^-), f (m^+)] .

Definition 3.12 ([26]). Let f : $(X;\ *,\gamma)$ $\rightarrow (Y;$ $*',\gamma')$ be a mapping from a set X into a set Y . let A be a an i-v fuzzy set of X, then the image of A, denoted by f (A), is the i-v fuzzy subset of Y with membership function denoted by :

$\tilde{\mu}_{f(A)}$ (x)=$\begin{cases} sup_{z\in f^{-1}(y)}\,\tilde{\mu}_A(z) & if\ \ f^{-1}(y) \neq \varphi, y \in Y\\ [\gamma,\gamma] & otherwise\end{cases}$,

where f^{-1} (y):={x$\in X$ | f (x) = y}.

Theorem 3.14. Let f : $(X;\ *,\gamma)$ $\rightarrow (Y;\ *',\gamma')$ be homomorphism from an AB-algebra X into an AB-algebra Y. If B is an i-v fuzzy AB-ideal of Y, then the pre-image f^{-1} (B) of B is an i-v fuzzy AB-ideal of X.

Proof. Since B= [μ_B^-, μ_B^+] is an i-v fuzzy AB-ideal of Y, it follows that from Theorem (5.3), that (μ_B^-) and (μ_B^+) are fuzzy AB-ideals of Y. Using Theorem (2.23(3)), we know f^{-1} (μ_B^-) and f^{-1} (μ_B^+) are fuzzy AB-ideals of X. Hence by Proposition (5.2), we conclude that f^{-1} (B) = [f^{-1} (μ_B^-) , f^{-1} (μ_B^+)] is an i-v fuzzy AB-ideal of X. ⌂

Theorem 5.7. Let f be an epimorphism from an AB-algebra X into an AB-algebra Y . If A is an i-v fuzzy AB-ideal of X with sup property, then f (A) of A is an i-v fuzzy AB-ideal of Y.

Proof. Assume that A= [μ_A^-, μ_A^+] is an i-v fuzzy AB-ideal of X. it follows that from Theorem (4.3), that (μ_A^-) and (μ_A^+) are fuzzy AB-ideals of X. Using Theorem (2.23(4)), that the images f (μ_A^-) and f (μ_A^+) are fuzzy AB-ideals of Y. Hence by Proposition (5.2), we conclude that f (A)= [f (μ_A^-),f (μ_A^+)] is an i-v fuzzy AB-ideal of Y .⌂

Acknowledgment

References

[1] A.T. Hameed and B.H. Hadi, **Anti-Fuzzy AT-Ideals on AT-algebras,** Journal Al-Qadisyah for Computer Science and Mathematics , vol.10, no.3(2018), 63-74.

[2] A.T. Hameed and B.H. Hadi, **Cubic Fuzzy AT-subalgebras and Fuzzy AT-Ideals on AT-algebra,** World Wide Journal of Multidisciplinary Research and Development, vol.4, no.4(2018), 34-44.

[3] A.T. Hameed and B.H. Hadi, **Intuitionistic Fuzzy AT-Ideals on AT-algebras,** Journal of Adv Research in Dynamical & Control Systems, vol.10, 10-Special Issue, 2018

[4] A.T. Hameed and B.N. Abbas**, AB-ideals of AB-algebras,** Applied Mathematical Sciences, vol.11, no.35 (2017), pp:1715-1723.

[5] A.T. Hameed and B.N. Abbas**, Derivation of AB-ideals and fuzzy AB-ideals of AB-algebra**, LAMBERT Academic Publishing, 2018.

[6] A.T. Hameed and B.N. Abbas**, On Some Properties of AB-algebras,** Algebra Letters, vol.7 (2017), pp:1-12.

[7] A.T. Hameed and B.N. Abbas**, Some properties of fuzzy AB-ideal of AB-algebras,** Journal of AL-Qadisiyah for Computer Science and Mathematics, vol.10, no. 1(2018), pp:1-7.

[8] A.T. Hameed and E.K. Kadhim, **Interval-valued IFAT-ideals of AT-algebra**, Journal of Physics: Conference Series (IOP Publishing), 2020, pp:1-5.

[9] A.T. Hameed and N.H. Malik, (2021), **(β, α)-Fuzzy Magnified Translations of AT-algebra**, Journal of Physics: Conference Series (IOP Publishing), 2021, pp:1-13.

[10] A.T. Hameed and N.J. Raheem, (2020), **Hyper SA-algebra**, International Journal of Engineering and Information Systems (IJEAIS), vol.4, Issue 8, pp.127-136.

[11] A.T. Hameed and N.J. Raheem, (2021), **Interval-valued Fuzzy SA-ideals with Degree (λ,κ) of SA-algebra**, Journal of Physics: Conference Series (IOP Publishing), 2021, pp:1-13.

[12] A.T. Hameed, , R.A. Flayyih and S.H. Ali, (2021), **Bipolar hyper Fuzzy AT-ideals of AT-Algebra**, International Journal of Engineering and Information Systems (IJEAIS), vol.4, Issue 3, pp.145-152.

[13] A.T. Hameed, A.H. Abed and I.H. Ghazi, (2020), **Fuzzy β-magnified AB-ideals of AB-algebras**, International Journal of Engineering and Information Systems (IJEAIS), vol.4, Issue 6, pp.8-13.

[14] A.T. Hameed, A.S. abed and A.H. Abed, (2018), **TL-ideals of BCC-algebras**, Jour of Adv Research in Dynamical & Control Systems, Vol. 10, 11-Special Issue, 2018

[15] A.T. Hameed, **AT-ideals and Fuzzy AT-ideals of AT-algebras**, Journal of Iraqi AL-Khwarizmi Society, vol.1, no.2, (2018).

[16] A.T. Hameed, F. F. Kareem and S.H. Ali, **Hyper Fuzzy AT-ideals of AT-algebra**, Journal of Physics: Conference Series (IOP Publishing), 2021, pp:1-15.

[17] A.T. Hameed, **Fuzzy ideals of some algebras**, PH. Sc. Thesis, Ain Shams University, Faculty of Sciences, Egypt, 2015.

[18] A.T. Hameed, H.A. Faleh and A.H. Abed, (2021), **Fuzzy Ideals of KK-algebra**, Journal of Physics: Conference Series (IOP Publishing), 2021, pp:1-7.

[19] A.T. Hameed, H.A. Mohammed and A.H. Abed, **Anti-fuzzy AB-ideals of AB-algebras,** (2022).

[20] A.T. Hameed, I.H. Ghazi and A.H. Abed, (2019), **Big Generalized fuzzy AB-Ideal of AB-algebras**, Jour of Adv Research in Dynamical & Control Systems, vol. 11, no.11(2019), pp:240-249.

[21] A.T. Hameed, I.H. Ghazi and A.H. Abed, (2020), **Fuzzy α-translation AB-ideal of AB-algebras**, Journal of Physics: Conference Series (IOP Publishing), 2020, pp:1-19.

[22] A.T. Hameed, S. Mohammed and A.H. Abed, (2018), **Intuitionistic Fuzzy KUS-ideals of KUS-Algebras**, Jour of Adv Research in Dynamical & Control Systems, Vol. 10, 11-Special Issue, 2018, pp:154-160.

[23] A.T. Hameed, S.H. Ali and , R.A. Flayyih, **The Bipolar-valued of Fuzzy Ideals on AT-algebra**, Journal of Physics: Conference Series (IOP Publishing), 2021, pp:1-9.

[24] S.M. Mostafa, A.T. Hameed and A.H. Abed, **Fuzzy KUS-ideals of KUS-algebra**, Basra Journal of Science (A), vol.34, no.2 (2016), 73-84.

[25] S.M. Mostafa, M.A. Abdel Naby, F. Abdel-Halim and A.T. Hameed, **On KUS-algebra**, International Journal of Algebra, vol.7, no.3(2013), 131-144.

[26] S.M. Mostafa, M.A. Abdel Naby, F. Abdel-Halim and A.T. Hameed, **Interval-valued fuzzy KUS-ideal** ,IOSR Journal of Mathematics (IOSR-JM), vol.5, Issue 4(2013) , 61-66 .

[27] Zadeh L. A., **Fuzzy sets**, Inform. And Control, vol. 8 (1965), 338-353.