International Journal of Engineering and Information Systems (IJEAIS)
ISSN: 2643-640X
Vol. 7 Issue 3, March - 2023, Pages: 1-6

Study New Types of Resolvable Spaces via Ideal bitopological space

Hawraa S. Abu Hamad Al-Ali[1] and Yiezi K. Al-Talkany[2]

Faculty of Computer Science and Maths :dept.of mathematics, University of Kufa, Najaf, Iraq
e-hauraas.alali@uokufa.edu.iq
Faculty of Education:dept.of mathematics, University of Kufa, Najaf, Iraq
e-yiezi k .Altalkany@uokufa.edu.iq

Abstract*: In this paper we define a new type of resolvable spaces depended on the lambda ideal open set,these new types includ: weakly-λI-resolvable space, weakly-λI-I-resolvable space ,Weakly-λI-T-resolvable space, λI-T-resolvable space,in addition to studying some properties and generating theorem for these sets and paces.*

Keywords: ideal space, bitopological space, I-density**,** λI-dense, weakly-λI-dense set and λI-resolvable space.

1. INTRODUCTION

Kuratowski [18] has introduced the concept of ideal in topological spaces. One of the most important concepts which they were defined in ideal topological spaces of local function was studied by Kuratowski [19] . After that, tis concept was developed by presenting a different of studies related to concept of ideal topological spaces, such as Jankovic and Hamlet [13], R.Vaidyanathaswamy [25], who were among the first to present a studies related to some topological concepts, Also A.Abdel Monsef, Radwan [2], Lashien.E.F, Nasef.A.A [22], and Al-Swidi,L.A, AL. Rubaye M.S [5] where they presented an important studied deals with the concept of I-open set in addition Al-Swidi .L.A., introduced a different studies with some of researchers in a different types of spaces and sets we can see that in [1 7,9,10,14,15].

Njasted.O [24] defined concept α-open sets and defined via ideal concept and this study introduced by Abdel-Monsef.M.E, Nasef, Radwan.A.E. and Esmaeel. A.B [2].

Kelly [21] has introduced the concepts of bitopological spaces by defining two topologies on a set X.

The investigation on various aspects of resolvability of topological spaces has been carried the concept of resolvable space was studied by Hewitt [14] called a resolvable space. After that Chandan Chattopadhyay and Uttam Kumar Roy [12] studies the resolvability, irresolvability space and properties of maximal spaces.

Al-Swidi , L.A Abdaalbaqi, L. S., Hawraa Abbas Al-Bawi [11] introduced the concept of the weakly-I-dense set and invested it with other topological spaces.

2. PRELIMINARIES

2.1 Definition [18]

A nonempty collection I of subsets of X is called to be an ideal on X, if it satisfies the following two conditions:

(1) A∈I, and, B⊆A→B∈I (heredity).

(2) A∈I, and , B∈I⟶A∪B∈I (finite additivity).

2.2 Definition [19]

Let (X,T) be a topological space with an ideal I on X, a set operator (.)*:p(X)→p(X), defined as follow A*(I,T)={x ∈ X: A∩U∉ I, for every U_x ∈T}, Which is called the local function of A with respect to I and T.

2.3 Remark [15]

Every I-dense is T*-dense and, then T- dense

2.4 Definition [15]

Let (X,T,I)be an ideal topological space ,then X is called a resolvable space iff there exist two disjoint dense set A,B ,such that $A\sqcup B=X$

2.5 Definition [15]

Let (X,T,I)be an ideal topological space ,then X is called I- resolvable space iff X has two disjoint I- dense set A,B ,such that $A\sqcup B=X$.

2.6 Theorem1[15]

If (X,T,I) is I-resolvable space, then I is condense.

2.7 Definition [17]

A subset A of X is called λI-open set iff for each x∈A ,and for each α_I-open set W_x such that A ⊆ W, satisfy that x∈{x:$U_x \sqcap int_T^{\alpha}$ (W) ∉ I, for each U_x∈ T }.The family of all λI-open set denoted by $O_{\lambda I}(x)$

2.8 Definition [17]

Let (X,T,I) be an ideal topological space an operator $(.)^{*\lambda I}$:P(X) $\longrightarrow$ P(X) called λI-local function of A with respect to I and λI-open set is define as follow, for each A⊆X

$A^{*\lambda I}$ (I, λI-open) = {x∈X:U⊓A∉I,for each subset U∈$O_{\lambda I}$(x)},when there is no chance for confusion $A^{*\lambda I}$ (I, λI-open) denoted by $A^{*\lambda I}$.

2.9 Theorem [17]

Let (X,T,T^{α},I) be an ideal bitopological space and let A,B are subset of X, then the following statement is hold:

1- If A∈ I, then $A^{*\lambda I}$(I)= ϕ.

2- If I={ϕ} , then A⊔ B⊆ $A^{*\lambda I}$⊔ $B^{*\lambda I}$

3- For any ideal , then $(A ⊔B)^{*\lambda I}$ =$A^{*\lambda I}$⊔ $B^{*\lambda I}$

4- For any ideal , then $(A⊓ B)^{*\lambda I}$.=$A^{*\lambda I}$⊓$B^{*\lambda I}$

5- If I⊆ J, then $A^{*\lambda J}$⊆ $A^{*\lambda I}$

6- If U∈ T ,then U⊓ $A^{*\lambda I}$⊂$(U⊔ A)^{*\lambda I}$

7- If U∈ T^{α} , then U ⊓ $A^{*\lambda I}$⊂$(U⊔A)^{*\lambda I}$

8- $A^{*\lambda I}$ ⊆ $Cl^{\lambda I}$(A).

9- $A^{*\lambda I}$⊆ $Cl^{\lambda I}$($A^{*\lambda I}$) .

10 - $(A^{*\lambda I})^{*\lambda I}$ ⊆ $A^{*\lambda I}$

2.10 Definition [17]

Let(X,T,T^{α},I) be an ideal bitopological space for any A⊆X we define:

$Cl^{*\lambda I}$ (A)(I,T) = A⊔$A^{*\lambda I}$

2.11 Theorem

Let(X,,T,T^{α},I) be an ideal bitopological space and let A, B are sub set of X , then the following statement is hold:

1-$Cl^{*\lambda I}$ (x) =X.

2- If A⊆ B, then $Cl^{*\lambda I}$ (A)⊆ $Cl^{*\lambda I}$(B)

3- A⊆ $Cl^{*\lambda I}$ (A) .

4- $Cl^{*\lambda I}$ (A⊔B)= $Cl^{*\lambda I}$ (A)⊔ $Cl^{*\lambda I}$(B) .

5- $Cl^{*\lambda I}$ (A⊓B)= $Cl^{*\lambda I}$ (A)⊓ $Cl^{*\lambda I}$(B) .

6- $Cl^{*\lambda I}$ (A) = $Cl^{*\lambda I}$ ($Cl^{*\lambda I}$ (A)) .

7- $Cl^{*\lambda I}$ (A) ⊆ Cl(A) . If I is condense.

8- $Cl^{*\lambda I}$ (A) ⊆ Cl^{*}(A) . If x is true space .

9- Cl($A^{*\lambda I}$) = $A^{*\lambda I}$

10 - Cl^{*}(A) ⊂ $Cl^{*\lambda I}$ (A) . If I is condense.

11- $A^{*\lambda I}$⊂ $Cl^{*\lambda I}$ (A) .

2.12 Proposition [17]

Let (X,T,T^{α},I) be an ideal bitopological space and let A, B are sub set of X then the following statement is hold:

1-If I is condense set, then $A^{*\lambda I}$ ⊆ A^{*} .

2-If X is true space, then A^{*} ⊆ $A^{*\lambda I}$

2.13 Definition[16]

Let (X,T,T^{α},I) be an ideal bitopological space and let A be subset of X is called λI-dense set iff $A^{*\lambda I}$= X

2.14 Proposition [16]

Let (X,T,T^{α},I) be an ideal bitopological space, then the following properties hold:

1- Let I is condense , then every λI- dense is I –dense set.
2- Let x is true space , then every λI-dense is T^{*}-dense set
3- Let I is condense , then every λI- dense is T- dense set .

2.15 Definition [11]

Let (X,T,I) be an ideal topological space and A ⊆ X , then A is called weakly –I-dense if A^{**}= X

2.16Remark [11]

Every weakly - I - dense is I - dense and hence T*- dense and T- dense.

2.17 Remark [11]

Let (X,T,I) be an ideal topological space and A⊂ X with I is condense , then if A is I-dense in X , then A is weakly -I-dense in X.

2.18 Lemma [11]

Let(X,T,I) be an ideal topological space , then the following statements are hold:

1- if A is weakly -I-dense of X , then A⊓ $U \neq$ ϕ for each U ϵ T

2- if A is weakly -I-dense of X , then A^{*} ⊓ $U \neq$ ϕ for each U ϵ T

2.19 Remark [11]

Let(X ,T,I) be an ideal topological space with I and J ideals on X and A subset of X , then :

If I ⊆ J, then A^{**}(J) ⊆ A^{**}(I) .

International Journal of Engineering and Information Systems (IJEAIS)
ISSN: 2643-640X
Vol. 7 Issue 3, March - 2023, Pages: 1-6

2.20 Corollary[11]
Let (X,T,I) be a topological space , I and J ideals on X such that I ⊆ J. If A is weakly -J-dense in X , then A is weakly -I-dense in X .

3. On λI-resolvable spaces

3.1 Definition:
Let (X,T,$T^{\propto}, I$) be an ideal bitopological space and A⊆X, then A is called weakly –λI-dense if $(A^{*\lambda I})^{*\lambda I}$=X .
3.2 Example:
Let X ={a,b,c} with a topology T ={X, Φ,{a}}and I= Φ, then weakly - λI-dense={X,{a,b},{a,c}}.
3.3 Definition:
Let (X,T,$T^{\propto}$,I) be an ideal bitopological space, then I is called λI- condense if I ⊓ $O_{\lambda I}(x)$=Φ.
3.4 Example:
Let X={a,b,c}, with a topology T={X,Φ,{a}} and I=Φ, then I ⊓ $O_{\lambda I}(x)$=Φ
3.5 Theorem :
Let (X,T,$T^{\propto}, I$) be an ideal bitopological space .If I is λI- condense , then $X^{*\lambda I}$=X
Proof
Suppose $X^{*\lambda I}$ ≠X , then there exist xϵX such that x ∉ $X^{*\lambda I}$,then there exist H ∈ $O_{\lambda I}(x)$
Such that H⊓ $x \in I$, then H∈I . Since H ∈ $O_{\lambda I}(x)$,then H ∈ I ⊓ $O_{\lambda I}(x)$
But I is λI- condense , then H∉ I ⊓ $O_{\lambda I}(x)$ and this is contradiction ,and then $X^{*\lambda I}$=X.
3.6 Remark :
Let (X,T,$T^{\propto}, I$) be an ideal bitopological space. I and J ideals on X such that I ⊆J.If A is weakly - λJ-dense then A is weakly-λI-dense
Proof:
Since I ⊆ J by theorem (2.9)(5)we have $(A^{*\lambda J})^{*\lambda J}$ (J)⊆ $(A^{*\lambda I})^{*\lambda I}$(I), and since($A^{*\lambda J})^{*\lambda J}$(J)=X , then X= $(A^{*\lambda I})^{*\lambda I}$(I), hence A is weakly - λI-dense.
3.7 Proposition:
Let (X,T,T^{α},I) be an ideal bitopological space, then :If I is condense, then every $Cl^{\lambda I}$ (A)⊂ Cl(A).
Proof:
Let x∈$Cl^{\lambda I}$ (A), then U⊓A≠ ϕ for each U∈$O_{\lambda I}$(x) if x∉ Cl(A),then there exist H ∈ T(x) , H⊓A=ϕ since I is condense, then H is λI -open set, ther exist λI –open set H , H⊓A=ϕ and this is contradiction.
3.8 Theorem :
Let (X,T,T^{α},I) be an ideal bitopological space, then the following properties hold :
1-If x is true space , then every weakly− λI-dense is dense set.
2- If I is condense , then every weakly− λI-dense is I- dense .
3- every weakly− λI-dense is λI-dense.
4- If I is condense and λI-codense , then every dense is weakly - λI-dense.
5- If I is λI-codense , then every I- dense is weakly− λI-dense.
6- If I is λI-codense , then every λI-dense is weakly− λI-dense
Proof:
1- since $(A^{*\lambda I})^{*\lambda I}$=X, by theorem (2.9)(10), $(A^{*\lambda I})^{*\lambda I} \subset A^{*\lambda I}$,
Then $A^{*\lambda I} = X$, by theorem (2.11)(8) ,then $A^{*\lambda I} \subset CL$(A) , then$A^{*\lambda I}$=X
2-since $(A^{*\lambda I})^{*\lambda I}$=X, by theorem (2.9)(10), $(A^{*\lambda I})^{*\lambda I} \subset A^{*\lambda I}$and by theorem (2.12)(1) ,then $A^{*\lambda I} \subset A^{*}$, then A^{*}=X
3- since $(A^{*\lambda I})^{*\lambda I}$=x by theorem(3.8)(2),by theorem (2.12)(2) then $A^{*\lambda I} = X$
4- Since A is dense set then Cl (A)=X by proposition (3.7), then $Cl^{\lambda I}$=X by theorem (2.9)(8), then $A^{*\lambda I}$=X, since I is λI-codense ,then $(A^{*\lambda I})^{*\lambda I}$=X.
5-Since A is I-dense, then A^{*}=X, by proposition (2.12)(2),then $(A^{*})^{*\lambda I} \subset (A^{*\lambda I})^{*\lambda I}$, then $X^{*\lambda I} \subset (A^{*\lambda I})^{*\lambda I}$ since I is λI-codense, then x⊂$(A^{*\lambda I})^{*\lambda I}$, then $(A^{*\lambda I})^{*\lambda I}$ =X.
6- Since A is λI-dense. Then $A^{*\lambda I}$ =X and I is λI-codense , then $(A^{*\lambda I})^{*\lambda I}$=X.
3.9 Theorem :
Let (X,T,T^{α},I) be an ideal bitopological space and A subset of X ,then the following properties hold :
1- if A is λI- dense , then A⊓U≠ Φ for each U∈ $O_{\lambda I}$
2- If A is weakly− λI-dense ,then $A^{*\lambda I}$ ⊓U≠ Φ for each U ∈ $O_{\lambda I}$.
Proof
1-Since $A^{*\lambda I}$= X for each xϵX , xϵ $A^{*\lambda I}$, then U⊓ $A \notin I$for each U ∈ $O_{\lambda I}$.
2-Since $(A^{*\lambda I})^{*\lambda I}$= X for each x$\epsilon$X , x$\epsilon$ $A^{*\lambda I}$ then $A^{*\lambda I}$ ⊓ U ∉ I for each U ∈ $O_{\lambda I}$.

3.10 Definition:

Let (X,T,T^{α},I) be an ideal bitopological space ,then X is called λI- resolvable space iff X has two disjoint λI-dense set A,B subset of X , such that A⊔ $B = X$.

3.11 Proposition:

Let (X,T,T^{α},I) be an ideal bitopological space and A,B subset of X ,then the following properties hold :

1-If I is condense, then every λI-resolvable space is I-resolvable space .

2-Let X is true space, then every I-resolvable space is λI-resolvable space.

3-every λI-resolvable space is resolvable space.

4-If X is resolvable space, then X is not necessary λI-resolvable space.

Proof:

1-Since X is λI-resolvable, then there exist A,B subset of x such that A⊔B=X and A⊓B=ϕ where A and B are λI-dense sets . So by proposition(3.8). We have that A and B are I-dense sets and, then X is I-resolvable space .

2- Since X is I-resolvable, then there exist A,B subset of X such that A⊔B=X and A⊓B=ϕ where A and B are I-dense sets . So by proposition(2.12)(2). We have that A and B are λI-dense sets and, then X is λI-resolvable space .

3-Since X is λI-resolvable space ,then there there exist A,B subset of X such that A⊔B=X and A⊓B=ϕ where A and B are λI-dense sets . So by proposition(3.8)(3). We have that A and B are dense sets and, then X is resolvable space .

4-Let X={a,b,c}, with a topology T={X,ϕ, {a}} and I={ϕ}.Clearly that x is resolvable space but not λI-resolvable space.

3.11 Proposition:

Let (X,T,T^{α},I) be an ideal bitopological space with tow ideals I,J and I ⊆J. If X is weakly –λJ-resolvable , then X is weakly –λI-resolvable.

Proof:

Since X is weakly-λJ-resolvable, then there exist A,B subset of x such that A⊔B=X and A⊓B=ϕ where A and B are weakly λJ-dense sets. So by proposition(3.6). We have that A,B are weakly –λI-dense set, and then X is weakly –λI-resolvable.

3.12 Definition:

A nonempty (X,T,T^{α},I) is called:

1- weakly- λI- resolvable , if X is the disjoint union of two weakly- λI-dense.

2- weakly- λI- I -resolvable , if X is the disjoint union of two weakly- λI-dense and I-dense.

3- weakly- λI-T-resolvable, if X is the disjoint union of two weakly- λI-dense and T-dense.

4- λI-T-resolvable , if X is disjoint union of two λI-dense and T- dense.

3.13 Theorem:

1- Let I is condense , if X is weakly - λI- resolvable ,then X is weakly- λI- I –resolvable.

2- Let X is true space , if X is weakly - λI- resolvable ,then X is weakly- λI-T-resolvable.

3- Let X is true space , if X is weakly - λI- resolvable, then X is λI-T-resolvable.

4-Let I is condense , if X is weakly- λI- I –resolvable space ,then X is weakly- λI-T-resolvable space.

5- If X is weakly- λI- I –resolvable space ,then X is λI-T-resolvable space.

6-If X is weakly- λI-T-resolvable space , then X is λI-T-resolvable space.

7- Let I is λI- condense, if X is weakly- λI- I –resolvable, then X is weakly - λI- resolvable space.

8- Let I is condense and I is λI- condense, if X is weakly- λI-T-resolvable space then X is weakly - λI- resolvable space.

9- Let I is condense and I is λI- condense, if X is λI-T-resolvable space then X is weakly - λI- resolvable space.

10- Let I is λI- condense, if X is λI-T-resolvable space, then X is weakly - λI-T- resolvable space.

Proof:

1- Since X is weakly - λI- resolvable, then there exist A,B subset of X where A and B $are\ weakly - \lambda$I-dense sets such that A⊔ B=X, and A⊓ $B = \Phi$. So by theorem (3.8)(2). We have that B is I-dense set and then X is weakly- λI- I –resolvable.

2- Since X is weakly - λI- resolvable, then there exist A,B subset of X where A and B $are\ weakly - \lambda$I-dense sets such that A⊔ B=x and A⊓ $B = \Phi$, and by theorem (3.8)(1). We have that B is T- dense set and, then x is weakly- λI-T-resolvable.

3- Since X is weakly - λI- resolvable, then there exist A,B subset of X where A and B $are\ weakly - \lambda$I-dense sets such that A⊔ B=X, A⊓ $B = \Phi$ and by theorem (3.8)(3), then A is λI- dense set and by theorem(3.8)(1) B is T-dense set such that A is λI-dense and B is T-dense where A⊔ B=X and A⊓ $B = \Phi$ then X is λI-T-resolvable.

4- Since X is weakly - λI-I- resolvable, then there exist A,B subset of X such that A is weakly - λI-dense set and B is I-dense where A⊓B =ϕ and A ⊔B =X. So by theorem(2.3) we have that B is T –dense set, then X is weakly – λI-T-resolvable space.

5- Since X is weakly - λI-I- resolvable, then there exist A,B subset of X such that A⊔B=X and A⊓B $= \Phi$,and A is weakly - λI-dense set and B is I-dense set by theorem(3.8)(3), then A is λI-dense and by theorem (2.3) B is T- dense set , then X is $\lambda I -$ T –resolvable space .

6- Since X is weakly - λI-I- resolvable, then there exist A,B subset of X such that A⊔B=X and A⊓B $= \Phi$, and A is weakly - λI-dense set and B is T-dense set by theorem(3.8)(3), then A is λI-dense, then X is $\lambda I - T$ –resolvable space .
7- Since X is weakly - λI-I- resolvable, then there exist A,B subset of X such that A⊔B=X and A⊓B $= \Phi$, A is weakly - λI-dense set and B is I-dense set , since I is λI- condense ,then B is weakly - λI-dense and A is weakly - λI-dense set , then X is weakly – λI-resolvable.
8-Since X is weakly- λI-T-resolvable, then there exist A,B subset of X such that A⊔B=X and A⊓B=Φ,A is weakly – λI- dense and B is T-dense by theorem (3.8)(4) ,then B is weakly – λI- dense ,then X is weakly – λI-resolvable.
9-Since X is λI-T-resolvable, then there exist A,B subset of X such that A⊔B=X and A⊓B=ϕ where A is λI-dense set and B is T-dense set by theorem(3.8)(4), then B is weakly λI-dense and by theorem (3.8)(6) A is weakly λI-dense set dense set , then X is weakly –λI-resolvable space .
10-Since X is λI-T-resolvable, then there exist A,B subset of X such that A⊔B=X and A⊓B=ϕ where A is λI-dense set and B is T-dense set by theorem(3.8)(6), then A is weakly λI-dense, then X is weakly –λI-resolvable space .

3.14 Remark:
1-Since not every T-dense set is I-dense set , then not necessary that weakly – λI-T- resolvable space is weakly – λI-I- resolvable space .
2-Since not every T-dense set is I-dense set , then not necessary that λI-T- resolvable pace is weakly – λI-I- resolvable space

Reference

[1] Al Talkany,Y.K, "Study Special Case of Bitopological Spaces" Journal ofbabylon ,No.1(2007), 17.
[2] Abd El-Monsef.M.E, Nasef.A.A, Radwan .A.E., Esmaeel.R.B "On - open sets with respect to an ideal "Journal of Advanced Studies in topology 5(3) (2014),1-9.
[3] Ali Abdulsada, D,AI-Swidi,L.A.A "Compatibility of center Topology", IOP conference Series : Materials Science and Engineering 928(4). 2020.
[4] Almohammed, R.,AL-Swidi, L.A, "New concepts of fuzzy local function" Baghdad Science Journal 17(2), pp.515-522,2020 .
[5] Al-Swidi, L.A., AL-Rubaye, M.S., "New classes of separation axiom via special case of local function," International Journal of Mathematical Analysis,8(21-24)pp.1119- 1131,2014.
[6] Altalkany,Y.K.,AL-Swidi, L.A.A., "On Some Types of Proximity W-set" Journal of physics: Conference Series, 2021.1963(1),012076.
[7] Ali, R.D., Al-Swidi, L.A., Hadi, M.H.," On fuzzy intense separation axioms in fuzzy ideal topological space", Journal of Inter disciplinary Mathematics, 2022,25(5), pp1357-1363.
[8] Al-Swidi, L.A.,Qahtan, G.A., Hadi, M.H., Omran, A.A., "On the soft Dsdense and Dc-dense in soft ideal topological spaces",Journal of Inter disciplinary Mathematics,2022,25(5),pp.1341- 1346.
[9] Al Talkany,Y.K.,Al-Swidi, L.A. "On proximity focal congested set intopological proximity spaces Journal of Inter disciplinary Mathematics 2022,25(5), pp. 1415-1420.
[10] Abdalbaqi,L.S., Hadi, M.H.,Al-Swidi, L.A., "On condensed set in ideal topologicalspaces" Mathematics,2022,25(5),pp1421-s1425
[11] Al-Swidi, L.A. Abdalbaqi,L.S., Hawraa Abbas Al-Bawi," Various Resolvable Space in Ideal topological spaces "Journal of the University of Babylon –Pure and Applied Sciences Volume (24) , Issue (4) of the Journal (2016).
[12] C.Chattopdhyay "Dense sets ,Nowhere Dense sets and Ideal in Generalized Closure spaces" 59(2007),181-188.
[13] Dragan Jankovic and T. R.Hamlett,"New Topologies from Old Via Ideals", The American Mathemtical Monthly. Vol.97. No. 4 (Apr.,1990), pp. 295-310.
[14] E. Hewitt, A problem of set –theoretic topology ,Duke Math. J., Vol.10,(1943),pp.309-333.
[15] J. Dontchev , M Ganster , D .Rose , Ideal resolvability ,Topology Appl. Vol.93(1999), pp. 1-16.
[16] Hawraa s. Abu Hamed Al-Ali,Yiezi K. Al-talkany "Some properties of density related to lambda ideal open sets"Internation Journal Of Engineering and Information Systems (IJEAIS).2023 .Vol.7 .Iss:2 .pp.42-45.
[17] Hawraa s. Abu Hamed Al-Ali,Yiezi K. Al-talkany "Specil Case Of Local Function"Internation Journal Of Engineering and Information Systems (IJEAIS).2023 .Vol.7 .Iss:2 .pp.73-78.
[18] Kuratowski. K., Topology I, Warszawa, 1933.
[19] Kuratowski. K., Topology ,Academic Press, New York ,1966.
[20] Kelley ,J. L., 1955 . General Topology ,D . Van Nastrand Company ,Inc.
[21] Kelly.J.C, Bi topological spaces, Proc. London Math. Soc. 13 (1963), no.3, 71-89.
[22] Lashien .E.F, A.A.Nasef., "On ideals in general topology" J.Sci. 15(2) 1991, 19-3.
[23] M.Ganster,"Preopen sets and resolvable spaces" Kyungpook Math.J., Vol .27 ,(2)(1987) , pp.135-143.
[24] Njåstad, Olav. "On some classes of nearly open sets." Pacific journal of mathematics 15.3(1965):961-970. : 10.2140/pjm.1965.15.961
[25] R.Vaidyanathaswamy , The localization theory in set topology ,Proc. Indian Acad . Sci.Vol . 20 (1945) ,pp 51-61.

[26] S.Modak ,"Remarks on Dense sets" International Mathematical forum ,Vol. 6 ,(2011), No. 44, 2153-2158 .
[27] V.R. Devi, D. Sivaraj, T.T. Chelvam, Codense and completely condense ideals , Acta Math . Hungar.Vol.108(2005) ,pp.197-205.
[28] W.W.Comfort and L.Feng,The union of resolvable spaces is resolvable, Math. Japan., Vol.38,(3)(1993),pp.413-414.