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On Fuzzy $\boldsymbol{SA}$-subalgebras of $\boldsymbol{SA}$- algebra

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***Abstract**: We use the idea of fuzzy SA-sub algebras of SA-algebras in this research and demonstrate their extensions. In addition, we investigate the relationship between fuzzy SA-subalgebras and their level cuts, stating and proving various theorems and characteristics..Key words: SA-algebra, SA-subalgebra, fuzzy SA-subalgebra.*

Keywords— SA-algebras, SA-subalgebra, homomorphism of SA-algebra.

1. Introduction

L.A. Zadeh established the idea of a fuzzy subset, and many writers have since used it in a variety of mathematical fields. in the field of fuzzy topology, in particular. Since then, a lot of research has been done. BCI-algebras and BCK − algebras are types of abstract algebra that S. Tanaka presented. and K. Is'eki It is well known that the BCK − algebra class is a proper subclass of the BCI-algebra class. To BCK-algebras, O.G. Xi utilized the idea of fuzzy subset and provided some of its features. *Moreover*, in BCI-*algebras* with *degree*, S. S. Ahn and Y. S. Hwang proposed *the concepts oγ an expanded* -ideal and a fuzzy -ideal. The algebras of *logic are one oγ the* significant and *extensively* researched classes of *algebras*. BCI algebras and BCK −algebra s are *examples* of these.

2. *Preliminaries*

Now *we give some* definitions and preliminary *results needed* in the later sections.

Definition* 2.1([2]).** Let $(X;\sigma,\#,ב)$ be an algebra with two operations (σ) *and* $(\#)$ and constant $(ב)$. X is called a ***SA-algebra *if it meets the criteria listed below*: for any $x,\ b,z\ \in\ X$

(SA_1) $\vartheta\#\vartheta = ב$,

(SA_2) $\vartheta\#ב=\vartheta$,

(SA_3) $(\vartheta\#b)\#z=\vartheta\#(z\sigma b)$,

(SA_4) $(\vartheta\sigma b)\#(\vartheta\sigma z)= b\#z$.

In X, we can define *a binary relation* $(\leq)$ *by* :

$$\vartheta\leq b \text{ if and only if } \vartheta\sigma b = ב \text{ and } \vartheta\# b = ב ,\vartheta, b\in X.$$

***Lemma* 2.2([22]).** Let $(X;\sigma,\#,ב)$ be a SA-algebra Then for any $x,\ b\in X$,

(L_1) $\vartheta\sigma b=\vartheta\#(\# b)$.

(L_2) $\vartheta\# b=\vartheta\sigma(\# b)$,

(L_3) $\vartheta\# b=\# b\sigma\vartheta$.

Proposition 2.3([2]).

Let $(X;\sigma,\#,ב)$ be an SA − algebra , the following is true.: for any ϑ, y, z ∈ X,

(a_1) $(\vartheta\# b)\#z=(\vartheta\#z)\# b$,

(a_2) $ב\#(\vartheta\# b)=(b\#\vartheta)$,

(a_3) $\vartheta\# b\leq z$ imply $\vartheta\#z\leq b$,

(a_4) $\vartheta\leq b$ imply $z\sigma b\leq z\sigma\vartheta$,

(a_5) $(\vartheta\# b)\#(z\# b)\leq\vartheta\#z$ and $(\vartheta\# b)\#(\vartheta\#z)\leq z\# b$,

$(a6)$ $\vartheta\leq b$ *and* $b\leq z$ *imply* $\vartheta\leq z$.

Definition 2.8 ([16]).

A fuzzy subset ∇ of a set X **has sup property** if for any subset T of X, there *exist* $\sigma_ב\in T$ such that $\nabla(\sigma_ב) = sup\{\nabla(\sigma)|\ \sigma\in T\}$.

3. The relation of fuzzy SA-subalgebras of SA-algebra :

The relationship between fuzzy SA-subalgebras and fuzzy SA-ideals of SA-algebra is now presented, along with some of its features.

Definition 3. 1:

Let $(X;\sigma,\#,ב)$ be a SA-algebra , a fuzzy subset ∇ of X is called **a *fuzzy SA*-subalgebra of X** if $\forall\, x\,,\ b\in X$,

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$1-\ \nabla(\vartheta\sigma\ b) \geq min\{\ \nabla(\vartheta), \nabla(\ b)\}$.

$2-\ \nabla(\vartheta\#\ b) \geq min\{\ \nabla(\vartheta), \nabla(\ b)\}$.

***Proposition* 3.2:**

Suppose is a fuzzy subset .of SA- algebra $(X;\sigma,\#,\beth)$. If μ *is* a $fuzzy\ \ SA\text{-}subalgebra$ of X, then for any t ∈ [ℶ,1], ∇_σ is either empty or a $SA\text{-}subalgebra$ of X .

Proof:

Consider that ∇ is a fuzzy SA-subalgebra of X ,let $\vartheta,\ b\ \in X$ be such that $\vartheta \in \nabla_\sigma$ and $b \in \nabla_\sigma$, then $\nabla\ (\vartheta)\ \geq \sigma\ and\ \ \nabla\ (\ b)\ \geq\ \sigma$. Since ∇ is a $fuzzy\ SA-subalgebra$, $it\ follows$ that $\nabla(\vartheta\sigma\ b) \geq$min$\{\ \nabla(\vartheta), \nabla(\ b)\}\ \geq\ \sigma\ \ and\ \nabla(\vartheta\#\ b) \geq min\{\ \nabla(\vartheta), \nabla(\ b)\} \geq$ t , then $(\vartheta\sigma\ b) \in \nabla_\sigma$.

Hence ∇_σ is a $SA-$ subalgebra of X . ⌂

***Corollary* 3.3:**

Let ∇ *be a* fuzzy subset of SA-algebra $(X;\sigma,\#,\beth)$. If ∇ is a fuzzy SA-subalgebra , then for *every* t ∈ Im (μ) , ∇_σ is a $SA\text{-}subalgebra$ of X ,when $\nabla_\sigma \neq \emptyset$.

Proof:

That is obvious by Proposition (3.2). ⌂

Proposition 3.4:

Let ∇ be a fuzzy subset of SA- algebra $(X;\neg,\#,\beth)$. If, ∀ t ∈ [ℶ,1], ∇_σ is either empty or a SA-subalgebra of X, then ∇ is a fuzzy SA- subalgebra of X.

***Proof*:**

Assume $\nabla(\vartheta\sigma\ b) \geq min\{\nabla(\vartheta), \nabla(\ b)\}$ is false, then they do not exist. ϑ' and y'∈Xsuch that , $\nabla(\vartheta'\sigma\ b') < min\{\nabla(\vartheta'), \nabla(\ b')\}$. $Putting\ \ \sigma^{'} = (\nabla(\vartheta^{'}\sigma\ b^{'})\sigma\, min\{\nabla(\vartheta), \nabla(\ b^{'})\})/2$, then

$\nabla\ (\vartheta'\sigma\ b')\ <\ \sigma'$ and $\beth\ \leq\ \sigma'\ < min\{\nabla(\vartheta'), \nabla(\ b')\} \leq$ 1, hence $\nabla(\vartheta') > \sigma'$ and $\nabla(\ b') > \sigma'$, which imply that $\vartheta' \in \nabla_{\sigma'}$ and $b' \in \nabla_{\sigma'}$, since $\nabla_{\sigma'}$ is a $SA\text{-}subalgebra$,it follows that $\vartheta'\sigma\ b' \in \nabla_{\sigma'}$ and that $\nabla(\vartheta'\sigma\ b') \geq \sigma'$, this is also a contradiction. $Therefore\ \ \nabla(\vartheta\sigma\ b) \geq min\{\ \nabla(\vartheta), \nabla(\ b)\}$.

Simareily, $\nabla(\vartheta\#\ b) \geq min\{\nabla(\vartheta), \nabla(\ b)\}$.

Hence μ is a fuzzy SA-subalgebra of X . ⌂

4. Image (Pre-image) SA-*ideals* under *homomorphism* of SA-algebras:

The homomorphism of the image and pre-image of fuzzy SA-ideals andSA-subalgebras of SA-algebra is discussed.

***Theorem* 4.1:**

pre-image of a homomorphic fuzzy SA-subalgebra of SA-algebra is also a$fuzzy\ SA\text{-}subalgebra$ of SA-algebra .

***Proof*:**

Let $\gamma: (X;\sigma,\#,\beth) \to (Y;\sigma',\#',\beth')$ pre-image of a homomorphic SA-algebras, δ a fuzzy $SA\text{-}subalgebra$ of Y and ∇ the pre− image of δ under γ , then $\beta(\gamma(\vartheta)) = \nabla(\vartheta)$, ∀ ϑ ∈ X.

$Let\ \ \vartheta\ , b\ \in\ X, then\ we\ get$

$$\nabla(\vartheta\sigma\ b) = \beta(\gamma(\vartheta\sigma\ b)) = \beta(\gamma(\vartheta)\sigma'\gamma(\ b))$$

$$\geq min\{\ \beta(\gamma(\vartheta)), \beta(\gamma(\ b))\}$$

$$= min\{\ \nabla(\vartheta), \nabla(\ b)\}$$

$i.e., \nabla(\vartheta\sigma\ b) \geq min\{\ \nabla(\vartheta), \nabla(\ b)\},\quad \gamma or\ all\ \vartheta\ ,\ b\ \in\ X.$

$Simeality,\quad \nabla(\vartheta\#\ b) \geq min\{\ \nabla(\vartheta), \nabla(\ b)\},\ \gamma or\ all\ \vartheta\ ,\quad b\ \in\ X.$

$Hence\ , \nabla\ \ is\ a\ \gamma uzzy\ SA-subalgebra\ \ o\gamma\ X.$ ☖

***Proposition* 4.2:**

Let $\gamma: (X;\sigma,\#,\beth) \to (Y;\sigma',\#',\beth')$ be an epimorphism of SA-algebras. For every fuzzy SA-subalgebra ∇ of X and with sup property , $\gamma\ (\nabla)$ is a fuzzy SA- subalgebra of Y.

***Proof*:**

By Definition (2.10), $\beta(\ b') = \gamma(\nabla)(\ b') = sup\{\nabla(\vartheta) : \vartheta \in \gamma^{-1}(\ b')\}$, ∀y'∈ Y $(sup(\emptyset) = \beth)$.

We have to prove that $\beta(\vartheta'\sigma'\ b') \geq$min{ δ(ϑ'),δ(y')} ,∀ $\vartheta',\ b' \in\ Y$.

(I) Let $\gamma: (X;\sigma,\#,\beth) \to (Y;\sigma',\#',\beth')$ be a epimorphism of SA-algebras , ∇ is a fuzzy SA-subalgebra of X with sup *property* and δ the *image* of μ under γ.

Since ∇ is a fuzzy SA- subalgebra of X,

For any $\vartheta', y' \in Y$, let $x_ℶ \in \gamma^{-1}(x')$, $b_ℶ \in \gamma^{-1}(b')$ be such that:

$$\nabla(\vartheta_ℶ \sigma z_ℶ) = \beta(\gamma(\vartheta_ℶ \sigma z_ℶ)) = \beta(\gamma(\vartheta' \sigma' z'))$$

$$= \sup_{x_ℶ \sigma z_ℶ \in \gamma^{-1}(x' \sigma' z')} \nabla(\sigma),$$

$\nabla(\vartheta_ℶ) = \beta(\gamma(\vartheta_ℶ)) = \sup_{x_ℶ \in \gamma^{-1}(\vartheta')} \nabla(\sigma)$ and

$\nabla(b_ℶ) = \beta(\gamma(b_ℶ)) = \sup_{b_ℶ \in \gamma^{-1}(b')} \nabla(\sigma)$, then

$$\beta(\vartheta' \sigma' b') = sup \nabla(\sigma) = \nabla_{\sigma \in \gamma^{-1}(x' \sigma' b')}(\vartheta_ℶ \sigma b_ℶ)$$

$$\geq \min\{\nabla(\vartheta_ℶ), \nabla(b_ℶ)\}$$

$$= min\{sup_{\sigma \in \gamma^{-1}(\vartheta')} \nabla(\sigma), sup_{\sigma \in \gamma^{-1}(b')} \nabla(\sigma)\}$$

$$= min\{\beta(\vartheta'), \beta(b')\}$$

Therefore, $\beta(x' \sigma' b') \geq min\{\beta(x'), \beta(b')\}$.

$$\beta(\vartheta' \#' b') = sup \nabla(\sigma) = \nabla_{\sigma \in \gamma^{-1}(x' \#' b')}(\vartheta_ℶ \# b_ℶ)$$

$$\geq min\{\nabla(\vartheta_ℶ), \nabla(b_ℶ)\}$$

$$= min\{sup_{\sigma \in \gamma^{-1}(\vartheta')} \nabla(\sigma), sup_{\sigma \in \gamma^{-1}(b')} \nabla(\sigma)\}$$

$$= min\{\beta(\vartheta'), \beta(b')\}$$

Therefore, $\beta(\vartheta' \#' b') \geq min\{\beta(\vartheta'), \beta(b')\}$.

Hence δ is a fuzzy SA- subalgebra of Y.

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