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On The ψ-subalgebras of ψ-algebra

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***Abstract**: In this article, we present the concept of -algebras, a novel type of two-operation algebraic structure, as well as its subalgebra and some of its features. In particular, we demonstrate that (X;¬) is a semigroup with identity ב if (X;¬,-,ב) is a -algebra. We also discussed the connection between congruences and subalgebras.*

Keywords— ψ-algebras, ψ-subalgebra, homomorphism of ψ-algebra.

1. Introduction

The abstract was introduced by Y. Imai and K. Iseki. algebras BCI – algebra and BCK – algebra, respectively.

The BCK –algebras are a istinctive subclass of the BCI –algebras, as is widely know. In this essay, we define the

terms algebra, subalgebra, and

homomorphism of algebras.

2. ψ -algebras

In this section, we introduced an algebraic structure called a ψ-algebra, as the following:

Definition 2.1. The *algebraic system* $(X; \neg, \wr, \beth)$ *with two operations* $(\neg)$ and $(\wr)$ and constant $(\beth)$ is called ψ-algebras, if it *satisfies* the *following properties*: for all $w,$ ɦu$, z \in X$,

(ψ_1) $w \wr w = \beth$,
(ψ_2) $(\beth \wr w) \neg w = \beth$,
(ψ_3) $(w \wr$ ɦu$) \wr z = w \wr (z \neg$ɦu$)$,
(ψ_4) $($ɦu$\neg w) \wr (w \wr z) =$ ɦu$\neg z$.

For brevity we *shall* call $(X; \neg, \wr, \beth)$ a ψ-algebra *unless* otherwise specified.

In X we can define a binary relation $(\leq)$ by: $w \leq$ ɦu $\leftrightarrow$ if $w \wr$ ɦu $= \beth$.

Lemma 2.2. Let $(X; \neg, \wr, \beth)$ be a ψ-algebra. Then for any $w,$ ɦu$, z \in X$,

(L_1) Since $w \in X$, *then* $(\wr w) \in X$,
(L_2) $w \neg$ ɦu $=$ ɦu$\neg w$,
(L_3) $w \wr$ ɦu $= \wr$ ɦu$\neg w$,
(L_4) $\beth \wr w = \wr w$, $w = \wr (\wr w)$
(L_5) $\beth \neg \beth = \beth$, $\beth \wr \beth = \beth$,
(L_6) $\wr (w \wr$ ɦu$) = \wr w \neg$ ɦu and $\wr (w \neg$ ɦu$) = \wr w \wr$ ɦu,
(L_7) $((w \wr z) \neg (z \wr$ ɦu$)) = w \wr$ ɦu and $((w \wr z) \wr ($ɦu $\wr z)) = w \wr$ ɦu,
(L_8) $(w \wr$ ɦu$) \wr z = (w \wr z) \wr$ ɦu.

Proposition 2.3. Let $(X; \neg, \wr, \beth)$ be a ψ -algebra. Hence the following is true: for any $w,$ ɦu$, z \in X$,

(a_1) $w \wr$ ɦu $= \beth$ *and* ɦu $\wr w = \beth$ *imply* $w =$ ɦu,
(a_2) $w \neg$ ɦu $= w \wr (\wr$ ɦu$)$,
(a_3) $w \wr$ ɦu $= w \neg (\wr$ ɦu$)$,
(a_4) $(w \wr$ ɦu$) \wr z = (w \wr z) \wr$ ɦu,
(a_5) $(w \wr$ ɦu$) \wr w = \beth \wr$ ɦu,
(a_6) $\beth \wr (w \wr$ ɦu$) =$ ɦu $\wr w$,
(a_7) $(\beth \wr w) = \beth$ *implies that* $w = \beth$,
(a_8) $w = (w \wr \beth) \wr \beth$,
(a_9) $z \wr w = z \wr$ ɦu *implies that* $\beth \wr w = \beth \wr$ ɦu.

Proof:

(a_1) $w \wr$ ɦu $= \beth$ *and* ɦu $\wr w = \beth$, then $w \leq$ ɦu and ɦu $\leq w$ imply $w =$ ɦu.

(a_2) It is clear by lemma (2.4(L_1)) .

(a_3) It is clear by lemma (2.4 (L_3) and (L_2)) .

(a_4) $(w \wr$ ɦu$) \wr z = w \wr (z \neg$ ɦu$)$ *by* (ψ_3)

$= w \wr ($ɦu$\neg z)$ *by lemma* $(2.2(L_2))$
$= (w \wr z) \wr$ ɦu *by* (ψ_3).

(a_5) $(w \wr$ ɦu$) \wr w = (w \wr w) \wr$ ɦu, *by* (a_4) $= \beth \wr$ ɦu, *by* (ψ_1).

(a_6) $\beth \wr (w \wr$ ɦu$) = \beth \wr (\wr$ ɦu$\neg w)$ *by lemma* $(2.2(L_3))$

$= \wr (\wr$ ɦu$\neg w) \neg \beth$ *by lemma* $(2.2(L_3))$
$= ($ɦu $\wr w) \neg 0$ *by lemma* $(2.2(L_6))$
$=$ ɦu $\wr w$ *by* (ψ_2).

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(a_7) $\beth \wr w = \beth$ *implies that* $w = \beth$, *by* (ψ_2).

(a_8) $(w \wr \beth) \wr \beth = w \wr (\beth \neg \beth)$, *by* (ψ_3)

$= w \wr \beth$ *and, by lemma* $(2.2(L_5))$
$= w$, *by* (ψ_2).

(a_9) $\beth \wr w = (z \wr z) \wr w$, *by* (ψ_1)

$= (z \wr w) \wr z$, *by* (a_4)
$= (z \wr \mu) \wr z$, *by assumption*
$= (z \wr z) \wr \mu$, *by* (a_4)
$= \beth \wr \mu$, *by* (ψ_1). ⌂

Proposition 2.4. Let $(X; \neg, \wr, \beth)$ be a ψ-algebra, then the following holds: for any $w, \mu, z \in X$,

(b_1) $w \wr \mu \leq z$ *imply* $w \wr z \leq \mu$,

(b_2) $w \leq \mu$ *implies that* $z \neg \mu \leq z \neg w$,

(b_3) $w \leq \mu$ *implies* $w \wr z \leq \mu \wr z$,

(b_4) $(w \wr \mu) \wr (z \wr \mu) \leq w \wr z$,

(b_5) $(w \wr \mu) \wr (w \wr z) \leq z \wr \mu$,

(b_6) $w \leq \mu$ *and* $\mu \leq z$ *imply* $w \leq z$.

Proof:

(b_1) $w \wr \mu \leq z$ *imply* $(w \wr \mu) \wr z = \beth$ *imply*

$(w \wr z) \wr \mu = \beth$ *by proposition* $(2.3(a_4))$ *imply* $w \wr z \leq \mu$.

(b_2) $(w \neg z) \wr (\mu \neg z) = (w \wr \mu)$, *by* (ψ_4)
$= \wr (\mu \wr w)$,
by lemma $(2.2(L_6)$ *and* $(L_2))$,
$= \beth$, *by assumption* $(w \leq \mu)$. Then $z \neg \mu \leq z \neg w$.

(b_3) $(w \wr z) \wr (\mu \wr z) = (w \wr \mu)$,
by lemma $(2.2(L_7))$
$= \beth$, *by assumption* $(w \leq \mu)$,

Then $w \wr z \leq \mu \wr z$.

(b_4) $[(w \wr \mu) \wr (z \wr \mu)] \wr (w \wr z)$

$= (w \wr \mu) \wr [(z \wr \mu) \neg (w \wr z)]$, *by* (ψ_3)

$= (w \wr \mu) \wr (w \wr \mu)$, *by lemma* $(2.2(L_7))$
$= \beth$, *by* (ψ_1)

Then $(w \wr \mu) \wr (z \wr \mu) \leq w \wr z$.

(b_5) $[(w \wr \mu) \wr (w \wr z)] \wr (z \wr \mu)$

$= (w \wr \mu) \wr [(w \wr z) \neg (z \wr \mu)]$ *by* (ψ_3)

$= (w \wr \mu) \wr (w \wr \mu)$, *by lemma* $(2.2(L_7))$
$= \beth$, *by* (ψ_1)

Then $(w \wr \mu) \wr (w \wr z) \leq z \wr \mu$.

(b_6) By applying (ψ_2), $(x \leq y)$ and (ψ_4),
$z \wr w = (z \wr \beth) \wr w$, *by* (ψ_2)
$= (z \wr (\mu \neg \mu)) \wr w$, *by* (ψ_1)
$= z \wr ((\mu \wr \mu) \wr w)$, *by* (ψ_3)
$= z \wr (\mu \neg (w \wr \mu))$, *by* (ψ_3)
$= z \wr (\mu \neg \beth)$, *by* $(w \leq \mu)$
$= (z \wr \beth) \wr \mu$, *by* (ψ_3)
$= z \wr \mu$, *by* (ψ_2)
$= \beth$, *by* $(\mu \leq z)$
Hence, $z \wr w = \beth$ and so $w \leq z$. ⌂

Proposition 2.5. *Let* $(X; \neg, \wr, \beth)$ be ψ-algebra and $(\leq)$ be a *relation* on X, then $(X, \leq)$ is a partially ordered set.

Proof:

Let $(X; \neg, \wr, \beth)$ bea ψ-*algebra* and *let* $w, \mu, z \in X$, since $x - x = \beth$.

Suppose that $w \leq \mu$ *and* $\mu \leq w$, then $x - y = \beth = y - x$ and $x = y$, by Proposition $(2.3(a_1))$.

Suppose that $w \leq \mu$ *and* $\mu \leq z$, then by Proposition $(2.6(b_6))$, $x \leq z$. Thus $(X, \leq)$ is a partiallly ordered set. ⌂

3. On ψ-subalgebras of ψ-algebras

We explain the idea of -subalgebra in -algebra in this section and provide some instances and results.

Definition 3.1.

Let $(X; \neg, \wr, \beth)$ be a ψ-*algebra* and S be a nonempty set of X. S is know **a ψ-*subalgebra* of X** if $w \neg \mu \in S$ *and* $w \wr \mu \in S$, whenever $w, \mu \in S$.

Example 3.2. Let $(Z_6; \neg_6, \wr_6, \beth)$ the following tables to form a set.:

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Then (Z_4; $\neg_4$, ≀$_4$, ב) is a ψ-algebra. It is easy to show that $I_1 = \{$ב̄$, \bar{2}\} =< \bar{2} >$ and $I_2 = Z_4 =< \bar{1} >$ are ψ-subalgebras of (Z_4; $\neg_4$, ≀$_4$, ב) .

Proposition 3.3. Let I be a ψ-subalgebra of ψ-algebra (X; ¬, ≀, ב) and J be a ψ-subalgebra of I. Then J is a ψ-subalgebra of X.

Proof:

Let w, ɥ $\in X$, such that $w\neg$ ɥ $\in J$ and w ≀ ɥ $\in J$, we see that $w\neg$ ɥ $\in I$ and w ≀ ɥ $\in I$, by assumption, I is a ψ-subalgebra of X, it follows that $w\neg$ ɥ $\in J \subseteq I$ and w ≀ ɥ $\in J \subseteq I$. Therefore J is a ψ-subalgebra of X. ⌂

Proposition 3.4. *Let* $\{I_i : i \in \Lambda\}$be a $family$ of ψ-subalgebras of ψ-algebra (X; ¬, ≀, ב), then $\bigcap_{i\in\Lambda} I_i$ is a ψ-subalgebra of X.

Proof:

Since $\{I_i : i \in \Lambda\}$ be a family of ψ-subalgebras of X , suppose $w \in \bigcap_{i\in\Lambda} I_i$ and $w \in \bigcap_{i\in\Lambda} I_i$, then $w \in I_i$, $w \in I_i$, for all i $\in\Lambda$, but I_i is a ψ-subalgebra of X, ∀i $\in\Lambda$. Then $w\neg$ ɥ $\in I_i$, and w ≀ ɥ $\in I_i$∀ i $\in\Lambda$, therefore, $w\neg$ ɥ $\in \bigcap_{i\in\Lambda} I_i$ and w ≀ ɥ $\in \bigcap_{i\in\Lambda} I_i$. Hence $\bigcap_{i\in\Lambda} I_i$ is a ψ-subalgebra of X . ⌂

Remark 3.5.

The union of ψ-subalgebra of ψ-algebra (X; ¬, ≀, ב), is not a ψ-subalgebra as seen in the following example.

Example 3.6.

Let (Z_6; $\neg_6$, ≀$_6$, ב) be a set $with$ the $following$ tables:

¬	ב̄	$\bar{2}$	$\bar{4}$	$\bar{3}$	$\bar{1}$	$\bar{5}$
ב̄	ב̄	$\bar{2}$	$\bar{4}$	$\bar{3}$	$\bar{1}$	$\bar{5}$
$\bar{1}$	$\bar{1}$	$\bar{3}$	$\bar{5}$	$\bar{4}$	$\bar{2}$	ב̄
$\bar{2}$	$\bar{2}$	$\bar{4}$	ב̄	$\bar{5}$	$\bar{3}$	$\bar{1}$
$\bar{3}$	$\bar{3}$	$\bar{5}$	$\bar{1}$	ב̄	$\bar{4}$	$\bar{2}$
$\bar{4}$	$\bar{4}$	ב̄	$\bar{2}$	$\bar{1}$	$\bar{5}$	$\bar{3}$
$\bar{5}$	$\bar{5}$	$\bar{1}$	$\bar{3}$	$\bar{2}$	ב̄	$\bar{4}$

≀	ב̄	$\bar{4}$	$\bar{5}$	$\bar{1}$	$\bar{2}$	$\bar{3}$
ב̄	ב̄	$\bar{2}$	$\bar{1}$	$\bar{5}$	$\bar{4}$	$\bar{3}$
$\bar{1}$	$\bar{1}$	$\bar{3}$	$\bar{2}$	ב̄	$\bar{5}$	$\bar{4}$
$\bar{2}$	$\bar{2}$	$\bar{4}$	$\bar{3}$	$\bar{1}$	ב̄	$\bar{5}$
$\bar{3}$	$\bar{3}$	$\bar{5}$	$\bar{4}$	$\bar{2}$	$\bar{1}$	ב̄
$\bar{4}$	$\bar{4}$	ב̄	$\bar{5}$	$\bar{3}$	$\bar{2}$	$\bar{1}$
$\bar{5}$	$\bar{5}$	$\bar{1}$	ב̄	$\bar{4}$	$\bar{3}$	$\bar{2}$

Then (Z_6; $\neg_6$, ≀$_6$, ב) is a ψ-algebra. It is easy to show that $I_1 = \{$ב̄$, \bar{3}\} =< \bar{3} >$ and $I_2 = \{$ב̄$, \bar{2}, \bar{4}\} =< \bar{2} >$ are ψ-

¬	$\bar{2}$	$\bar{3}$	$\bar{1}$	ב̄
ב̄	$\bar{2}$	$\bar{3}$	$\bar{1}$	ב̄
$\bar{1}$	$\bar{3}$	ב̄	$\bar{2}$	$\bar{1}$
$\bar{3}$	$\bar{1}$	$\bar{2}$	ב̄	$\bar{3}$
$\bar{2}$	ב̄	$\bar{1}$	$\bar{3}$	$\bar{2}$

≀	$\bar{1}$	$\bar{3}$	$\bar{2}$	ב̄
ב̄	$\bar{3}$	$\bar{1}$	$\bar{2}$	ב̄
$\bar{1}$	ב̄	$\bar{2}$	$\bar{3}$	$\bar{1}$
$\bar{3}$	$\bar{2}$	ב̄	$\bar{1}$	$\bar{3}$
$\bar{2}$	$\bar{1}$	$\bar{3}$	ב̄	$\bar{2}$

subalgebras of (Z_6; $\neg_6$, ≀$_6$, ב), but the union $I \cup$ J = {ב, 2,3} is not a ψ-subalgebra of X, since

$(2\neg 1) = 3 \in (I \cup \mathrm{J})$, but (2 ≀ 1) $= 1 \notin (I \cup \mathrm{J})$.

Proposition 3.7. assume$\{I_i : i \in \Lambda\}$ the family of ψ-subalgebras on ψ-algebra (X; ¬, ≀, ב), where $I_i \subseteq I_{i\neg 1}$, ∀ i $\in \Lambda$. Then $\bigcup_{i\in\Lambda}^{\infty} I_i$ is a ψ-subalgebra of X.

Proof:

Since $\{I_i : i \in \Lambda\}$ be a family of ψ-subalgebras on X , for any w, ɥ $\in X$, suppose $w\neg z \in \bigcup_{i\in\Lambda}^{\infty} I_i$ and ɥ$\neg z \in \bigcup_{i\in\Lambda}^{\infty} I_i$, ∀i $\in\Lambda$. It follows that $w\neg z \in I_i$, ɥ$\neg z \in I_i$, for some i $\in\Lambda$. By assumption $I_i \subseteq I_k$. Hence $\neg z \in I_k$, ɥ$\neg z \in I_k$, but I_k is a ψ-subalgebra of X , it follows that w ≀ ɥ $\in I_k$, therefore, w ≀ ɥ $\in \bigcup_{i\in\Lambda}^{\infty} I_i$. Hence $\bigcup_{i\in\Lambda}^{\infty} I_i$ is a ψ-subalgebra of X. ⌂

4. Homomorphism of ψ-algebras

We examine and discuss the characteristics of -algebra homomorphism in this section.

Definition 4.1 . assume(X; ¬, ≀, ב) & (Y; ¬′, ≀′, ב′) be ψ - algebras , the mapping f: (X; ¬, ≀, ב) → (Y; ¬′, ≀′, ב′) is called **a homomorphism** if it satisfies: ∀ w, ɥ $\in X$

1- $f(w\neg$ ɥ$) = f(w)\neg' f($ ɥ$)$,
2- $f(w$ ≀ ɥ$) = f(w)$ ≀′ $f($ ɥ$)$,

We specify $(ker\ f)(w) = \{w \in X : f(w) =$ ב′ $\}$.

Theorem 4.2. Let f: (X; ¬, ≀, ב) → (Y; ¬′, ≀′, ב′) be a homomorphsm of ψ-algebras, then:

(A_1) f(ב) = ב′.

(A_2) f is injective ↔ if ker f = {ב}.

(A_3) If $x \leq y \rightarrow f(x) \leq f(y)$.

Proof:

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(A_1) f(ב) = f(ב¬ב) = f(ב)¬′f(ב) = ב′¬′ב′ = ב′ and

f(ב) = f(ב ≀ ב) = f(ב) ≀ ′f(ב) = ב′ ≀ ′ב′ = ב′ , hence (ב) = ב′.

(A_2) Suppose that f is injective and x ∈ ker f.
It follows that

$\rho(w)$ = ב′ . Since f(ב) = ב′, so $f(w)$ = f(ב) . By assumption, x = ב. Thus $(ker f)$ = {ב}.

Conversely, suppose that $(ker f)$ = {ב} . Let w, ɥu ∈ X be such that $f(w)$ = f(ɥu). We get that $f(w$ ≀ ɥu) = $f(w)$ ≀′ f(ɥu) = ב′ and f(ɥu ≀ w) = f(ɥu) ≀′ $f(w)$ = ב′, then w ≀ ɥu, ɥu ≀ w ∈ $(ker f)$, this means that w ≀ ɥu = ב = ɥu ≀ w by Proposition (2.5(a1)), then x = y.

Hence f is injective.

(A_3) Let x ≤ y. It follows that x ≀ y = ב. So, from (A_1) implies

$f(w)$ ≀ ′f(ɥu) = $f(w$ ≀ ɥu) = f(ב) = ב′. Hence $(w) \le f($ ɥu$)$. ⌂

Theorem 4.3. Let f: (X; ¬,≀,ב) → (Y; ¬′,≀′,ב′) be a homomorphism of ψ-algebras, then

$(\mathbf{F_1})$ If S is a ψ-subalgebra of X, then f (S) is a ψ-subalgebra of Y.

$(\mathbf{F_2})$ If K is a ψ-subalgebra of Y, then $f^{≀1}$ (K) is a ψ-subalgebra of X .

Proof:

Since f: (X; ¬,≀,ב) → (Y; ¬′,≀′,ב′) is a homomorphism of ψ- algebras,

$(\mathbf{F_1})$ Let S be a ψ-subalgebra of X and a , $b \in S$, since S is a ψ-subalgebra we have $a¬b \in S$ and $a ≀ b \in S$. Then there exist x, y ∈f (S) such that $w = f(a)$ and ɥu = $f(b)$.

Hence $f(a¬b) = f(a)¬′f(b) = w¬′$ ɥu ∈ $f(S)$ and

$f(a ≀ b) = f(a) ≀′ f(b) = w ≀′$ ɥu ∈ $f(S)$.

Thus $f(S)$ is a ψ-subalgebra of X.

$(\mathbf{F_2})$ Let K be a ψ-subalgebra of Y and w, ɥu ∈ $f^{≀1}(K)$.

Let $f^{≀1}(a) = w$ and $f^{≀1}(b)$ = ɥu, for some $a, b \in K$, thus

$f(w¬$ ɥu) = $f(w)¬′f($ ɥu) = $a¬′b \in K$, and

$f(w ≀$ ɥu) = $f(w) ≀′ f($ ɥu) = $a ≀′ b \in K$, as K is a ψ-subalgebra. Thus $w¬$ ɥu ∈ $f^{≀1}(K)$ and $w ≀$ ɥu ∈ $f^{≀1}(K)$.

Hence $f^{≀1}(K)$ is a ψ-subalgebra of X .

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