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# On The Generalized Fuzzy of KK-algebras

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***Abstract**: This paper introduces the notion of a generalized fuzzy q-ideal of a KK-algebra, and investigates several of their fundamental properties. We define big generalized fuzzy q-ideals of KK-algebras. In addition to these findings, several theorems and properties are stated and proven.*

**Keywords—**KK-algebra, generalized fuzzy q-ideals, big generalized fuzzy q-ideals of KK-algebra.

**Introduction** As a significant category of logical algebras, BCK-algebras were first described by K. Is'eki [13] and have since been the subject of much study by epperts in the field. The concept of fuzzy sets was first suggested by L.A. Zadeh [14], who also described some of its characteristics. Fuzzy set theory was applied to BCK-algebras by J. Meng and Y. B. Jun in [11]. More characteristics of fuzzy BCK-algebras and fuzzy ideals were eppfored by K. B. Lee, Y.B. Jun, and M. I. Doh, who then introduced fuzzy translations and fuzzy multiplications of BCK/BCI-algebras ([12]). The concept of KUS-ideals in KUS-algebras was first presented by S.M. Mostafa and A.T. Hameed [16], and the concept of fuzzy KUS-subalgebras by A.T. Hameed [9]. In [15]. Several scholars have eptensively studied KK-algebras, a significant class of logical algebras developed by A.T. Hameed and B.N. Abbas [3,4,5]. Fuzzy Q-ideal translation in AB-algebra was discussed by A.T. Hameed and el at. Fuzzy translation of fuzzy QS-ideal and fuzzy multiplication of fuzzy QS-ideal in QS-algebra were studied by A.T. Hameed and A.K. Alkurdi in [1,2]. Fuzzy CI-ideal translation and multiplication were studied by A.T. Hameed and N.Z. Mohammed in [6]. Several theorems and features of large generalized fuzzy Q-ideals of KK-algebras are outlined by A.T. Hameed and el at in [7,8].

In this paper, we introduce the notions of generalized fuzzy KK-subalgebras and generalized fuzzy q-ideals of KK-algebras and gave some properties of it. Also, we introduce the notions of big generalized fuzzy KK-subalgebras and big generalized fuzzy q-ideals of KK-algebras and gave some theorems and properties of it.

## 2. Preliminaries

Now, we give some definitions and preliminary results needed in the later sections.

**Definition 2.1([1,2]).**

Let $(X; *, ב)$ be *an algebra with a binary* operation $*$ *and a nullary operation* 0. Then $X$ is called a **KK-algebra** *if it satisfies the following*: for all $,y,z \in X$,

$(KK_1)$ : $(p * y) * ((y * z) * (p * z)) = ב$ ,
$(KK_2)$ : $ב * p = p$ ,
$(KK_3)$ : $p * y = ב$ and $y * p = ב$ if *and only* if $p = y$.

**Definition 2.2 ([1,2]).**

*Define a binary relation* $\leq$ on $KK - algebra$ $(X; *, ב)$ by letting $p \leq y$ if and only if $y * p = 0$.

**Example 2.3 ([3]).**

Let $*$ be *defined on an abelian* group G by letting $x * y = x^{-1} \cdot y$, where $x, y \in G$, *with e is unity element of* G. Then $(G; \cdot, e)$ is a $KK - algebra$.

**Example 2.4([1,2]).**

Let $X = \{ב, 1\}$ and let $*$ be defined by:

| $*$ | ב | 1 |
|---|---|---|
| ב | ב | 1 |
| 1 | 1 | ב |

Then $(X; *, ב)$ is a KK-algebra.

**Proposition 2.5 ([3,4]).**

*In any* $KK - algebra$ $(X; *, ב)$, the *following properties* hold: for all $x, y, z \in X$

$(P_1)$ $p * ((p * y) * y) = ב$,
$(P_2)$ $p * p = ב$,
$(P_3)$ $p * (y * z) = y * (p * z)$,
$(P_4)$ $((p * y) * y) * y = p * y$,
$(P_5)$ $(p * y) * ב = (p * ב) * (y * ב)$,
$(P_6)$ $(p * y) * ((z * p) * (z * y)) = ב$,
$(P_7)$ If $p \leq y$, then $y * z \leq p * z$,
$(P_8)$ If $p \leq y$, then $z * p \leq z * y$.

**Definition 2.6([1,2]).**

Let $(P; *, ב)$ be a KK-algebra *and let* $S$ *be a nonempty* subset of X. S is called a **KK-subalgebra of X** if $p * y \in S$ *whenever* $p \in S$ and $y \in S$.

**Definition 2.7([3,4]).**

*A nonempty subset* $I$ of a KK-algebra $(X; *, ב)$ is called **an ideal of X** if it satisfies the *following conditions*: for any $p, y \in X$,

$(I_1)$ $ב \in I$,

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($I_2$) $p * y \in I$ *and* $x \in I$ *imply* $y \in I$.

**Epamples 2.8 ([3,4]).**

Let $X = \{ב, 1, 2, 3\}$ and let $*$ *be defined by the table*

| * | ב | 1 | 2 | 3 |
|---|---|---|---|---|
| ב | ב | 1 | 2 | 3 |
| 1 | ב | ב | 3 | 3 |
| 2 | 3 | 3 | ב | ב |
| 3 | 3 | 2 | 1 | ב |

*Thus, it can be easily shown that* $(X;\ *, ב)$ is a $KK-algebra$. And we see that $I = \{ב, 1\}$ and $J = \{ב, 3\}$ are ideals of $X$.

**Proposition 2.9 ([6]).** Every *ideal of* $KK - algebra$ *is* a KK-subalgebra.

**Proposition 2.8 ([1,2]).** Let $\{I_i \mid i \in \Lambda\}$ be a family of ideals of KK-algebra P. The intersection of any set of ideals of KK-algebra P is also an ideal.

**Definition 2.9 ([8]).** Let $(X;\ *, ב)$ and $(Y;\ *`, ב`)$ be nonempty sets. The mapping $f: (X;\ *, ב) \to (Y;\ *`, ב`)$ is called **a homomorphism** if it satisfies: $f(p * y) = f(p) *` f(y)$, for all $p, y \in X$.

The set $\{p \in P \mid f(p) = ב'\}$ is *called* ***the kernel of*** $f$ denoted by $\ker f$.

**Theorem 2.10 ([1,2]).** Let $f: (X;\ *, ב) \to (Y;\ *`, ב`)$ be a *homomorphism of a* $KK - algebra$ $X$ *into a* $KK - algebra$ $Y$, then :

A. $f(ב) = ב'$.

B. $f$ is *injective if and only if* $\ker f = \{ב\}$.

C. $p \leq y$ implies $f(p) \leq f(y)$.

**Theorem 2.11 ([1,2]).** Let $f: (X;\ *, ב) \to (Y;\ *`, ב`)$ be a homomorphism of a KK-algebra $X$ into a KK-algebra $Y$, then:

($F_1$) If $S$ is a KK-subalgebra of $X$, then $f$ (S) is a KK-subalgebra of $Y$, where $f$ is onto.

($F_2$) *If* $I$ *is ideal* of $X$, then $f$ (I) is ideal of $Y$, where $f$ is onto.

($F_3$) If $H$ is *a* $KK - subalgebra$ of Y, then $f^{-1}$ (H) is a KK-subalgebra of $X$.

($F_4$) If *is ideal of* $Y$, then $f^{-1}$ (J) *is ideal* of $X$.

($F_5$) $\ker f$ is ideal of $X$.

($F_6$) $\mathrm{Im}(f)$ *is a* $KK - subalgebra$ of $Y$.

**Definition 2.12([25]).** Let $(X;\ *, ב)$ be a nonempty set, a fuzzy subset $\mu$ of $X$ is a function $\mu: X \to [ב, 1]$.

**Definition 2.13 ([25]).** Let $X$ *be a nonempty set* and $\mu$ be a fuzzy subset of $(X;\ *, ב)$, for $t \in [ב, 1]$, the set $\mu_t = \{p \in X \mid \mu(p) \geq t\}$ *is called* ***a level subset of*** $\boldsymbol{\mu}$.

**Definition 2.14([1-4]).** Let $(X;\ *, ב)$ be a KK-algebra, a fuzzy subset $\mu$ of $X$ is called a **fuzzy KK-subalgebra of X** if for all $p, y \in X$,

$\mu(p * y) \geq \min\{\mu(p), \mu(y)\}$.

**Definition 2.15([1-4]).** Let $(X;\ *, ב)$ be a KK-algebra, a *fuzzy subset* $\mu$ of $X$ is *called* ***a fuzzy ideal*** **of X** if it satisfies the *following conditions, for* all $x, y, z \in X$,

($FKK_1$) $\mu(ב) \geq \mu(p)$,

($FKK_2$) $\mu(y) \geq \min\{\mu(p * y), \mu(p)\}$.

**Example 2.16([17]).**

Let $X = \{ב, 1, 2, 3\}$ in which ($*$) is defined by the following table:

| * | ב | 1 | 2 | 3 |
|---|---|---|---|---|
| ב | ב | 1 | 2 | 3 |
| 1 | ב | ב | 3 | 3 |
| 2 | 3 | 3 | ב | ב |
| 3 | 3 | 2 | 1 | ב |

Then $(X;\ *, ב)$ is *a* $KK - algebra$. *Define a fuzzy* subset $\mu: X \to [ב, 1]$ by $\mu(x) = \begin{cases} 0.7 & if\ \ x \in \{ב, 1\} \\ 0.3 & otherwise \end{cases}$

$I_1 = \{ב, 1\}$ is ideal of $X$.

*Routine calculation gives* that $\mu$ is a fuzzy ideal of KK-algebras $X$.

**Lemma 2.17([1-4]).** Let $\mu$ be a fuzzy ideal of KK-algebra $(P;\ *, ב)$ and if $p \leq y$, then $\mu(p) \geq \mu(y)$, for all $p, y \in P$.

**Proposition 2.18([1-4]).**

1- *Let* $\mu$ *be a fuzzy* subset *of* $KK - algebra$ $(X;\ *, ב)$. If $\mu$ is a fuzzy KK-subalgebra of $X$ *if and only if for every* $t \in [ב, 1]$, $\mu_t$ *is a* $KK - subalgebra$ of $X$.

2- Let $\mu$ *be a fuzzy ideal of* $KK - algebra$ $(X; *, ב)$, $\mu$ is *a fuzzy ideal* of $X$ if *and only if for every* $t \in [ב, 1]$, $\mu_t$ is an ideal of $X$.

3- Let A be a nonempty subset of a KK-algebra $(X;\ *, ב)$ *and* $\mu$ *be a fuzzy subset* of X *such that* $\mu$ *is into* $\{ב, 1\}$, so that $\mu$ is the characteristic function of A. Then $\mu$ is a fuzzy ideal of $X$ if and only if A is an ideal of .

**Proposition 2.19([1-4]).**

1- The *intersection* of any *set of fuzzy ideals of KK*-algebra is also fuzzy ideal.

2- The union of any set of fuzzy ideals of KK-algebra is also fuzzy ideal where is chain.

**Proposition 2.20([1-4]).** Every fuzzy ideal of KK-algebra is a fuzzy KK-subalgebra.

**Definition 2.21 ([22]).** Let $f: (X;\ *, ב) \to (Y;\ *`, ב`)$ be a *mapping nonempty* sets $X$ and $Y$ *respectively*. If $\mu$ is a *fuzzy subset* of $X$, then the *fuzzy* subset $\beta$ of $Y$ *defined* by: $f(\mu)(y) =$

$$\begin{cases} \sup\{\mu(x): x \in f^{-1}(y)\} & if\ \ f^{-1}(y) = \{x \in X, f(x) = y\} \neq \emptyset \\ ב & otherwise \end{cases}$$

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*is said to be* ***the image*** **of** $\mu$ ***under*** $f$.

Similarly if $\beta$ is *a fuzzy subset* of , then the fuzzy subset $\mu = (\beta \circ f)$ of $X$ ( i.e *the fuzzy subset defined by* $\mu(x) = \beta(f(x))$, for all $x \in X$) is *called* ***the pre − image*** **of** $\beta$ **under** $f$ .

**Definition 2.22 ([22]).** A f*uzzy subset* $\mu$ of a set $X$ has sup property if for any subset T of $X$ , there exist $t_0 \in$T such that $\mu(t_0) = sup\{\mu(t)/t \in T\}$.

**Proposition 2.23([5,6]).** Let $f: (X; *, ב) \to (Y; *`, ב`)$ be a *homomorphism between* KK-algebras $X$ and $Y$ r*espectiv*ely.

1- For every fuzzy KK-subalgebra $\beta$ of $Y$, $f^{-1}(\beta)$ is a fuzzy KK-subalgebra of $X$.

2- *For every fuzzy KK − subalgebra* $\mu$ of $X$ , $f(\mu)$ is a fuzzy KK-subalgebra of $Y$, where $f$ is onto.

3- *For every fuzzy idea*l $\beta$ of $Y$, $f^{-1}(\beta)$ is a fuzzy ideal of $X$.

4- For every fuzzy *ideal* $\mu$ *of* $X$ *with sup property,* $f(\mu)$ *is a fuzzy ideal* of $Y$, where $f$ is onto.

**Definition 2.24[1,18]:** Let $X$ be a *nonempty* set and $\mu$ be a fuzzy *subset of* $X$ *and le*t $\alpha \in [ב, T]$. A mapping $\mu_\alpha^T : X \to [ב, 1]$ is called a ***α-translation fuzzy subs*et of** $\mu$ if it satisfies: $\mu_\alpha^T(p) = \mu(p) + \alpha$, for all $x \in X$, whereT $= 1 - \sup\{\mu(p): p \in X\}$.

**Definition 2.25([6]).** Let $(X; *, ב)$ *be a KK − algebra* , a fuzzy subset μ in X is called **a fuzzy** $q-$ ***ideal of P*** *if it satisfies* the following conditions: , for all $x, y, z \in X$ ,

(1) $\mu(ב) \geq \mu(x)$ ,

(2) μ (p*z) ≥ min {μ ((p*y)*z), μ (y)} .

**Lemma 2.26([6]).** Let $\mu$ be a fuz*zy q − ideal of KK −*algebra $(X; *, ב)$ and if $x \leq y$, then $\mu$ (p) ≥ $\mu$ (y), for all p, y∈ P.

**Theorem 2.27([6]).** *Let A be a nonempty subset* of a KK-*algebra* $(X; *, ב)$ and $\mu$ be *a fuzzy subset* of $X$ such that μ is into {ב, 1} , so that $\mu$ is the characteristic function of $A$ . Then $\mu$ is a fuzzy q-ideal in P if and only if , $A$ is a q-ideal of $X$ .

***Theorem*** **2.28**([**6**]). *Let* $\mu$ *be a fuzzy q*-ideal *f KK − algebra* $(X; *, ב)$. μ is a fuzzy q-ideal of $P$ *if and only* if , *for every* t ∈[ב,1] , $\mu_t$ is a q-ideal of $X$ .

**Proposition 2.29[5].** *Let* $\mu$ *be a fuzzy* q-*ideal of KK − algebra* $(X; *, ב)$. $\mu$ *is a fuzzy q − ideal of* P if and only if , for every t ∈[ב,1] , $\mu_t$ is a q-ideal of $X$ .

**Proposition 2.30([6]).** Every fuzzy q-ideal of KK-algebra $(X; *, ב)$ is a fuzzy ideal of $X$**.**

**Theorem 2.31([6]).** Le*t A be a q − ideal of KK − a*lgebra $(X; *, ב)$. *Then for any fixed number* (t) in an open interval (ב,1), there epists a fuzzy q-ideal μ of X such that $\mu_t = A$ .

Now, w*e give α − translation* on KK-algebras and β-*magnified on KK − algebras*.

*In what follows, let* $(X; *, ב)$ *denote a KK − algebra, and* for any fuzzy subset $\mu$ of $X$, we denote $T = 1 - sup\{\mu(p) \mid p \in X\}$.

**Definition 2.32([6]).** Let μ be a fuzzy subset of a KK-algebra P and let α ∈[ב,T] . A mapping $\mu_\alpha^T$ :P→[ב,1] is called a **α-translation** of μ if it satisfies: $\mu_\alpha^T$,(p) = $\mu$ (p)+α, for all $p \in X$.

**Definition 2.33([6]).** Let $(X; *, ב)$ be a KK-algebra, a fuzzy subset $\mu$ of $X$ is called a **α-translation fuzzy KK-subalgebra** of $X$, if for all $x, y \in X$, $\mu_\alpha^T(p * y) \geq \min\{\mu_\alpha^T(p), \mu_\alpha^T(y)\}$.

**Definition 2.34([6]).** For a *fuzzy subset* $\mu$ *of an KK − algebra* $(X; *, ב)$, α ∈ [ב,T] and t∈ Im(μ) with t ≥ α, then

$\mathbf{U_\alpha}$ **(μ; t)** = {p ∈ $X$ | $\mu$ (p) ≥ t − α }.

**Definition 2.35([6]).** Let $(X; *, ב)$ *be a KK − algebra, a* α-*translation fuzzy su*bset μ of $X$ is *called a* ***α − translation*** **fuzzy q-ideal** of $X$ if it satisfies the following *conditions*: *for all* $p, y, z \in X$ ,

(1) $\mu_\alpha^T(ב) \geq \mu_\alpha^T$ (p) ,

(2) $\mu_\alpha^T(p * z) \geq \min\{\mu_\alpha^T((p * y) * z), \mu_\alpha^T(y)\}$.

**Definition 2.36([6]).** Let $\mu$ be a fuzzy subset of a KK-algebra $(X; *, ב)$ *and let* β ∈(ב,1] *A mapping* $\mu_\beta^M$:P→[ב,1] *is called* a **β-magnified** of $\mu$ if it satisfies:

$\mu_\beta^M$(p) = β . $\mu$ (p), for all $p \in X$.

**Definition 2.37([6]).** Let $(X; *, ב)$ be a KK-algebra, a fuzzy subset $\mu$ of X is called a **β-m*agnified fuzzy KK − subalgebra*** of $X$ , if for all$p, y \in X$, $\mu_\beta^M(p * y) \geq \min\{\mu_\beta^M(p), \mu_\beta^M,(y)\}$.

**Definition** 2.38([6]). For a fuzzy subset μ of a KK-algebra $(X; *, ב)$, β ∈ (ב,1] and t∈ Im(μ) with t ≤ β, then $\mathbf{U_\beta}$ **(μ; t)** = {p ∈ $X$ | $\mu$(p) ≥ t/β }.

**Definition 2.39([6]).** Let $(X; *, ב)$ *be a KK − algebra, a* α-*translation fuzzy subset* $\mu$ of $X$ is called a **β-magnified fuzzy q-ideal** of $X$ if it satisfies the *following conditions*: for all $p, y, z \in X$ ,

(1) $\mu_\beta^M(ב) \geq \mu_\beta^M(p)$ ,

(2) $\mu_\beta^M(p * z) \geq \min\{\mu_\beta^M((p * y) * z), \mu_\beta^M(y)\}$.

### *3. Generalized fuzzy q-ideals*

In this section, we will discuss and investigate new notions called generalized fuzzy q-ideal of KK-algebras and study several basic properties of them.

**Definition 3.1:**

Let μ be fuzzy *ubset of a KK − algebra* $(X; *, ב)$, λ ,δ ∈(ב,1] and λ < δ. $\mu$ *is said to be* ***a generalized fuzzy*** **KK-subalgebra of** ***P***, *if for all* $p, y \in X$, $\mu(p * y) \vee \lambda \geq \mu(x) \wedge \mu(y) \wedge \delta$.

**Definition 3.2 :**

For a fuzzy subset μ of a KK-algebra (P;∗,ב), α ∈[ב,T], β ∈ (ב,1] and t∈ Im(μ) with t ≤ β, then $\boldsymbol{U_{(\beta,\alpha)}}$ **(μ; t)** = {p ∈ $X$ | $\mu$ (p) ≥ (t − α)/β }.

**Definition 3.3:**

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*Let* $\mu$ *be fuzzy subset* of an KK-algebra $(X;\ *,ב)$, $\lambda,\delta \in (ב,1]$ and $\lambda < \delta$ **.** $\mu$ is said to *be* ***a generalized fuzzy* q-ideal of *X***, *if for all* $p, y, z \in X$,

G1) $\mu(ב) \vee \lambda \geq \mu(p) \wedge \delta$,

G2) $(p*z) \vee \lambda \geq \mu((p*y)*z) \wedge \mu(y) \wedge \delta$.

**Proposition 3.4:**

Let $\mu$ be a generalized fuzzy q-ideal of an KK-algebra $(X;\ *,ב)$. If the inequality $y = ב$, then $\mu(p*z) \vee \lambda \geq \mu(p) \wedge \delta$.

**Proof**.

For all $p, y, z \in X$, $\mu(p*z) \vee \lambda \geq \mu((p*y)*z) \wedge \mu(y) \wedge \delta$, by Definition (3.6).

But y=ב, then we have $\mu(p*z) \vee \lambda \geq \mu(p) \wedge \delta$. ■

**Proposition 3.5:**

Let $\mu$ be a *generalized fuzzy* q-ideal *of an* $KK-$ *algebra* $(X;\ *,ב)$. If the inequality $x*y \leq z$, for all $x, y, z \in X$ *hold* of $X$, then $\mu(p*z) \vee \lambda \geq \mu(y) \wedge \delta$.

**Proof**.

*For all* $p, y, z \in X$, $\mu(p*z) \vee \lambda \geq \mu((p*y)*z) \wedge \mu(y) \wedge \delta$, *by Definition* (3.3). But $p*y \leq z$, we have $(p*y)*z = ב$, *then we have* $\mu(p*z) \vee \lambda \geq \mu(y) \wedge \delta$. ■

**Proposition 3.6:**

*Let* $\mu$ *be a fuzzy* subset of *an* $KK-$ *algebra* $(X;\ *,ב)$, then $\mu$ is a generalized fuzzy q-ideal of $X$ if and *only if, for all* $t \in (\lambda,\delta]$, $\alpha \in [ב,T]$, $\beta \in (ב,1]$, $U_{(\beta,\alpha)}(\mu;\ t)$ is an q-ideal of $X$.

**Proof.**

**(⇒)** Suppose that $\mu$ *is a generalized fuzzy* $q-$ *ideal* of $X$ and $U_{(\beta,\alpha)}(\mu; t) \neq \emptyset$, for any $t \in (\lambda,\delta]$, $\alpha \in [ב,T]$, $\beta \in (ב,1]$. It is clear that $ב \in U_{(\beta,\alpha)}(\mu; t)$.

Let $x, y, z \in X$ be *such that* $((p*y)*z) \in U_{(\beta,\alpha)}(\mu; t)$ and $y \in U_{(\beta,\alpha)}(\mu; t)$, then $\mu((p*y)*z) \geq (t-\alpha)/\beta$, and $\mu(y) \geq (t-\alpha)/\beta$.

*It follows from* (G2) that

$\mu(x*z) \vee \lambda \geq \mu((x*y)*z) \wedge \mu(y) \wedge \delta \geq t$.

Namely, $\mu(x*z) \geq (t-\alpha)/\beta$, and $(x*z) \in U_{(\beta,\alpha)}(\mu; t)$.

*This shows that* $U_{(\beta,\alpha)}(\mu; t)$ *is an* $q-$ *ideal of* $X$.

(⇐) Conversely, suppose that for each $t \in (\lambda,\delta]$, $U_{(\beta,\alpha)}(\mu; t)$ is an q-ideal of $X$. *If there exist* $x \in X$, *such that*

$\mu(ב) \vee \lambda < c = \mu(x) \wedge \delta$, then $p \in \mu_c$, $c \in (\lambda,\delta]$ and

$\mu(ב) < c$. Since $\mu_c$ is an q-ideal of P, so $ב \in U_{(\beta,\alpha)}(\mu; c)$ and

$\mu(ב) \geq (c-\alpha)/\beta$. This is a *contradiction with* $\mu(ב) < (c-\alpha)/\beta$.

*Therefore* $\mu(ב) \vee \lambda \geq \mu(p) \wedge \delta$, for all $x \in X$.

*Now, we only* need to show *that* $\mu$ *satisfies* (G2).

Assume $\mu(p*z) \vee \lambda \geq \mu((p*y)*z) \wedge \mu(y) \wedge \delta$ is not true, then *there* exist p', y', z'∈ X, *such that* $\mu(p^\wedge{}'*z') \vee \lambda < \mu((p^\wedge{}'*y^\wedge{}')*z') \wedge \mu(y') \wedge \delta$.

Putting $t` = 1/n\ (\mu(p`*z`) + \min\{\mu((p`*y`)*z'), \mu(y`)\})$ *where* $n \in N$, $n \geq 2$, *then* $\mu(p`*z`) < t`$ and $ב \leq t` < \min\{\mu((p`*y`)*z'), \mu(y`)\} \leq 1$, *hence* $\mu((p`*y`)*z`) > t`$ and $\mu(y`) > t`$, which imply that $((p`*y`)*z`) \in U_{(\beta,\alpha)}(\mu; t')$ and $y` \in U_{(\beta,\alpha)}(\mu; t')$, since $U_{(\beta,\alpha)}(\mu; t')$ is an q-ideal, it follows that $(p`*z`) \in U_{(\beta,\alpha)}(\mu; t')$ and that $\mu(p`*z`) \geq t`$, this is also a *contradiction*. Thus $\mu(p`*z`) \vee \lambda \geq \mu((p`*y`)*z`) \wedge \mu(y`) \wedge \delta \geq t$, *for all* $p, y \in X$.

*Hence* $\mu$ *is a generalize*d fuzzy q-ideal of $X$. ■

**Proposition 3.7:**

*Any generalized* fuzzy q-ideal $\mu$ of an $KK-algebra$ $(X;\ *,ב)$ *must be a* generalized *fuzzy* $KK-$ *subalgebra* of $X$.

**Proof**.

Let $\mu$ *be a generalized fuzzy* q-ideal of $X$, *then by* Proposition (3.3), for all $t \in (\lambda,\delta]$, $\mu_t$ is an q-ideal of $X$. By Proposition (2.11), for all $t \in (\lambda,\delta]$, $\mu_t$ *is an* $KK-$ *subalgebra of* $X$. Hence, by Proposition (3.4), $\mu$ is a generalized fuzzy KK-subalgebra of $X$. ■

In general, the converse of the Proposition (3.7) is not *true. For example*:

**Example 3.8:**

Let $X = \{ב, 1, 2, 3\}$ be an KK-*algebra which is given in example* (3.4).

*Define a fuzzy subsets* $\mu$, $\nu$ of X by:

| P | ב | 1 | 2 | 3 |
|---|---|---|---|---|
| μ | 0.9 | 0.6 | 0.6 | 0.7 |
| ν | 0.9 | 0.6 | 0.7 | 0.6 |
| μ∪ν | 0.9 | 0.6 | 0.7 | 0.7 |

Then $(X;\ *,ב)$ is an KK-algebra.

*Define a fuzzy subset* $\mu: X \to [ב,1]$ by: $\mu(x) = \begin{cases} 0.7 & if\ \ p \in \{ב,1\} \\ 0.3 & otherwise \end{cases}$.

$I = \{ב, 1\}$ *is not* $q-$ *ideal* of $X$, then μ *is not a generalized* fuzzy q-ideal of $X$, but $\mu$ is a *generalized fuzzy* $KK-$ *subalgebra* of $X$.

**Proposition 3.9:**

*Let* $\mu$ *and* $\nu$ *be* two generalized *fuzzy* $q-$ *ideals of* $KK$-algebra $(X;\ *,ב)$, then $(\mu \cap \nu)$ *is also a generalized fuzzy* q-ideal of $X$.

**Proof.**

For all $p, y, z \in X$,

$(\mu \cap \nu)(ב) \vee \lambda = (\mu(ב) \wedge \nu(ב)) \vee \lambda$

$= (\mu(ב) \vee \lambda) \wedge (\nu(ב) \vee \lambda)$

$\geq (\mu(p) \wedge \delta) \wedge (\nu(p) \wedge \delta)$, by (G1).

$= (\mu(p) \wedge \nu(p)) \wedge \delta$

$= (\mu \cap \nu)(p) \wedge \delta$, *and*

$(\mu \cap \nu)(p*z) \vee \lambda = (\mu(p*z) \wedge \nu(p*z)) \vee \lambda$

$= (\mu(p*z) \vee \lambda) \wedge (\nu(p*z) \vee \lambda)$

$\geq (\mu((p*y)*z) \wedge \mu(y) \wedge \delta) \wedge (\nu((p*y)*z) \wedge \nu(y) \wedge \delta)$, by ($G_2$).

$= (\mu((p*y)*z) \wedge \nu((p*y)*z)) \wedge (\mu(y) \wedge \nu(y)) \wedge \delta$

$= (\mu \cap \nu)((p*y)*z) \wedge (\mu \cap \nu)(y) \wedge \delta$.

*Hence* $(\mu \cap \nu)$ is a generalized *fuzzy q − ideal of X*. ■

*The union of generalized* fuzzy *q − ideals of KK − algebra* $(X; *, ב)$ is *not a fuzzy* generalized *fuzzy q − ideals* as seen *in the following example*.

**Example 3.10:**

Let $X=\{ב, 1, 2, 3\}$ *be an KK* −algebra *which is given in example* (3.4).

*Define a fuzzy subset*s $\mu, \nu$ of X by:

| $X$ | ב | 1 | 2 | 3 |
|---|---|---|---|---|
| $\mu$ | 0.9 | 0.6 | 0.6 | 0.7 |
| $\nu$ | 0.9 | 0.6 | 0.7 | 0.6 |

And let $\lambda, \delta \in (ב, 1]$, such that $0.2 = \lambda < \delta = 0.7$, then $\mu, \nu$ are generalized fuzzy q-ideals of KK-algebra $X$.

But the union $\mu \cup \nu$ is not a generalized fuzzy q-ideal, since

$(\mu\cup\nu)(1*2) \vee \lambda = (\mu\cup\nu)(3) \vee 0.2 = 0.6 < 0.7$

$= (\mu\cup\nu)((1*3)*2) \wedge (\mu\cup\nu)(2) \wedge \delta$

$= (\mu\cup\nu)(1*1) \wedge (\mu\cup\nu)(2) \wedge 0.7$

$= (\mu\cup\nu)(ב) \wedge (\mu\cup\nu)(2) \wedge 0.7$.

## 4. *Big generalized fuzzy q-ideals*

In this part, we address the ideas of big generalized fuzzy q-ideals of KK-algebras. Also, we included some interesting examples. This comprises the results and various theorems and properties which have been obtained and proved.

**Definition 4.1:**

*Let μ be a fuzzy* subset of *an KK − algebra* $(X; *, ב)$ and let $\lambda, \delta, \beta \in (ב,1]$, $\alpha \in [ב,T]$ and $\lambda<\delta$, $\mu$ is called a **Big generalized fuzzy KK-subalgebra** of P, if for all p, $y \in X$, $\mu^C_{(\beta,\alpha)}(p*y) \vee \lambda \geq \mu^C_{(\beta,\alpha)}(x) \wedge \mu^C_{(\beta,\alpha)}(y) \wedge \delta$. (i.e., $(\beta.\mu(p*y)+\alpha) \vee \lambda \geq (\beta.\mu(p)+\alpha) \wedge (\beta.\mu(y)+\alpha) \wedge \delta$).

**Definition 4.2:**

Let $\mu$ be a fuzzy subset of an KK-algebra $(X; *, ב)$ and let $\lambda, \delta \in (ב,1]$ and $\lambda<\delta$, $\alpha \in [ב,T]$, $\beta \in (ב,1]$.

*μ is called a* ***Big* generalized fuzzy q-ideal** of *X if for all* $p, y, z \in X$

(1) $\mu^C_{(\beta,\alpha)}(ב) \vee \lambda \geq \mu^C_{(\beta,\alpha)}(x)$,

(2) $\mu^C_{(\beta,\alpha)}(p*z) \vee \lambda \geq \mu^C_{(\beta,\alpha)}(p*(y*z)) \wedge \mu^C_{(\beta,\alpha)}(y) \wedge \delta$.

i.e.,

($B_1$) $(\beta.\mu(ב)+\alpha) \vee \lambda \geq (\beta.\mu(p)+\alpha) \wedge \delta$,

($B_2$) $(\beta.\mu(p*z)+\alpha) \vee \lambda \geq (\beta.\mu((p*y)*z)+\alpha) \wedge (\beta.\mu(y)+\alpha) \wedge \delta$.

**Proposition 4.3:**

*Any big generalized* fuzzy q-ideal of an KK-algebra $(X; *, ב)$ *must be a generalized fuzzy* q-ideal of P, if $\beta = 1, \alpha=0$.

**Proof:**

Let $\mu$ be a *big generalized fuzzy q − ideal* of $X$, and let $p, y, z \in X$ and $\delta, \lambda \in (ב,1]$, and $\lambda < \delta$, then

($B_1$) $(\beta.\mu(ב)+\alpha) \vee \lambda \geq (\beta.\mu(p)+\alpha) \wedge \delta$,

($B_2$) $(\beta.\mu(p*z)+\alpha) \vee \lambda \geq (\beta.\mu((p*y)*z)+\alpha) \wedge (\beta.\mu(y)+\alpha) \wedge \delta$.

Since $\beta = 1$, $\alpha=ב$, then $\mu(ב) \vee \lambda \geq \mu(p) \wedge \delta$, and $\mu(p*z) \vee \lambda \geq \mu((p*y)*z) \wedge \mu(y) \wedge \delta$.

Hence $\mu$ be a generalized fuzzy q-ideal of $X$. ■

**Theorem 4.3:**

Let $\sqrt[3]{}(X; *, ב)$. Then $\mu$ is a *big generalized fuzzy q − ideal* of $X$ if and only if, $\forall t \in \mathrm{Im}(\mu)$, $t > \alpha$, $\alpha \in [ב,T]$, $\beta \in (ב,1]$ imply $U_{(\beta,\alpha)}(\mu; t)$ *is q − ideal* of $X$, where $\beta \neq 0$.

**Proof:**

($\Rightarrow$)

*Suppose that μ is a big generalized fuzzy q* −ideal of $X$, let $t \in \mathrm{Im}(\mu)$ *be such that* $t > \alpha$. Since $(\beta.\mu(ב)+\alpha) \vee \lambda \geq (\beta.\mu(p)+\alpha) \wedge \delta$, for all $p \in X$, *we have* $(\beta.\mu(ב)+\alpha) \vee \lambda \geq (\beta.\mu(p)+\alpha) \wedge \delta \geq t$, for all $p \in U_{(\beta,\alpha)}(\mu; t)$.

Hence $ב \in U_{(\beta,\alpha)}(\mu; t)$ and $U_{(\beta,\alpha)}(\mu; t) \neq \emptyset$, *for any* $t \in (\lambda, \delta]$.

*Now, assume that there exist* $p, y, z \in P$ *such that* $\beta.\mu((p*y)*z)+\alpha \geq t$ and $\beta.\mu(y)+\alpha \geq t$, then $((p*y)*z) \in U_{(\beta,\alpha)}(\mu; t)$, and $y \in U_{(\beta,\alpha)}(\mu; t)$, *since μ is a big generalized fuzzy q − ideal of X, it follows that*

$(\beta.\mu(p*z)+\alpha) \vee \lambda \geq (\beta.\mu((p*y)*z)+\alpha)) \wedge (\beta.\mu(y)+\alpha) \wedge \delta \geq t$, that is $\mu(p*z) \geq t$, then $(p*z) \in U_{(\beta,\alpha)}(\mu; t)$. *Therefore* $U_{(\beta,\alpha)}(\mu; t)$ *is an q − ideal* of $X$.

($\Leftarrow$) *Conversely, suppose* that $U_{(\beta,\alpha)}(\mu; t)$ is q-ideal of $X$, *for every* $t \in \mathrm{Im}(\mu)$ with $t > \alpha$. If there exists $p \in X$ *such that*

$(\beta.\mu(ב)+\alpha) \vee \lambda < c = (\beta.\mu(p)+\alpha) \wedge \delta$, then $p \in U_{(\beta,\alpha)}(\mu; t)$ and

$ב \notin U_{(\beta,\alpha)}(\mu; t)$. *This is a contradiction*, and so

$(\beta.\mu(ב)+\alpha) \vee \lambda > c = (\beta.\mu(p)+\alpha) \wedge \delta$, for all $p \in X$.

*Now, assume that there exist* $p, y, z \in X$ such that $(\beta.\mu(p*z)+\alpha) \vee \lambda < c = (\beta.\mu((p*y)*z)+\alpha) \wedge (\beta.\mu(y)+\alpha) \wedge \delta \geq t$. Then $((p*y)*z) \in \mu_t$ and $y \in \mu_t$, but $(p*z) \notin \mu_t$.

Hence $((p*y)*z) \in U_{(\beta,\alpha)}(\mu; t)$, $t \in (\lambda, \delta]$ and $y \in U_{(\beta,\alpha)}(\mu; t)$, but

$(p*z) \notin U_{(\beta,\alpha)}(\mu; t)$. *This is a contradiction, therefore* $(\beta.\mu(p*z)+\alpha) \vee \lambda \geq (\beta.\mu((p*y)*z)+\alpha) \wedge (\beta.\mu(y)+\alpha)) \wedge \delta$.

*Hence μ is a big* generalized *fuzzy q − ideal* of $X$. ■

**Corollary 4.4:**

Let $\mu$ be a *uzzy subset of an KK − algebra* $(X; *, ב)$. *Then μ is a big generalized* fuzzy *q − ideal of X if and only* if, for all $t \in (\lambda, \delta]$, $\mu_t$ *is an q − ideal* of $X$.

**Proof:**

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By Theorem (4.4), *and by Proposition* (4.3) when $\beta = 1$, $\alpha=0$. Then $\mu$ is a *big generalized fuzzy* $q-$*ideal* of $X$ if and only if, $\mu_t$ is an q-ideal of $X$. ■

**Proposition 4.5:**

*Any big generalized* fuzzy q-*ideal of an KK*$-$*algebra* $(X; *, ב)$ must be a big generalized fuzzy KK-subalgebra of $X$.

**Proof.** Let $\mu$ be a big *generalized fuzzy* $q-$*ideal* of $X$, *then by Proposition* (4.3), for every $t \in \mathrm{Im}(\mu)$, $t > \alpha$, $U_{(\beta,\alpha)}(\mu; t)$ is an q-ideal of X. *By Proposition* (2.11), for every $t \in \mathrm{Im}(\mu)$, $t > \alpha$, $U_{(\beta,\alpha)}(\mu; t)$ *is an KK*$-$*subalgebra of* $X$. Hence $\mu$ is a *big generalized* fuzzy KK-subalgebra of $X$ by Proposition (4.4). ■

In general, the *converse* of the *Proposition* (4.4.5) is not true. For example:

**Example 4.6:**

Let $X = \{ב, 1, 2, 3\}$ be an KK-*algebra which is given in* example (2.2).

Then $(X; *, ב)$ is an AB-*algebra*. *Define a fuzzy subset* $\mu : X \to [ב, 1]$ by $\mu(p) = \begin{cases} 0.7 & if\ x \in \{ב,1\} \\ 0.3 & otherwise \end{cases}$.

$I = \{ב,1\}$ is not an q-ideal of $X$, then $\mu$ is not a big generalized fuzzy q-ideal of $X$, but $\mu$ is a big generalized fuzzy KK-subalgebra of $X$.

**Proposition 4.7:**

*Let* $\mu$ *and* $\nu$ *be two* big generalized fuzzy q-ideals of an KK-algebra $(X; *, ב)$, then $(\mu \cap \nu)$ is *also a big generalized* fuzzy q-ideal of $X$.

**Proof.**

For all $p, y, z \in X$,

$(\beta.(\mu \cap \nu)(ב) + \alpha) \vee \lambda = (\beta.[\mu(ב) \wedge \nu(ב)] + \alpha) \vee \lambda$
$= [(\beta.\mu(ב) + \alpha) \wedge (\beta.\nu(ב) + \alpha)] \vee \lambda$
$= ((\beta.\mu(ב) + \alpha) \vee \lambda) \wedge ((\beta.\nu(ב) + \alpha) \vee \lambda)$
$\geq ((\beta.\mu(p) + \alpha) \wedge \delta) \wedge ((\beta.\nu(p) + \alpha) \wedge \delta)$
$= ((\beta.\mu(p) + \alpha) \wedge (\beta.\nu(p) + \alpha)) \wedge \delta$
$= (\beta.[\mu(p) \wedge (\nu(p)] + \alpha) \wedge \delta$
$= (\beta.(\mu \cap \nu)(p) + \alpha) \wedge \delta$, and

$(\beta.(\mu \cap \nu)(p * z) + \alpha) \vee \lambda = (\beta.[\mu(p * z) \wedge \nu(p * z)] + \alpha) \vee \lambda$
$= ((\beta.\mu(p * z) + \alpha) \vee \lambda) \wedge ((\beta.\nu(p * z) + \alpha) \vee \lambda$
$\geq ((\beta.\mu((p * y) * z) + \alpha) \wedge (\beta.\mu(y) + \alpha) \wedge \delta) \wedge ((\beta.\nu((p * y) * z) + \alpha) \wedge (\beta.\nu(y) + \alpha) \wedge \delta)$
$= ((\beta.\mu((p * y) * z) + \alpha) \wedge (\beta.\nu((p * y) * z) + \alpha)) \wedge ((\beta.\mu(y) + \alpha) \wedge (\beta.\nu(y) + \alpha)) \wedge \delta$
$= (\beta.[\mu((p * y) * z) \wedge \nu((p * y) * z)] + \alpha) \wedge (\beta.[\mu(y) \wedge \nu(y)] + \alpha) \wedge \delta$
$= (\mu \cap \nu)((p * y) * z) \wedge (\mu \cap \nu)(y) \wedge \delta$.

Hence $(\mu \cap \nu)$ is *big generalized fuzzy* $q-$*ideal of* $X$. ■

*The union of big generalized* fuzzy q-ideals of KK-algebra $(X; *, ב)$ is not a big fuzzy generalized fuzzy q-ideals as seen in the following example.

**Example 4.8:**

Let $X = \{ב, 1, 2, 3\}$ be an KK-algebra *which is given in* example (4.1.4). *Define a fuzzy subsets* $\mu, \nu$ of X by:

| P | ב | 1 | 2 | 3 |
|---|---|---|---|---|
| μ | 0.9 | 0.7 | 0.6 | 0.6 |
| ν | 0.9 | 0.6 | 0.7 | 0.6 |
| μ∪ν | 0.9 | 0.7 | 0.7 | 0.6 |

Then $\mu, \nu$ is not a big *generalized fuzzy* $q-$*ideals of KK*-algebra $(X; *, ב)$.

*But the union* $\mu\cup\nu$ is not a *big generalized fuzzy* $q-$*ideal*, since let $\lambda$, $\delta \in (ב, 1]$, such that $0.2 = \lambda < \delta = 0.35$, then $\mu, \nu$ are big generalized fuzzy KK-subalgebras of $X$. If $\beta = 0.5$ and $\alpha = 0$.

But the union $\mu\cup\nu$ is not a *big generalized fuzzy* KK-subalgebra, since

$(\beta.(\mu \cup \nu)(3 * 2) + \alpha) \vee \lambda = 0.5(\mu \cup \nu)(1) + ב) \vee 0.2 = 0.3 \vee 0.2 = 0.3$
$< 0.35 = \beta.(\mu \cup \nu)((3 * ב) * 2) + \alpha) \wedge \beta.(\mu \cup \nu)(2) + \alpha) \wedge \delta$
$= 0.35 \wedge 0.35 \wedge 0.35$.

**Proposition 4.9:**

*Let* $I$ *and* $J$ *be nonempty* subsets of $KK-$*algebra* $(X; *, ב)$ such that $I \subseteq J$. If $I$ is an q-ideal of $X$, then $J$ is an q-ideal of $X$.

**Proof.**

*To prove that* J is an q-ideal of $X$. *It suffices to show* that J is an $q-$*ideal of* $X$. Since $ב \in I$ and $I \subseteq J$, then $ב \in J$.

Let $x, y, z \in X$ *be such that* $((p * y) * z) \in I$ and $y \in I$, then $(p*z) \in I$, but $I \subseteq J$, then $((p * y) * z) \in J$ and $y \in J$, then $(p * z) \in J$. Hence J *is an* $q-$*ideal* of $X$. ■

**Proposition 4.10:**

Let $\mu$ and $\nu$ *be two fuzzy* q-*ideals of an KK*$-$*algebra* $(X; *, ב)$ such that $\mu \subseteq \nu$. If $\mu$ *is a big generalized* fuzzy q-ideal of $X$, then so is $\nu$.

**Proof.**

To prove that $\nu$ is a big generalized fuzzy q-ideal of $X$, it suffices to show that for any $t \in \mathrm{Im}(\mu)$, $t > \alpha$ imply $U_{(\beta,\alpha)}(\nu; t)$, where $\beta \neq ב$ is an Q-ideal of $X$.

*Since the level subset* $U_{(\beta,\alpha)}(\mu; t)$ *is nonempty, then* $U_{(\beta,\alpha)}(\nu; t) \neq \emptyset$ and $U_{(\beta,\alpha)}(\mu; t) \subseteq U_{(\beta,\alpha)}(\nu; t)$. By the *hypothesis*, $\mu$ is *a big generalized* fuzzy q-ideal of $X$, it follows *from Proposition* (4.3), then $U_{(\beta,\alpha)}(\mu; t)$ is an q-ideal of $X$. *By Proposition* (4.9), $U_{(\beta,\alpha)}(\nu; t)$ *is also an* $q$-ideal of $X$.

Hence ν is a $big\ generalized\ fuzzy\ an$ q-ideal of $X$ by $Proposition$ (4.3). ■

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