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Analysis of Type II Censored Exponentially Distributed Survivor and Hazard Functions on Chronic Granulomatous Disease Data with Placebo and Interferon Gamma Treatment Methods

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Abstract: *Chronic granuloma is a disease associated with primary immunodeficiency that affects the phagocytes of the innate immune system. Several treatment methods have been investigated, including the administration of Placebo and Interferon Gamma. Placebo in the medical world, commonly referred to as a fake drug made similar to the original drug. Interferon as one of the treatment methods that works with the immune system to fight certain diseases. In general, interferons are proteins produced naturally in parts of the body`s immune system that prevent viruses from multiplying. Based on the description above, a study can be conducted to test the survival of GCD patients who are given treatment with Placebo and Interferon Gamma drugs until they recover or die. Based on the results, the proportion of chronic granuloma patients treated with Interferon Gamma (rIFN-g) who survive up to two months is 0.956243, while the proportion of chronic granuloma patients treated with Placebo who survive up to two months is 0.88475. So, it can be concluded that the Interferon Gamma (rIFN-g) treatment method is more effective in healing chronic granuloma patients compared to the Placebo treatment method.*

Keywords— Chronic Granuloma, Interferon Gamma (rIFN-g), Placebo, survive

1. INTRODUCTION

Chronic granuloma is a disease associated with primary immunodeficiency that affects the phagocytes of the innate immune system. Chronic Granulomatous Disease (CGD) is caused by mutations in one of the four genes that encode the phagocyte oxidizing NADPH sub-unit, an enzyme that produces microbicidal and pro-inflammatory oxygen radicals. In the last 50 years, since its first discovery in 1954, the disease has changed from a fatal disease with early complications to a chronic disease with increased survival rates.

Several treatment methods have been investigated, including the administration of Placebo and Interferon Gamma. Placebo in the medical world, commonly referred to as a fake drug made similar to the original drug. This drug is often called an empty drug because it does not contain ingredients that affect health at all. The form of Placebo can vary, such as tablets, capsules, or injectable liquids that do not contain medical ingredients. Placebo administration activities aim to test the effectiveness of certain drugs or medical treatments by giving fake drugs that actually have no medical effects.

Interferon as one of the treatment methods that works with the immune system to fight certain diseases. In general, interferons are proteins produced naturally in parts of the body`s immune system that prevent viruses from multiplying. There are 3 types of interferons: interferon-alpha, beta, and gamma. Interferon-alpha and beta are responsible for giving warning signals to the immune system which then triggers the immune system to release interferon-gamma which will fight the virus or disease. The treatment method with interferon-gamma will help the immune system to fight and stop infection from viruses in patients with chronic granuloma disease.

Based on the description above, a study can be conducted to test the survival of GCD patients who are given treatment with Placebo and Interferon Gamma drugs until they recover or die.

2. Literature Review

2.1 Life Test Data Analysis

Survival analysis is a method used to describe the analysis of time-related data, starting from the known time of the beginning of the study, to the time of an event or the end of the study. An event that occurs can include various things, such as death, illness, recurrence, new disease, accident, response to an experiment, or other events of interest to the researcher (Kleinbaum & Klein, 2005).

According to (Collet, 1994) if T is a random variable that represents the *survivor* time and has a probability distribution function, then the cumulative distribution function (CDF) (T) is the probability distribution function. $f(t)$, then the cumulative distribution function (CDF) is expressed as follows:

$$F(t) = P(T < t)$$

or

$$F(t) = \int_0^t f(x)\, dx \text{ , for } t > 0$$

2.2 Survivor Function

The survival function or survivor function is the probability of an individual surviving beyond t time. The survivor function is denoted by $S(t)$ and is formulated as follows (Lawless, 1982).

$$F(t) = P(T \geq t)$$

Or

$$S(t) = \int_t^\infty f(x)dx, \text{ for } t > 0$$

Survivor function $S(t)$ has a relationship with the cumulative distribution function (CDF) which is formulated as:

$$S(t) = 1 - F(t)$$

Thus, it can also be obtained

$$S'(t) = -f(t)$$

2.3 Failure Function (Hazard Function)

The failure function or hazard function is the probability of an individual dying within an interval of time t to $t + \Delta t$, if it is known that the individual can still survive until time t (Francis & Lawless, 1983). The hazard function is defined as follows:

$$h(t) = \lim_{\Delta t \to 0} \left[\frac{P(t \leq T < t + \Delta t | T \geq t}{\Delta t} \right]$$

The hazard function has a relationship with the probability distribution, if $f(t)$ is the probability density function at time t, then we get:

$$h(t) = \frac{f(t)}{S(t)}$$

2.4 Exponential Distribution

If T is an exponentially distributed random variable with parameters θ, then the exponential probability distribution function (t, θ) is formulated as follows:

$$f(t) = \frac{1}{\lambda} e^{-t/\lambda}, \text{ for } t > 0 \text{ and } \lambda > 0$$

With θ is the average failure time and t is the trial time. Thus, the exponential cumulative distribution function (CDF) is obtained as follows:

$$F(t) = \int_0^t f(x) = 1 - e^{-t/\lambda}$$

Then, the survivor function and hazard function are formulated as follows:

$$S(t) = 1 - F(t) = e^{-t/\lambda}$$

So,

$$h(t) = \frac{f(t)}{S(t)} = \frac{1}{\lambda}$$

2.5 Censorship

Data censoring is one of the important things in the analysis of life test data. Censoring is done to shorten the experiment time in measuring the time of failure or death of individuals, according to (Lawless, 1982) censoring is done because of time and cost limitations.

2.5.1 Type I Sensor

Type I censorship is a statistical sampling method based on a predetermined length of observation. If the sample lifetime exceeds a predetermined length of time, the observation of the sample will be stopped or a censored sample will be obtained.

2.5.2 Type II Sensor

Type II censoring is the type in which the th sample is the smallest observation in a random sample of the same size. r is the smallest observation in a random sample of size n $(1 \leq r \leq n)$. From the total sample, the experiment will be stopped until a certain damage or death is reached (Francis & Lawless, 1983). In other words, the experiment will be terminated if sample ke r Retrieved from n experiences damage or death.

2.6 Mean Time To Failure (MTTF)

Mean Time To Failure (MTTF) is a measure of the average time until something breaks down. The MTTF function value is formulated as follows

$$MTTF = \int_0^\infty S(t)\, dt$$

So, for exponentially distributed data with one parameter, the following formula is obtained

$$\begin{aligned} MTTF &= \int_0^\infty S(t)\, dt \\ &= \int_0^\infty exp\left(-\frac{t}{\lambda}\right) dt \\ &= -\lambda e^{-\frac{\infty}{\lambda}} - \left(-\lambda e^{-\frac{0}{\lambda}}\right) \\ &= \lambda \end{aligned}$$

2.7 Confidence interval for parameters $\hat{\boldsymbol{\lambda}}$

Confidence interval for parameters $\hat{\boldsymbol{\lambda}}$ can be calculated using the following equation:

$$\hat{\lambda}_{min} \leq \hat{\boldsymbol{\lambda}} \leq \hat{\lambda}_{max}$$

with

$$\hat{\lambda}_{min} = \frac{2T}{X^2_{\left(2r\,;\frac{\alpha}{2}\right)}} \quad , \hat{\lambda}_{max} = \frac{2T}{X^2_{\left(2r\,;1-\frac{\alpha}{2}\right)}} \quad ,$$

and $T = \sum_{i=1}^{r} t_{(i)} + (n-r)t_{(r)}$

3. Research Methods

3.1 Data Source

Data sources are everything that can provide information about data. The data used in this study is secondary data obtained from RStudio Datasets with package (survival) data (cgd) which is data on chronic granuloma disease. Data totalling 203 data with 83 data is patient data with interferon Gamma treatment and 120 data is patient data with placebo treatment.

3.2 Population and Sample

Population is a generalization area consisting of objects or subjects that have certain quantities and characteristics set by researchers to study and then draw conclusions (Sugiyono, 2006). The population in this study were all RStudio package (survival) datasets (cgd), data on chronic granuloma patients undergoing treatment with interferon gamma method and placebo method.

The research sample is a part or representative of the population to be studied (Arikunto, 2006). The samples in this study were 20 patient data who were still undergoing treatment with the interferon gamma method and 56 patient data who were still undergoing treatment with the placebo method.

3.3 Research Methods

The type of research used in this study is quantitative research with a type of survivor analysis method and type II censored one-parameter exponentially distributed data. This study was conducted to determine and estimate the survivor function and hazard function of type II censored exponentially distributed data and to analyze the Confidence Interval for the parameter λ.

3.4 Data Analysis Steps

1. Determining the number of censored samples
2. Testing the hypothesis to identify that the data is exponentially distributed
3. Calculating parameter estimates $\hat{\lambda}$
4. Calculating the survivor function $S_{(t)}$
5. Calculating the hazard function $h_{(t)}$
6. Calculating MTTF Value
7. Calculating the Confidence Interval for parameters $\hat{\lambda}$

4. Results and Discussion

4.1 Survival Analysis of Chronic Granuloma Patients with Interferon Gamma (rIFN-g) Treatment Method

The following table shows the survival data of 20 out of 83 interferon gamma (rIFN-g)-treated chronic granuloma patients sorted by shortest survival time

Table 1. Survival data of chronic granuloma patients with Interferon Gamma (rIFN-g) treatment method

Order	1	2	3	4	5
Survival	32	42	65	82	87
Order	6	7	8	9	10
Survival	113	118	122	146	154
Order	11	12	13	14	15
Survival	165	167	187	207	219
Order	16	17	18	19	20
Survival	223	265	267	274	373

By using the following hypothesis:

H0: Chronic Granuloma Patient Data with Interferon Gamma (rIFN-g) treatment method is exponentially distributed

H1: Chronic Granuloma Patient Data with Interferon Gamma (rIFN-g) treatment method is not exponentially distributed.

With the critical region, H0 will be rejected if the p-value or Sig. (2-tailed) is smaller than the 5% alpha significance level.

Table 2. KS Exponential Test of Interferon Gamma (rIFN-g) treatment method

N	20
Asymp. Sig. (2-tailed)	0,181

Based on the output above, the p-value is 0.181 which is greater than alpha 0.05 so that the decision to fail to reject H0 is obtained, which means that the Chronic Granuloma Patient Data with Interferon Gamma (rIFN-g) treatment method is exponentially distributed.

Finding parameter estimators

$$L(\lambda|\underline{t}) = \frac{n!}{(n-r)!}\prod_{i=1}^{r} f(t_{(i)})\, S(t_{(r)})^{n-r}$$

$$= \frac{n!}{(n-r)!}\prod_{i=1}^{r}\frac{1}{\lambda}\exp\left(\frac{-t_{(i)}}{\lambda}\right)\left[\exp\left(\frac{-t_{(r)}}{\lambda}\right)\right]^{n-r}$$

$$= \frac{n!}{(n-r)!}\frac{1}{\lambda^r}\exp\left(-\frac{1}{\lambda}\sum_{i=1}^{r} t_{(i)}\right)\exp\left(\frac{-t_{(r)}}{\lambda}\right)^{n-r}$$

$$ln\, L(\lambda|\underline{t}) = ln\frac{n!}{(n-r)!} + ln\frac{1}{\lambda^r} + ln\left[\exp\left(-\frac{1}{\lambda}\sum_{i=1}^{r} t_{(i)}\right)\right] + ln\left[\exp\left(\frac{-t_{(r)}}{\lambda}\right)^{n-r}\right]$$

$$= ln\frac{n!}{(n-r)!} - r ln\lambda - \frac{1}{\lambda}\sum_{i=1}^{r} t_{(i)} - (n-r)\frac{t_{(r)}}{\lambda}$$

$$\frac{d\, ln\, L(\lambda|\underline{t})}{d\,\lambda} = 0 - \frac{r}{\lambda} + \frac{\sum_{i=1}^{r} t_{(i)}}{\lambda^2} + \frac{(n-r)t_{(r)}}{\lambda^2}$$

$$0 = -\frac{r}{\lambda} + \frac{\sum_{i=1}^{r} t_{(i)}}{\lambda^2} + \frac{(n-r)t_{(r)}}{\lambda^2}$$

$$\frac{r}{\lambda} = \frac{\sum_{i=1}^{r} t_{(i)} + (n-r)t_{(r)}}{\lambda^2}$$

$$r = \frac{\sum_{i=1}^{r} t_{(i)} + (n-r)t_{(r)}}{\lambda}$$

$$\lambda = \frac{\sum_{i=1}^{r} t_{(i)} + (n-r)t_{(r)}}{r}$$

$$\hat{\lambda} = \frac{\sum_{i=1}^{r} t_{(i)} + (n-r)t_{(r)}}{r}$$

So

$$\hat{\lambda} = \frac{(32+42+\cdots+373)+((83-20)373)}{20}$$

$$\hat{\lambda} = \frac{3308+23499}{20}$$

$$\hat{\lambda} = \frac{26807}{20}$$

$$\hat{\lambda} = 1340{,}35 \approx 1341$$

From the results of these calculations, the parameter point estimate value for chronic granuloma patients with interferon gamma (rIFN-g) treatment is obtained $\hat{\lambda}$ which is 1341

The probability density function (PDF) of the type II censored exponential distribution for the data of chronic granuloma patients treated with Interferon Gamma (rIFN-g) is as follows:

$$f(t) = \frac{1}{\lambda}\exp\left(\frac{-t}{\lambda}\right)$$

$$f(t) = \frac{1}{1341}\exp\left(\frac{-t}{1341}\right)$$

The cumulative probability density function (CDF) of the above probability density function can be calculated with the following formula

$$F(t) = \int_0^t f(x)\,dx = \int_0^t \frac{1}{1341}\exp\left(\frac{-t}{1341}\right)dx$$
$$= \frac{1}{1341}\int_0^t \exp\left(\frac{-t}{1341}\right)dx$$
$$= 1 - \exp\left(\frac{-t}{1341}\right)$$

The survivor function of the odds density function above can be calculated with the following formula

$$S_{(t)} = \int_t^\infty f(t)dt$$
$$= \int_t^\infty \frac{1}{1341}\exp\left(\frac{-t}{1341}\right)dt$$
$$= \frac{1}{1341}\int_t^\infty \exp\left(\frac{-t}{1341}\right)\frac{d\left(\frac{-t}{1341}\right)}{-\frac{1}{1341}}$$
$$= \frac{1}{1341}(-1341)\int_t^\infty \exp\left(\frac{-t}{1341}\right)d\left(\frac{-t}{1341}\right)$$
$$= -\int_t^\infty \exp\left(\frac{-t}{1341}\right)d\left(\frac{-t}{1341}\right)$$
$$= -\left[\exp\left(\frac{-t}{1341}\right)\right]_t^\infty$$
$$= -\left[0 - \exp\left(\frac{-t}{1341}\right)\right]$$
$$= \exp\left(\frac{-t}{1341}\right)$$

The hazard function of chronic granuloma patient data with Interferon Gamma treatment (rIFN-g) can be calculated with the following formula

$$h(t) = \frac{f(t)}{S(t)} = \frac{\frac{1}{1341}\exp\left(\frac{-t}{1341}\right)}{\exp\left(\frac{-t}{1341}\right)} = \frac{1}{1341} = 0{,}000745712$$

The Mean Time to Failure (MTTF) value of chronic granuloma patient data with Interferon Gamma (rIFN-g) treatment can be calculated with the following formula

$$MTTF = \int_0^\infty S(t)\,dt$$
$$= \int_0^\infty exp\left(-\frac{t}{\lambda}\right)dt$$
$$= -\lambda e^{-\frac{\infty}{\lambda}} - \left(-\lambda e^{-\frac{0}{\lambda}}\right)$$
$$= \lambda$$
$$= 1341$$

From the result $\hat{\lambda}$ =1341 obtained, the confidence interval for the parameter estimate $\hat{\lambda}$ can be calculated. The confidence level used is 95%

$$T = \sum_{i=1}^{r} t_{(i)} + (n-r)t_{(r)}$$
$$= (32 + 42 + \cdots + 373) + ((83-20)373) = 26807$$

So that,

$$\hat{\lambda}_{min} \le \hat{\lambda} \le \hat{\lambda}_{max}$$
$$\frac{2T}{X^2{}_{\left(\frac{a}{2};\,2r\right)}} \le \hat{\lambda} \le \frac{2T}{X^2{}_{\left(1-\frac{a}{2};\,2r\right)}}$$
$$\frac{2(26807)}{X^2_{(0{,}025\,;\,2(20))}} \le \hat{\lambda} \le \frac{2(26807)}{X^2_{(1-0{,}025\,;\,2(20))}}$$
$$\frac{53614}{X^2_{(0{,}025\,;\,40)}} \le \hat{\lambda} \le \frac{53614}{X^2_{(0{,}975\,;\,40)}}$$
$$\frac{53614}{59{,}342} \le \hat{\lambda} \le \frac{53614}{24{,}433}$$
$$903{,}47477 \le \hat{\lambda} \le 2194{,}32734$$

So, the confidence interval value for the parameter estimated $\hat{\lambda}$ with 95% confidence level is $903{,}47477 \le \hat{\lambda} \le 2194{,}32734$

The researcher wanted to know the proportion of chronic granuloma patients treated with Interferon Gamma (rIFN-g) who died before two months or 60 days since the last treatment. Thus, the case can be calculated as follows

$$F(t) = 1 - \exp\left(\frac{-t}{1341}\right)$$
$$F(60) = 1 - \exp\left(\frac{-60}{1341}\right) = 0{,}0437565$$

Thus, it can be concluded that the proportion of chronic granuloma patients treated with Interferon Gamma (rIFN-g) who died before two months or 60 days since the last treatment is 0,0437565

The researcher wants to know the proportion of chronic granuloma patients treated with Interferon Gamma (rIFN-g) who survive up to four months or 120 days since the last treatment. Thus, the case can be calculated as follows

$$S(t) = \exp\left(\frac{-t}{1341}\right)$$
$$S(120) = \exp\left(\frac{-120}{1341}\right) = 0{,}914402$$

Thus, it can be concluded that the proportion of chronic granuloma patients treated with Interferon Gamma (rIFN-g) who survive up to four months or 120 days since the last treatment is 0,914402

4.2 Survival Analysis of Chronic Granuloma Patients with Placebo Treatment Method

The following table shows the survival data of 56 out of 120 Placebo-treated chronic granuloma patients sorted by shortest survival time

Table 3. Survival data of chronic granuloma patients with Placebo treatment method

2	4	4	5	6	7	8
8	11	11	13	14	18	18
19	23	28	30	34	36	38
49	52	54	57	64	67	73
82	89	91	99	104	105	120
126	130	146	147	155	168	175
181	190	206	207	211	236	246
264	280	292	294	304	318	334

By using the following hypothesis:
H0: Chronic Granuloma Patient Data with Placebo treatment method is exponentially distributed
H1: Chronic Granuloma Patient Data with Placebo treatment method is not exponentially distributed.

With the critical region, H0 will be rejected if the p-value or Sig. (2-tailed) is smaller than the 5% alpha significance level.

Table 4. KS Exponential Test of Placebo treatment method

N	56
Asymp. Sig. (2-tailed)	0,547

Based on the output above, the p-value is 0,547 which is greater than alpha 0.05 so that the decision to fail to reject H0 is obtained, which means that the Chronic Granuloma Patient Data with Placebo treatment method is exponentially distributed.

Finding parameter estimators

$$L(\lambda|\underline{t}) = \frac{n!}{(n-r)!}\prod_{i=1}^{r} f(t_{(i)})\, S(t_{(r)})^{n-r}$$

$$= \frac{n!}{(n-r)!}\prod_{i=1}^{r}\frac{1}{\lambda}\exp\left(\frac{-t_{(i)}}{\lambda}\right)\left[\exp\left(\frac{-t_{(r)}}{\lambda}\right)\right]^{n-r}$$

$$= \frac{n!}{(n-r)!}\frac{1}{\lambda^r}\exp\left(-\frac{1}{\lambda}\sum_{i=1}^{r} t_{(i)}\right)\exp\left(\frac{-t_{(r)}}{\lambda}\right)^{n-r}$$

$$ln\, L(\lambda|\underline{t}) = ln\frac{n!}{(n-r)!} + ln\frac{1}{\lambda^r} + ln\left[\exp\left(-\frac{1}{\lambda}\sum_{i=1}^{r} t_{(i)}\right)\right] + ln\left[\exp\left(\frac{-t_{(r)}}{\lambda}\right)^{n-r}\right]$$

$$= ln\frac{n!}{(n-r)!} - rln\lambda - \frac{1}{\lambda}\sum_{i=1}^{r} t_{(i)} - (n-r)\frac{t_{(r)}}{\lambda}$$

$$\frac{d\, ln\, L(\lambda|\underline{t})}{d\,\lambda} = 0 - \frac{r}{\lambda} + \frac{\sum_{i=1}^{r} t_{(i)}}{\lambda^2} + \frac{(n-r)t_{(r)}}{\lambda^2}$$

$$0 = -\frac{r}{\lambda} + \frac{\sum_{i=1}^{r} t_{(i)}}{\lambda^2} + \frac{(n-r)t_{(r)}}{\lambda^2}$$

$$\frac{r}{\lambda} = \frac{\sum_{i=1}^{r} t_{(i)} + (n-r)t_{(r)}}{\lambda^2}$$

$$r = \frac{\sum_{i=1}^{r} t_{(i)} + (n-r)t_{(r)}}{\lambda}$$

$$\lambda = \frac{\sum_{i=1}^{r} t_{(i)} + (n-r)t_{(r)}}{r}$$

$$\hat{\lambda} = \frac{\sum_{i=1}^{r} t_{(i)} + (n-r)t_{(r)}}{r}$$

So

$$\hat{\lambda} = \frac{(2+4+4+\cdots+334)+((120-56)334)}{56}$$

$$\hat{\lambda} = \frac{6053+21376}{56}$$

$$\hat{\lambda} = \frac{27429}{56}$$

$$\hat{\lambda} = 489{,}804 \approx 490$$

From the results of these calculations, the parameter point estimate value for chronic granuloma patients with Placebo treatment is obtained $\hat{\lambda}$ which is 490

The probability density function (PDF) of the type II censored exponential distribution for the data of chronic granuloma patients treated with Placebo is as follows:

$$f(t) = \frac{1}{\lambda}\exp\left(\frac{-t}{\lambda}\right)$$

$$f(t) = \frac{1}{490}\exp\left(\frac{-t}{490}\right)$$

The cumulative probability density function (CDF) of the above probability density function can be calculated with the following formula

$$F(t) = \int_0^t f(x)\,dx = \int_0^t \frac{1}{490}\exp\left(\frac{-t}{490}\right)dx$$

$$= \frac{1}{490}\int_0^t \exp\left(\frac{-t}{490}\right)dx = 1 - \exp\left(\frac{-t}{490}\right)$$

The survivor function of the odds density function above can be calculated with the following formula

$$S_{(t)} = \int_t^\infty f(t)dt$$

$$= \int_t^\infty \frac{1}{490}\exp\left(\frac{-t}{490}\right)dt$$

$$= \frac{1}{490}\int_t^\infty \exp\left(\frac{-t}{490}\right)\frac{d\left(\frac{-t}{490}\right)}{-\frac{1}{490}}$$

$$= \frac{1}{490}(-490)\int_t^\infty \exp\left(\frac{-t}{490}\right)d\left(\frac{-t}{490}\right)$$

$$= -\int_t^\infty \exp\left(\frac{-t}{490}\right)d\left(\frac{-t}{490}\right)$$

$$= -\left[\exp\left(\frac{-t}{490}\right)\right]_t^\infty$$

$$= -\left[0 - \exp\left(\frac{-t}{490}\right)\right]$$

$$= \exp\left(\frac{-t}{490}\right)$$

The hazard function of chronic granuloma patient data with Placebo treatment can be calculated with the following formula

$$h(t) = \frac{f(t)}{S(t)} = \frac{\frac{1}{490}\exp\left(\frac{-t}{490}\right)}{\exp\left(\frac{-t}{490}\right)} = \frac{1}{490} = 0{,}00204082$$

The Mean Time to Failure (MTTF) value of chronic granuloma patient data with Placebo treatment can be calculated with the following formula

$$MTTF = \int_0^\infty S(t)\,dt$$

$$= \int_0^\infty exp\left(-\frac{t}{\lambda}\right)dt$$

$$= -\lambda e^{-\frac{\infty}{\lambda}} - \left(-\lambda e^{-\frac{0}{\lambda}}\right)$$

$$= \lambda$$

$$= 490$$

From the result $\hat{\lambda}$ = 490 obtained, the confidence interval for the parameter estimate $\hat{\lambda}$ can be calculated. The confidence level used is 95%

$$T = \sum_{i=1}^{r} t_{(i)} + (n-r)t_{(r)}$$

$$= (2+4+4+\cdots+334) + ((120-56)334) = 27429$$

So that,

$$\hat{\lambda}_{min} \le \hat{\lambda} \le \hat{\lambda}_{max}$$

$$\frac{2T}{X^2_{\left(\frac{a}{2};\,2r\right)}} \le \hat{\lambda} \le \frac{2T}{X^2_{\left(1-\frac{a}{2};\,2r\right)}}$$

$$\frac{2(27429)}{X^2_{(0{,}025\,;\,2(56))}} \le \hat{\lambda} \le \frac{2(27429)}{X^2_{(1-0{,}025\,;\,2(56))}}$$

$$\frac{54858}{X^2_{(0{,}025\,;\,112)}} \le \hat{\lambda} \le \frac{54858}{X^2_{(0{,}975\,;\,112)}}$$

$$\frac{54858}{143{,}180} \le \hat{\lambda} \le \frac{54858}{84{,}604}$$

$$383{,}1401 \le \hat{\lambda} \le 648{,}40906$$

So, the confidence interval value for the parameter estimated $\hat{\lambda}$ with 95% confidence level is $383{,}1401 \le \hat{\lambda} \le 648{,}40906$

The researcher wanted to know the proportion of chronic granuloma patients treated with Placebo who died before two months or 60 days since the last treatment. Thus, the case can be calculated as follows

$$F(t) = 1 - \exp\left(\frac{-t}{490}\right)$$

$$F(60) = 1 - \exp\left(\frac{-60}{490}\right) = 0{,}115249$$

Thus, it can be concluded that the proportion of chronic granuloma patients treated with Placebo who died before two months or 60 days since the last treatment is 0,115249

The researcher wants to know the proportion of chronic granuloma patients treated with Placebo who survive up to four months or 120 days since the last treatment. Thus, the case can be calculated as follows

$$S(t) = \exp\left(\frac{-t}{490}\right)$$

$$S(120) = \exp\left(\frac{-120}{490}\right) = 0{,}782784$$

Thus, it can be concluded that the proportion of chronic granuloma patients treated with Placebo who survive up to four months or 120 days since the last treatment is 0,782784

To conclude the best treatment method for chronic granuloma patients, it is necessary to compare the probability of death and survival of patients. The following table is a comparison of the probability of death and survival of chronic granuloma patients with interferon gamma (rIFN-g) and Placebo treatment

Table 5. Comparison Table of Survival Data

	interferon gamma (rIFN-g)	**Placebo**
Died Before t Day		
t (Day)	Probability	
60	0,0437565	0,115249
Still Survive Up to t Day		
t (Day)	Probability	
120	0,914402	0,782784
Death Rate	0,000745712	0,00204082

Based on Table 5, it can be seen that the probability of chronic granuloma patients with interferon gamma (rIFN-g) treatment to die before 60 days is smaller than that of chronic granuloma patients with placebo treatment. In addition, the probability of chronic granuloma patients treated with interferon gamma (rIFN-g) to survive up to 120 days is greater than that of chronic granuloma patients treated with placebo.

5. Conclusion

Based on the results and discussion, it can be concluded that treatment with Interferon Gamma (rIFN-g) is more effective than placebo treatment in treating chronic granuloma patients. This statement can be supported by data that the probability of survival time of chronic granuloma patients undergoing treatment with Interferon Gamma (rIFN-g) is greater than that of patients undergoing placebo treatment during time t.

Suggestions that can be given to future researchers are to develop more complex survival analysis research by refining the survival test of chronic granuloma patients based on gender and causal factors so that other factors can be known from the data explored with the same or different case studies.

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