International Journal of Engineering and Information Systems (IJEAIS)
ISSN: 2643-640X
Vol. 7 Issue 7, July - 2023, Pages: 39-43

A method mtm For Calculating Triple Integrals with Numerically Continuous Integrals

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Abstract: *The main objective of this research is to derive a rule for numerically calculating the values of triple integrals with continuous integrals in the region of integration using the trapezoidal rule on the dimensions y, ,and the mid point rule on two the dimension x ,zand how to find the correction limits for it (error formula) and improve these results using the Romberg acce leration method Through the correction limits that we found, when the number of partial periods into which the integration period i s divided on the inner dimension x is equal to twice the number of partial periods into which the integration period is divided on the middle dimension Y and on the outer dimension Z. We will symbolize this method with the symbolRMTM and it is reliable be cause it has given high accuracy through the integrations we reviewed in the results compared to the analytical values of the integrals with a few partial periods.*

Keywords—Numerical integration 65D30 ; tripleintegers 32A55 ; Romberg Accelerating 65B99;

1.introducation

Numerical analysis is characterized by the creation of diverse methods for finding approximate solutions to certain mathemati cal issues in an effective manner. The efficiency of these methods depends on both the accuracy and the ease with which they can be implemented. The numerical analysis is the numerical interface of the wide field of applied analysis. Tripartite integrations are of great importance in finding sizes and middle positions and the determination of the inertia of the volumes and finding the blocks with variable density, for example the size of the inside and above and below and the calculation of the average position of the size of the impact in and above the level and below the level, Such as a piece of thin wire or a thin sheet of metal. Frank Ayers [8], prompting a number of researchers to work in the field of triple integrals. Which led many researchers to work in the field o f tripartite integrations, including Hans Jarr and Jacobsen [1] in 1973, Frank Ayers [2] in 1975, Muhammad [3] in 1984and Hilal [4] in 2081.

In this paper we present a theorem with proof to derive a new base for calculating approximate values of triple integrations with constant inversions and their error formula. This rule is the result of the application of the Rumbark acceleration method to the values resulting from the use of the two point bases on the internal x and outer dimensions and the trapezoid base on the middle dimension (The number of partial periods divided by the internal dimension period, the number of partial periods divided by the middle dimension period and the number of partial periods divided by the external dimension period and we will mark this method with the symbol where the method of accelerating Rumbark and Several derivative we have got good results in terms of accuracy and speed of approaching the number of partial periods of relatively few and very short time

1.1 Triple Integrals For Continuous Integrands With Singularity In Partial Derivatives

Let's say the triple integral is defined by the formula:

$$I=\int_{z_0}^{z_n}\int_{y_0}^{y_n}\int_{x_0}^{x_n} f(x,y,z)dxdydz$$

Which can be written using the trapezoid rule on the three dimensions in the following form:

$$I=\int_{z_0}^{z_n}\int_{y_0}^{y_n}\int_{x_0}^{x_n} f(x,y,z)dxdydz = mtm(h)+E_{mtm}(h) \qquad \cdots(1-1)$$

Whereas $x_i = a+ih,(i=1,2,3\cdots,n-1)$ & $h=\dfrac{b-a}{n}$ & $y_j = c+jh,\ j=1,2,3,\cdots$ $z_k = e+kh, k=1,2,3,\cdots$

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Note that $mtm(h)$ it represents the approximate value of the integral using the trapezoidal rule, and that $E_{mtm}(h)$

A string of correction terms that can be added to values $mtm(h)$.

Theorem :- Let the $f(x,y,z)$ function be continuous and differentiable at every point in the region $[a,b]\times[c,d]\times[e,g]$ the approximate value of the triple integral $I=\int_e^g\int_c^d\int_a^b f(x,y,z)dxdydz$ can be calculated from the following

$$I = mtm = I = \int_e^g\int_c^d\int_a^b f(x,y,z)dxdydz$$

$$\frac{h^3}{2}\sum_{i=0}^{n-1}\sum_{k=0}^{n-1}\left(f(x_i+\frac{h}{2},c,z_k+\tfrac{h}{2},)+f(x_i+\frac{h}{2},d,z_k+\tfrac{h}{2})+2\sum_{j=1}^{n-1} f(x_i+\frac{h}{2},y_j,z_k+\tfrac{h}{2})\right)$$

It can be calculated from the following rule:-

And that the error formula is

$$E_{mtm}(h) = I - mtm(h) = A_{mtm}h^2 + B_{mtm}h^4 + C_{mtm}h^6 + \cdots$$

Since $A_{mtm}, B_{mtm}, \cdots$ the constants depend on the values of the partial derivatives of the function f

Proof:

for a monointegral $\int_a^b f(x,y,z)dx$ It can be calculated numerically using the base of the mid point over the dimension χ and (dealing with z and y as constants) and its value:

$$\int_a^b f(x,y,z) = h\sum_{i=0}^{n-1} f(x_i+\frac{h}{2},y,z) + \frac{b-a}{6}h^2\frac{\partial^2 f(\mu_1,y,z)}{\partial x^2} - \frac{7(b-a)}{360}h^4\frac{\partial^4 f(\mu_2,y,z)}{\partial x^4} + \cdots$$

using the base of the the trapezoid over the dimension y :

$$tm = \int_c^d f(x,y,z)dxdy = \frac{h^2}{2}\sum_{i=0}^{n-1}\ [f(x_i+\frac{h}{2},c,z)+f(x_i+\frac{h}{2},d,z)+2\sum_{j=1}^{n-1} f(x_i+\frac{h}{2},y_j,z)] + \int_c^d\ \frac{b-a}{-12}h^2\frac{\partial^2 f(\mu_1,y,z)}{\partial x^2} +$$

$$+\frac{b-a}{720}h^4\frac{\partial^4 f(\mu_2,y,z)}{\partial x^4}+\cdots]dy + h\sum_{i=0}^{n-1}(\frac{d-c}{-12}h^2\frac{\partial^4 f(x_i+\frac{h}{2},\lambda_1,z)}{\partial y^2} + \frac{d-c}{720}h^4\frac{\partial^4 f(x_i+\frac{h}{2},\lambda_2,z)}{\partial y^4}+\cdots)$$

using the base of the mid point over the dimension z:

$$mtm=\int_e^g\int_c^d f(x,y,z)dxdydz=\frac{h^3}{2}\sum_{i=0}^{n-1}\sum_{k=0}^{n-1}\ [f(x_i+\frac{h}{2},c,z_k+\frac{h}{2})+f(x_i+\frac{h}{2},d,z_k+\frac{h}{2})+2\sum_{j=1}^{n-1}f(x_i+\frac{h}{2},y_j,z_k+\frac{h}{2})]+$$

$$\int_e^g\int_c^d[\frac{b-a}{-12}h^2\frac{\partial^2 f(\mu_1,y,z)}{\partial x^2}++\frac{b-a}{720}h^4\frac{\partial^4 f(\mu_2,y,z)}{\partial x^4}++\cdots]dydz+$$

$$+h\sum_{i=0}^{n-1}\int_e^g\ (\frac{d-c}{-12}h^2\frac{\partial^4 f(x_i+\frac{h}{2},\lambda_1,z)}{\partial y^2}+\frac{d-c}{720}h^4\frac{\partial^4 f(x_i+\frac{h}{2},\lambda_2,z)}{\partial y^4}+\cdots)]dz+\frac{h^2}{2}\sum_{i=0}^{n-1}[\frac{(g-e)}{6}h^2\frac{\partial^2 f(x_i+\frac{h}{2},c,\beta_{11})}{\partial z^2}-$$

$$\frac{7(g-e)}{360}\frac{\partial^4 f(x_i+\frac{h}{2},c,\beta_{12})}{\partial z^4}+\cdots+\frac{g-e}{6}h^2\frac{\partial^2 f(x_i+\frac{h}{2},d,\beta_{21})}{\partial z^2}-\frac{7(g-e)}{360}h^4\frac{\partial^4 f(x_i+\frac{h}{2},d,\beta_{22})}{\partial z^4}+\cdots$$

$$+2\sum_{j=1}^{n-1}(\frac{g-e}{6}h^2\frac{\partial^2 f(x_i+\frac{h}{2},y_j,\beta_{31})}{\partial z^2}-\frac{7(g-e)}{360}h^4\frac{\partial^4 f(x_i+\frac{h}{2},y_j,\beta_{32})}{\partial z^4}+\cdots]$$

applying the two theorems (the middle value theorem and the mean value theorem in integration),

we get

$$mtm=\frac{h^3}{2}\sum_{i=0}^{n-1}\sum_{k=0}^{n-1}\ [f(x_i+\frac{h}{2},c,z_k+\frac{h}{2})+f(x_i+\frac{h}{2},d,z_k+\frac{h}{2})+2\sum_{j=1}^{n-1}f(x_i+\frac{h}{2},y_j,z_k+\frac{h}{2})]+$$

$$h^2[\frac{(g-e)(d-c)(b-a)}{6}\frac{\partial^2 f(\mu_1,\alpha_1,\delta_1)}{\partial x^2}+h\sum_{i=0}^{n-1}(\frac{(g-e)(d-c)}{-12}\frac{\partial^2 f(x_i+\frac{h}{2},\lambda_1,\gamma_{21})}{\partial y^2}+\frac{h^2}{2}\sum_{i=0}^{n-1}(\frac{(g-e)}{6}(\frac{\partial^2 f(x_i+\frac{h}{2},c,\beta_{11})}{\partial z^2}+\frac{\partial^2 f(x_i+\frac{h}{2},d,\beta_{21})}{\partial z^2}$$

$$+\frac{\partial^2 f(x_i+\frac{h}{2},y_j,\beta_{31})}{\partial z^2})]+h^4[-\frac{7(g-e)(d-c)(b-a)}{360}\frac{\partial^4 f(\mu_2,\alpha_2,\delta_2)}{\partial x^4}+h\sum_{i=0}^{n-1}(\frac{(g-e)(d-c)}{720}\frac{\partial^4 f(x_i+\frac{h}{2},\lambda_2,\gamma_{22})}{\partial y^4}+\frac{h^2}{2}\sum_{i=0}^{n-1}(-\frac{7(g-e)}{360}$$

$$(\frac{\partial^4 f(x_i+\frac{h}{2},c,\beta_{12})}{\partial z^4}+\frac{\partial^4 f(x_i+\frac{h}{2},d,\beta_{22})}{\partial z^4}+\frac{\partial^4 f(x_i+\frac{h}{2},y_j,\beta_{32})}{\partial z^4})]+h^6[.....]$$

Since $A_{mtm},B_{mtm},\cdots$ the constants depend on the values of the partial derivatives of the function f

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2.EXAMPLS

	Integrals	Values of Approximate Integrals
1	$\int_2^3\int_1^2\int_0^1 xe^{-(x+y+z)}dxdydz$	**0.0052567434550** rounded to thirteen decimal places
2	$\int_0^1\int_0^1\int_0^1 \sin(\frac{\pi}{2}(x+y+z)\ )dxdydz$	**0.5160245509312** rounded to thirteen decimal places

3.Results

1- Integral function $\int_2^3\int_1^2\int_0^1 xe^{-(x+y+z)}dxdydz$ continuous and differentiable for each $(x,y,z)\in[0,1]\times[1,2]\times[2,3]$

From table (1) we notice that the value of integration using the rule is true for five decimal places $n=16$, and when using the Rumberck acceleration with the mentioned rule, it is true for thirteen decimal places (partial period 2^{12}), which is identical to the real value rounded to thirteen decimal places.

2- Integral function $\int_0^1\int_0^1\int_0^1 \sin(\frac{\pi}{2}(x+y+z)\ )dxdydz$ continuous and differentiable for each $(x,y,z)\in[0,1]\times[0,1]\times[0,1]$

from table (2), we conclude when $n=32$ the value of the integration using the rule ttm is true to two decimal places, and when using the Rumberck acceleration with the mentioned rule, the value is identical to the real value, rounded to thirteen decimal places with (partial period 2^{15})

n	mtm	K=2	K=4	K=6	K=8
1	0.0037989419941				
2	0.0048955533469	0.0052610904646			
4	0.0051667211074	0.0052571103609	0.0052568450207		
8	0.0052342562678	0.0052567679879	0.0052567451631	0.0052567435780	
16	0.0052511228273	0.0052567450138	0.0052567434822	0.0052567434555	0.0052567434550
table (1)				exat value	0.0052567434550

n	mtm	K=2	K=4	K=6	K=8	K=10
1	0.5000000000000					
2	0.5151650429450	0.5202200572599				
4	0.5159727873197	0.5162420354447	0.5159768339903			
8	0.5160213453397	0.5160375313463	0.5160238977397	0.5160246447834		
16	0.5160243510413	0.5160253529418	0.5160245410481	0.5160245512594	0.5160245508926	
32	0.5160245384452	0.5160246009132	0.5160245507780	0.5160245509325	0.5160245509312	0.5160245509312

table (2) $\int_0^1\int_0^1\int_0^1 \sin(\frac{\pi}{2}(x+y+z)\ )dxdydz$	exat value **0.5160245509312**

4.Conclusion

It is clear from the results of the tables of this research that when calculating the approximate values of triple integrals with continuous integrals of the compound base from the bases of the trapezoidal on the y and using middle point dimension on the two dimensions x,zwhen the number of partial periods to which the period is divided into the internal dimension is equal to twice the number of partial periods to which the period is divided into the middle dimension and equal to twice the number of partial periods into which the period of the outer dimension is divided. This rule (base) gives integer values (for several decimal places) compared with the real values of the integrals, and by using a number of partial periods without using the process of external adjustment on them, for example, in the first and second integrations, we get On the integer value of five decimal places and two decimal places respectively and in the third integral the real value is unknown.

However, when using the Romberg acceleration method with the mentioned rule, it gave better results in terms of the speed of approaching with a relatively small number of partial periods to the values of the real integrals, as they were identical to the real value in the first and second integrals when at n = 16 and n = 128.

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