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A numerical method for calculating triple integrals numerically when the value of the integrals is not defined at the upper limit of the integral

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Abstract: *The main objective of this study is to derive a new rule numerically for finding triple integrals when the value of the integrality is not defined at the upper limit of the integralwhen* $(x, y, z) = (x_n, y_n, z_n)$ *by using Mid-point rule on the x and y and tripsinol rule dimensions z and how to find the correction terms (error formula) and we use Romberg acceleration to improve the results when the number of subintervals of on the three dimensions are equal .*

Keywords:Region,integral,dimensions,objective,subintervals,finding
.

1.introducation

The importance of the subject of numerical analysis lies in devising certain methods that contribute to finding approximate solutions to problems in mathematics, including the integrals that constitute an important part of this topic, as this importance is more evident in the practical applications practiced by engineers and physicists, and finding the approximate value of the integration came as a result:

1. The impossibility of finding the analytical value of the integration.

2. When the process of finding the analytical value of the integration is possible, but with difficulty and requires a long time.

3. The analytic integration value may be approximate mainly because it contains terms that take their values from tables (such as the logarithm or the inverse tangent).

4. The problem may be to find an area under a curve defined by a table of values (that is, the function is defined at a few points in the integration period), as is the case when analyzing the results of experiments.

The process of finding a numerical value for the triple integration constitutes a more complex issue than the problem of finding the value of the unitary and binary integration, since the integrator here depends on three variables, and the issue of continuity or defectiveness in the integrator or impairment in the partial derivatives of the integrator poses great difficulties, as well as here we will deal with regions of integration (regions) or surfaces (Surfaces) and not with integration intervals as in the case of unary integration.

Therefore, finding the values of integrals of this kind is not an easy matter for some cases. Therefore, there has become an urgent need to find approximate values for these integrals. The importance of triple integrals lies in finding volumes, average centers, and inertial inertia of volumes, which prompted many researchers to work in the field of triple integrals and researchers who shed light on the calculation of integrals. Continuous integrals of the form $f(x,y,z) = f_1(x) f_2(y) f_3(z)$

Hilal [1], Hassan [2], Muhammad [3] and others.

In this paper, we present a theorem with proof to derive a new rule for finding triple integrals when the value of the integrality is not defined at the upper limit of the integralwhen $(x, y, z) = (x_n, y_n, z_n)$

by using Mid-point rule on the x and y and tripsinol rule dimensions z and the error formula for them. we will symbolize this method with the symbolTMM, and in order to improve the results we use the method of accelerating Ro Mubarak, then we symbolize this rule with the symbol RTMM and we have obtained good results in terms of accuracy and speed of approach and with a relatively small number of partial periods.

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1-2 The base of the middle point on the two dimensions x,y and the trapezoid on the dimension z:-

Thearme :-Let the function $f(x,y,z)$ be continuous and differentiable at each point of the region $[x_0,x_n]\times[y_0,y_n]\times[z_0,z_n]$ Except for at least one of its derivatives that is not differentiable at the point $(x,y,z)=(x_n,y_n,z_n)$, the approximate value of the triple integral $I=\int_{z_0}^{z_n}\int_{y_0}^{y_n}\int_{x_0}^{x_n} f(x,y,z)dxdydz$ It can be calculated from the following rule:-

$$TMM=\int_{z_0}^{z_n}\int_{y_0}^{y_n}\int_{x_0}^{x_n} f(x,y,z)dxdydz=\frac{h^3}{2}\sum_{j=0}^{n-1}\sum_{i=0}^{n-1}\ [f(x_i+.5h,y_j+.5h,z_0)+f(x_0+.5h,y_0+.5h,z_n)+$$
$$2\sum_{k=1}^{n-1}(f(x_i+.5h,y_j+.5h,z_k)]$$

The error formula (correction limits) is :- $E_{TMM}(h)=I-TMM(h)=$

$$\{h^5[\frac{1}{24}D_x^{\ 2}+\frac{1}{24}D_y^{\ 2}-\frac{1}{12}D_z^{\ 2}]+h^6[(\frac{1}{48}D_x^{\ 3}+\frac{1}{48}D_y^{\ 3}-\frac{1}{24}D_z^{\ 3})+\frac{1}{48}(D_x^{\ 2}D_y+D_y^{\ 2}D_x+$$
$$D_x^{\ 2}D_z+D_y^{\ 2}D_z)-\frac{1}{24}(D_z^{\ 2}D_x+D_z^{\ 2}D_y)]$$
$$+h^7[\cdots]+\cdots\}f(x_{n-1},y_{n-1},z_{n-1})+A_{TMM}h^2+B_{TMM}h^4+\cdots$$

Whereas, $A_{TMM},B_{TMM},\cdots$ the constants depend on partial derivatives f.

Proofe:-The integration $\int_{z_0}^{z_n}\int_{y_0}^{y_n}\int_{x_0}^{x_n} f(x,y,z)dxdydz$ **can be written as follows:**

$$I=\int_{z_0}^{z_n}\int_{y_0}^{y_n}\int_{x_0}^{x_n} f(x,y,z)dxdydz=$$
$$\int_{z_0}^{z_{n-1}}\int_{y_0}^{y_{n-1}}\int_{x_0}^{x_{n-1}} f(x,y,z)dxdydz+\sum_{t=0}^{n-2}\int_{z_t}^{z_{t+1}}\int_{y_{n-1}}^{y_n}\sum_{r=0}^{n-2}\int_{x_r}^{x_{r+1}} f(x,y,z)dxdydz+\sum_{t=0}^{n-2}\int_{z_t}^{z_{t+1}}\int_{y_{n-1}}^{y_n}\int_{x_{n-1}}^{x_n} f(x,y,z)dxdydz+$$
$$\int_{z_{n-1}}^{z_n}\sum_{s=0}^{n-2}\int_{y_s}^{y_{s+1}}\sum_{r=0}^{n-2}\int_{x_r}^{x_{r+1}} f(x,y,z)dxdy\sum_{t=0}^{n-2}\int_{z_t}^{z_{t+1}}\sum_{s=0}^{n-2}\int_{y_s}^{y_{s+1}}\int_{x_{n-1}}^{x_n} f(x,y,z)dxdydz+\int_{z_{n-1}}^{z_n}\sum_{s=0}^{n-2}\int_{y_s}^{y_{s+1}}\int_{x_{n-1}}^{x_n} f(x,y,z)dxdydz+$$
$$\int_{z_{n-1}}^{z_n}\int_{y_{n-1}}^{y_n}\sum_{r=0}^{n-2}\int_{x_r}^{x_{r+1}} f(x,y,z)dxdydz+\int_{z_{n-1}}^{z_n}\int_{y_{n-1}}^{y_n}\int_{x_{n-1}}^{x_n} f(x,y,z)dxdydz \qquad (1)$$

Since the function is defined at the point $(x_{n-1},y_{n-1},z_{n-1})$ We can spread the function $f(x,y,z)$ with the Tyler series about the point $(x_{n-1},y_{n-1},z_{n-1})$ and we get

To find the integral of the base of the middle point on the two dimensionsx,y and the trapezoid on the dimension z of the above equation we get

$$i)\,f(x_{n-1}+.5h,y_{n-1}+.5h,z_{n-1})=[1+\frac{h}{2}D_x+\frac{h}{2}D_y+\frac{h^2}{8}D_x{}^2+\frac{h^2}{8}D_y{}^2+\frac{h^2}{4}D_xD_y+\frac{h^3}{48}D_x{}^3+\frac{h^3}{48}D_y{}^3$$
$$+\frac{h^3}{16}D_x{}^2D_y+\frac{h^3}{16}D_y{}^2D_x+\frac{h^4}{384}D_x{}^4+\frac{h^4}{348}D_y{}^4+\cdots]f(x_{n-1},y_{n-1},z_{n-1})\cdots \qquad (4-131)$$

$$ii)\,f(x_{n-1}+.5h,y_{n-1}+.5h,z_n)=[1+\frac{h}{2}D_x+\frac{h}{2}D_y+hD_z+\frac{h^2}{8}D_x{}^2+\frac{h^2}{8}D_y{}^2+\frac{h^2}{2}D_z{}^2+\frac{h^2}{4}D_xD_y+\frac{h^2}{2}D_xD_z$$
$$+\frac{h^2}{2}D_yD_z+\frac{h^3}{48}D_x{}^3+\frac{h^3}{48}D_y{}^3+\frac{h^3}{6}D_z{}^3+\frac{h^3}{16}D_x{}^2D_y+\frac{h^3}{16}D_y{}^2D_x+\frac{h^2}{8}D_x{}^2D_z$$
$$+\frac{h^3}{4}D_xD_yD_z+\frac{h^2}{8}D_y{}^2D_z+\frac{h^3}{4}D_xD_z{}^2+\frac{h^2}{4}D_yD_z{}^2+\frac{h^4}{384}D_x{}^4$$
$$+\frac{h^4}{348}D_y{}^4+\frac{h^4}{24}D_z{}^4+\cdots]f(x_{n-1},y_{n-1},z_{n-1}) \qquad (2)$$

Multiplying (2)by $\frac{-h^3}{2}$ and adding them with other equntion gives us:

$$\int_{z_{n-1}}^{z_n}\int_{y_{n-1}}^{y_n}\int_{x_{n-1}}^{x_n} f(x,y,z)dxdydz=\frac{h^3}{2}\sum_{j=0}^{n-1}\sum_{i=0}^{n-1}\;[f(x_{n-1}+.5h,y_{n-1}+.5h,z_{n-1})+f(x_{n-1}+.5h,y_{n-1}+.5h,z_n)]+$$
$$h^5[\frac{1}{24}D_x{}^2+\frac{1}{24}D_y{}^2-\frac{1}{12}D_z{}^2]+h^6[(\frac{1}{48}D_x{}^3+\frac{1}{48}D_y{}^3-\frac{1}{24}D_z{}^3)+\frac{1}{48}(D_x{}^2D_y+D_y{}^2D_x+$$
$$D_x{}^2D_z+D_y{}^2D_z)-\frac{1}{24}(D_z{}^2D_x+D_z{}^2D_y)]+$$
$$h^7[\cdots]+\cdots]f(x_{n-1},y_{n-1},z_{n-1})$$

As for the other seven integrals in which the function is continuous derivatives in their integration areas, there is no prob lem

with morbidity and their values can be calculated halil[1] , and for the base TMMwe get

$$\sum_{t=0}^{n-2}\int_{z_t}^{z_{t+1}}\sum_{s=0}^{n-2}\int_{y_s}^{y_{s+1}}\int_{x_{n-1}}^{x_n} f(x,y,z)dxdydz=\frac{h^3}{2}\sum_{t=0}^{n-2}\sum_{s=0}^{n-2}\;[f(x_{n-1}+.5h,y_s+.5h,z_t)+f(x_{n-1}+.5h,y_s+.5h,z_{t+1})]+$$
$$A_1h^2+B_1h^4+\cdots$$

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$$\sum_{t=0}^{n-2}\int_{z_t}^{z_{t+1}}\int_{y_{n-1}}^{y_n}\sum_{r=0}^{n-2}\int_{x_r}^{x_{r+1}} f(x,y,z)dxdydz=\frac{h^3}{2}\sum_{t=0}^{n-1}\sum_{r=0}^{n-1}\ [f(x_r+.5h,y_{n-1}+.5h,z_t)+$$

$$f(x_r+.5h,y_{n-1}+.5h,z_{t+1})]+A_2h^2+B_2h^4+\cdots$$

$$\sum_{t=0}^{n-2}\int_{z_t}^{z_{t+1}}\int_{y_{n-1}}^{y_n}\int_{x_{n-1}}^{x_n} f(x,y,z)dxdydz=\frac{h^3}{2}\sum_{t=0}^{n-2}\ [f(x_{n-1}+.5h,y_{n-1}+.5h,z_t)+$$

$$f(x_{n-1}+.5h,y_{n-1}+.5h,z_{t+1})]+A_3h^2+B_3h^4+\cdots$$

$$\int_{z_{n-1}}^{z_n}\sum_{s=0}^{n-2}\int_{y_s}^{y_{s+1}}\sum_{r=0}^{n-2}\int_{x_r}^{x_{r+1}} f(x,y,z)dxdydz=\frac{h^3}{2}\sum_{t=0}^{n-2}\sum_{s=0}^{n-2}\ [f(x_r+.5h,y_s+.5h,z_{n-1})+$$

$$f(x_r+.5h,y_s+.5h,z_n)]+A_4h^2+B_4h^4+\cdots$$

$$\int_{z_{n-1}}^{z_n}\sum_{s=0}^{n-2}\int_{y_s}^{y_{s+1}}\int_{x_{n-1}}^{x_n} f(x,y,z)dxdydz=\frac{h^3}{2}\sum_{s=0}^{n-2}\ [f(x_{n-1}+.5h,y_s+.5h,z_{n-1})+f(x_{n-1}+.5h,y_s+.5h,z_n)]+$$

$$+A_5h^2+B_5h^4+\cdots$$

$$\int_{z_{n-1}}^{z_n}\int_{y_{n-1}}^{y_n}\sum_{r=0}^{n-2}\int_{x_r}^{x_{r+1}} f(x,y,z)dxdydz=\frac{h^3}{2}\sum_{r=0}^{n-2}\ [f(x_r+.5h,y_{n-1}+.5h,z_{n-1})+f(x_r+.5h,y_{n-1}+.5h,z_n)]+$$

$$A_6h^2+B_6h^4+\cdot\cdot$$

$$\int_{z_0}^{z_{n-1}}\int_{y_0}^{y_{n-1}}\int_{x_0}^{x_{n-1}} f(x,y,z)dxdydz=\frac{h^3}{2}\sum_{j=0}^{n-2}\sum_{i=0}^{n-2}\ [f(x_i+.5h,y_j+.5h,z_0)+f(x_i+.5h,y_j+.5h,z_{n-1})+$$

$$2\sum_{k=1}^{n-2} f(x_i+.5h,y_j+.5h,z_k)]+A_7h^2+B_7h^4+\cdots$$

Adding the eight integrals gives

$$\int_{z_0}^{z_n}\int_{y_0}^{y_n}\int_{x_0}^{x_n} f(x,y,z)dxdydz=\frac{h^3}{2}\sum_{j=0}^{n-1}\sum_{i=0}^{n-1}\ [f(x_i+.5h,y_j+.5h,z_0)+f(x_0+.5h,y_0+.5h,z_n)]+2\sum_{k=1}^{n-1}(f(x_i+.5h,y_j+.5h,z_k)$$

$$+[h^5[\frac{1}{24}D_x{}^2+\frac{1}{24}D_y{}^2-\frac{1}{12}D_z{}^2]+h^6[(\frac{1}{48}D_x{}^3+\frac{1}{48}D_y{}^3-\frac{1}{24}D_z{}^3)+$$

$$\frac{1}{48}(D_x{}^2D_y+D_y{}^2D_x+D_x{}^2D_z+D_y{}^2D_z)-\frac{1}{24}(D_z{}^2D_x+D_z{}^2D_y)]+h^7[\cdots]+\cdots]f(x_{n-1},y_{n-1},z_{n-1})+A_{TMM}h^2+B_{TMM}h^4+\cdots$$

Whereas, $A_{TMM}, B_{TMM},\cdots$ the constants not depend on partial derivatives.

3.example:

Integrals	Values of Approximate Integrals
$f(x,y,z)=xy\sqrt{1-xyz}$	0.2185673851875 rounded to thirteen decimal places
$\int_0^1\int_0^1\int_0^1\frac{xy^2}{\sqrt{1-xyz}}dxdydz$	**0. 2** rounded to thirteen decimal places

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1- $f(x,y,z)=xy\sqrt{1-xyz}$ Continuous in the region $[0,1]\times[0,1]\times[0,1]$ and can be differentiated in the region $[0,1)\times[0,1)\times[0,1)$, that is, partial derivatives are not defined at the point $(1,1,1)$ in the sense of the diseased integrator derived at the upper end and the type of radical morbidity, so the error formulas

$$E_{TMM}(h)=A_{TMM}h^2+e_1h^{3.5}+B_{TMM}h^4+C_{TMM}h^{4.5}+\cdots$$

We conclude from the table(1) when n=128 the value of the integral by using the TMM rule is true for five decimal places and when using the Rumbrk acceleration RTMM, the value is true for thirteen decimal places and is identical to the real value rounded to thirteen decimal places

2- $\int_0^1\int_0^1\int_0^1 \frac{xy^2}{\sqrt{1-xyz}}dxdydz$ Continuous in the region $[0,1]\times[0,1]\times[0,1]$ and can be differentiated in the region $[0,1)\times[0,1)\times[0,1)$, that is, partial derivatives are not defined at the point $(1,1,1)$ in the sense of the diseased integrator derived at the upper end and the type of radical morbidity, so the error formulas

$$E_{TMM}(h)=A_{TMM}h^2+e_1h^{2.5}+e_2h^{3.5}+B_{TMM}h^4+e_3h^{4.5}+e_4h^{5.5}+C_{TMM}h^6+\cdots$$

We conclude from the table(2) when n=256 the value of the integral by using the TMM rule is true for sixe decimal places and when using the Rumbrk acceleration RTMM, the value is true for thirteen decimal places and is identical to the real value rounded to thirteen decimal places.

n	TMM	K=2	K=3.5	K=4	K=4.5	K=5.5	K=6	K=6.5
1	0.2332531754731							
2	0.2225856144101	0.2190297607224						
4	0.2195797975011	0.2185778585314	0.2185340428475					
8	0.2188196334472	0.2185662454293	0.2185651194422	0.2185671912152				
16	0.2186302671671	0.2185671450738	0.2185672323018	0.2185673731591	0.2185673815717			
32	0.2185830827696	0.2185673546371	0.2185673749560	0.2185673844663	0.2185673849891	0.2185673850663		
64	0.2185713070950	0.2185673818701	0.2185673845106	0.2185673851476	0.2185673851791	0.2185673851834	0.2185673851852	
128	0.2185683654142	0.2185673848540	0.2185673851433	0.2185673851855	0.2185673851872	0.2185673851874	0.2185673851875	0.2185673851875
table(1) $\int_0^1\int_0^1\int_0^1 xy\sqrt{1-xyz}\,dxdydz$							True Value	0.2185673851875

n	TMM	K=2	K=2.5	K=3.5	K=4	K=4.5	K=5.5	K=6	K=6.5
1	0.1346687836487								

2	0.1817417241732	0.1974327043481							
4	0.1952762759668	0.1997877932313	0.2002935185035						
8	0.1988135817466	0.1999926836732	0.2000366812799	0.2000117787704					
16	0.1997049965722	0.2000021348474	0.2000041643664	0.2000010115807	0.2000002937680				
32	0.1999268084418	0.2000007457317	0.2000004474368	0.2000000870495	0.2000000254141	0.2000000130061			
64	0.1999818293348	0.2000001696324	0.2000000459225	0.2000000069923	0.2000000016551	0.2000000005566	0.2000000002753		
128	0.1999954826250	0.2000000337217	0.2000000045366	0.2000000005239	0.2000000000927	0.2000000000204	0.2000000000083	0.2000000000041	
256	0.1999988753957	0.2000000063192	0.2000000004349	0.2000000000372	0.2000000000047	0.2000000000007	0.2000000000002	0.2000000000001	0.2000000000000

$$\int_0^1\int_0^1\int_0^1 \frac{xy^2}{\sqrt{1-xyz}}\,dxdydz$$

table(2)

4.Conclusion

It is clear from the results of the tables of this research that when calculating the approximate values of triple integrals with continuous integrals when the value of the integrality is not defined at the upper limit of the integralwhen $(x,y,z)=(x_n,y_n,z_n)$ by using Mid-point rule on the x and y and tripsinol rule dimensions z and formula) and we use Romberg acceleration from the bases of the trapezoidal on the z and using middle point dimension on the two dimensions x,y when the number of partial periods to which the period is divided into the internal dimension is equal to twice the number of partial periods to into which the period of the middle and outer dimension is divided., we get On the integer value of five decimal places and sixe decimal places respectively and.

However, when using the Romberg acceleration method with the mentioned rule, it gave better results in terms of the speed of approaching with a relatively small number of partial periods to the values of the real integrals, as they were identical to the real value in the first and second integrals when at n = 128,n=256

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n	TMM	K=2	K=2.5	K=3.5	K=4	K=4.5	K=5.5	K=6	K=6.5
1	0.1346687836487								
2	0.1817417241732	0.1974327043481							
4	0.1952762759668	0.1997877932313	0.2002935185035						
8	0.1988135817466	0.1999926836732	0.2000366812799	0.2000117787704					
16	0.1997049965722	0.2000021348474	0.2000041643664	0.2000010115807	0.2000002937680				
32	0.1999268084418	0.2000007457317	0.2000004474368	0.2000000870495	0.2000000254141	0.2000000130061			
64	0.1999818293348	0.2000001696324	0.2000000459225	0.2000000069923	0.2000000016551	0.2000000005566	0.2000000002753		
128	0.1999954826250	0.2000000337217	0.2000000045366	0.2000000005239	0.2000000000927	0.2000000000204	0.2000000000083	0.2000000000041	
256	0.1999988753957	0.2000000063192	0.2000000004349	0.2000000000372	0.2000000000047	0.2000000000007	0.2000000000002	0.2000000000001	0.2000000000000
جدول (4-12) حساب التكامل $\int_0^1\int_0^1\int_0^1 \frac{xy^2}{\sqrt{1-xyz}}\,dxdydz$								القيمة الحقيقية	0.2