

# Mathematical Based Model for Stabilization of Human Heart Using Two-DOF PID Controller

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**Abstract:** In this paper, a mathematical based model for stabilization of human heart using two-Degree of Proportional Integral and Derivative (2-DOF PID) controller has been presented. The mathematical model representing the periodic action of the human heart is presented in transfer function based on hydro-electromechanical characteristics of the cardiovascular system in terms of four components: peacemaker, heartbeat, circulatory system, and feedback sensor. Simulation conducted in MATLAB/Simulink revealed that the mathematical model produced a response similar to electrocardiogram (ECG) recorded for normal heartbeat. With the 2-DOF controller, the system was able to stabilize the heart against the effect of noise and disease.

**Keywords—**ECG, Heartbeat, Human heart, Stabilization, two-DOF PID controller

## 1. INTRODUCTION

The human heart has been described as a pump for circulating oxygen and nutrient carrying blood around the body so as to maintain functioning [1]. During routine exercises, the heart rate provides a good indication of how healthy an individual is. Heartbeat rate significantly contributes in determining the chances of heart attack, which has been shown by the trend of cardiovascular diseases. Currently, cardiovascular disease is considered a major public health challenge globally. Globally,

cardiovascular disease is attributed to one-third of all deaths according to estimate by World Health Organization (WHO) [2].

The operation of the human heart can be related to the electrocardiogram (ECG) signal. The ECG is a continuous time-varying signal that describes the electrical potential produced by electrical activity in the cardiac tissue. It can be measured by taking note of the potential between two electrodes mounted on the skin surface at some set point. The heart's contraction and relaxation that results in its pumping action is described by a single cycle of the ECG [3]. The state of the cardiac heart including a potential heart problem can be represented using characteristic information obtained from ECG signal [4].

Considering the vital function of the human heart, improving the quality of life of a patient, particularly developing an optimized system such as hydro-

electromechanical (HEM) model with a controller as physical (or mathematical) model to represent and describe the heartbeat operation is of the essence. This is because the real physiological data can be simulated by such physical model to appropriate experimental based system [2]. For instance, the human heart was simulated in terms of three major functions, namely hydraulic, electrical and mechanical parameters using a hydro-electromechanical model formulated as Laplace transform equations [5]. Other studies have formulated different approaches involving the use of control theory to model human heartbeat. A nonlinear control theory was applied in heartbeat models by [3]. In the study, a nonlinear feedback linearization approach was applied to force output of the systems to produce artificial ECG signal using discrete data as reference inputs. Mathematical models were formulated as transfer functions for the heartbeat, the peacemaker, the circulatory system and the measurement (sensor) with the integration of Proportional-Integral-Derivative (PID) controller to simulate a system for human heart stabilization through monitoring and control in Aabid et al [2]. The comparison and validation of different probabilistic models for human heartbeat dynamics characterization based on the assessment of ECG data recorded with varying conditions in posture and pharmacological autonomic blockade control was presented by Chen et al. [6]. For the stabilization of some pathological characteristics of a nonlinear heartbeat model, with and without additive random noise, a discretized feedback control system was implemented by Brandt et al. [7]. The Zeeman nonlinear heart model was adopted to describe the stability and control of the operation of heart, which was presented as control system for nonlinear heartbeat models

under time-delay-switched feedback using emotional learning control (ELC) by Sargolzaei et al. [8].

With the previous studies on the mathematical models and application of control theory to simulate the characteristics of human heartbeat and its stabilization considered, this paper presents the application of nonlinear control system theory based on feedback linearization using two-degree of freedom (2-DOF) PID controller to analyze and stabilized human heart. The proposed system is based on the use of the mathematical model of human heart developed by Yalcinkaya et al. [5]. It is based on symmetry characteristics of human heart and circulatory system. Therefore it is aimed at stimulating and stabilizing the dynamic behaviour of human heart when subject to attack in the form of noise and cardiovascular disease.

## 2. SYSTEM DESIGN

In this paper, the main tools utilized for the human heart simulation and stabilization are the MATLAB codes, the Simulink embedded blocks that were employed for the modelling and simulations, and data from previous literature. The MATLAB codes were used to compute the cascade combinations of the different gains of the various Laplace transfer function models of the human heart representing the heartbeat, the peacemaker, the circulatory system and the feedback system. The Simulink blocks were used to develop the architecture model of the human heart stabilization system including the 2-DOF PID controller. The approaches to the modelling of the proposed system are also shown in this section. The proposed system architecture is shown in Fig. 1.

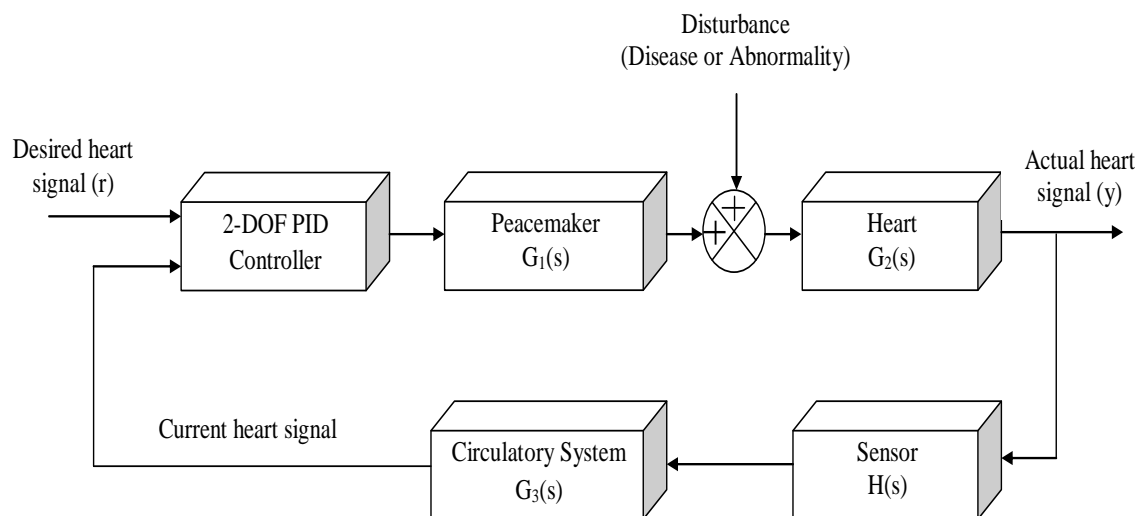


Fig. 1. Human heart stabilization system

### 2.1 System description

The system described in this subsection is a cardiovascular system, represented as electromechanical system. The behaviour of the heart is considered as that of hydro-electromechanical system shown in Fig. 2, which described the common function of cardiovascular system. The figure is characterized by pressure  $P_{IN}(t)$ , right-atrium and left-atrium systemic capacitances  $C_{BL}$  and  $C_{CL}$ , right-ventricle and left-ventricle systemic capacitances  $C_{DL}$  and  $C_{EL}$ , mutual inductance of ventricle and atrium  $L_F$ , inductances of left-ventricles and right-ventricles  $L_E$  and  $L_D$ , right-atrium and left-atrium inductances  $L_B$  and  $L_C$ .  $P_{IN}(t)$  represents the system input,  $C_{BL}$  is considered in the circuit as capacitor and acts as a first pump; the same holds for the  $C_{CL}$ . Pressure from the

right-atrium serves as an input to the right-ventricle and the same goes for the left chamber.

Figure 3 shows the normal heartbeat from ECG, which describes the periodic action of the human heart that is mainly caused by the frequently occurring possible actions initiated by the natural peacemaker at the senatorial node, such that through the myocardium, each impulse will be transmitted to the atria ventricular [2]. The figure describes periodic cycle of three waves, which are the P wave, complex QRS wave and T wave. Looking at the ECG signal, the complex QRS waveform is of higher amplitude among the three waves. Thus, the complex QRS wave must be detected with high accuracy so as to precisely monitor the heart beat rate [2].

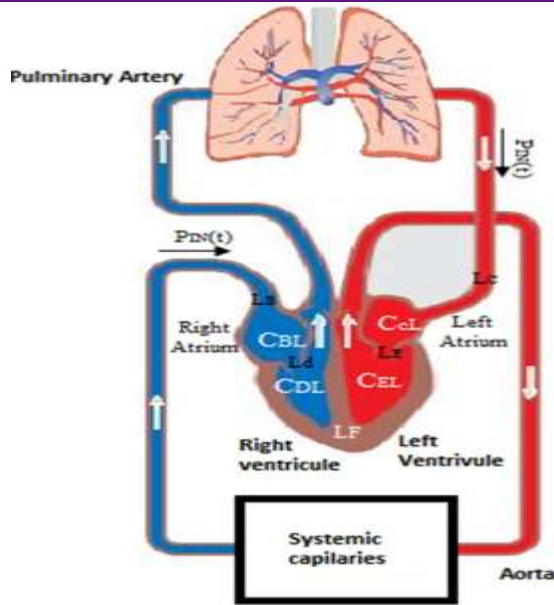


Fig. 2. Hydro-electromechanical characteristic of the system [2]

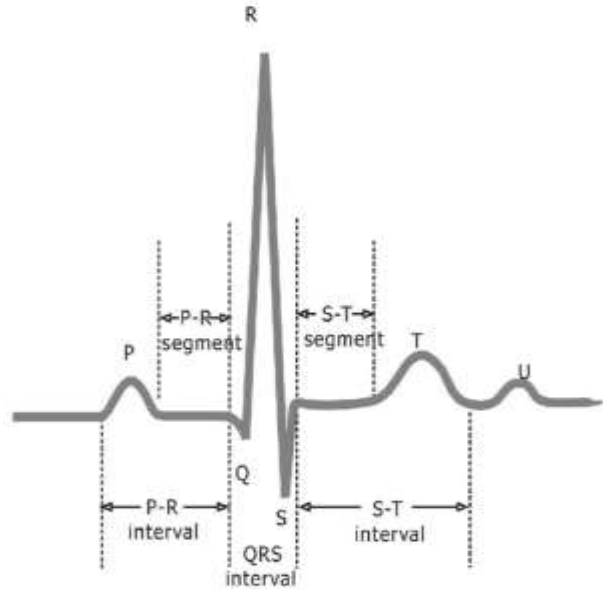


Fig. 3. ECG recorded normal heart beat [2]

## 2.2 Mathematical Model of Human Heart

The mathematical model characterizing human heart periodic actions are presented in this subsection. According to the model, which is based on hydro-electromechanical behaviour of the cardiovascular system, there are four components defined independently by Laplace transfer function given by [5]:

$$G_1(s) = \frac{40s^2 + 30s + 7}{3s^2 + 0.5s + 20} \quad (1)$$

$$G_2(s) = \frac{7}{s + 32} \quad (2)$$

$$G_3(s) = \frac{3s^2 + 25s + 99}{9s^2 + 0.5s + 10} \quad (3)$$

$$H(s) = \frac{15}{2s + 20} \quad (4)$$

where  $G_1(s)$  is the transfer function for the pacemaker medical devices that transmits electrical impulse to specified parts of the body,  $G_2(s)$  is the transfer function for the heartbeat,  $G_3(s)$  is the transfer function for the circulatory system, and  $H(s)$  is the transfer function for the feedback sensor. Figure 4 is the MATLAB/Simulink representation of the human heart model.

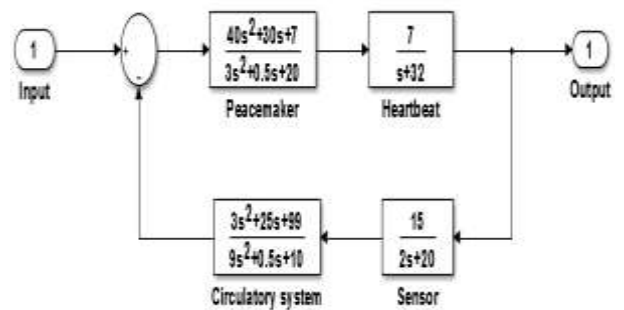


Fig. 4 Simulink model of human heart

In the absence of controller and with negative feedback, the response of the system shown in Figure 4 is given by:

$$Y(s) = R(s) \frac{G_1(s) \times G_2(s)}{1 + [G_1(s) \times G_2(s)][G_3(s) \times H(s)]} \quad (5)$$

where  $R(s)$  is the reference pulse signal.

So far, the cardiovascular system has been modelled considering normal working condition of the human heart. The fictitious noise/disease is represented as  $D(s)$ . However, the response of the system with fictitious noise/disease (disturbance) is given by:

$$Y(s) = R(s) \frac{G_1(s) \times G_2(s)}{1 + [G_1(s) \times G_2(s)][G_3(s) \times H(s)]} + D(s) \frac{G_1(s) \times G_2(s)}{1 + [G_1(s) \times G_2(s)][G_3(s) \times H(s)]} \quad (6)$$

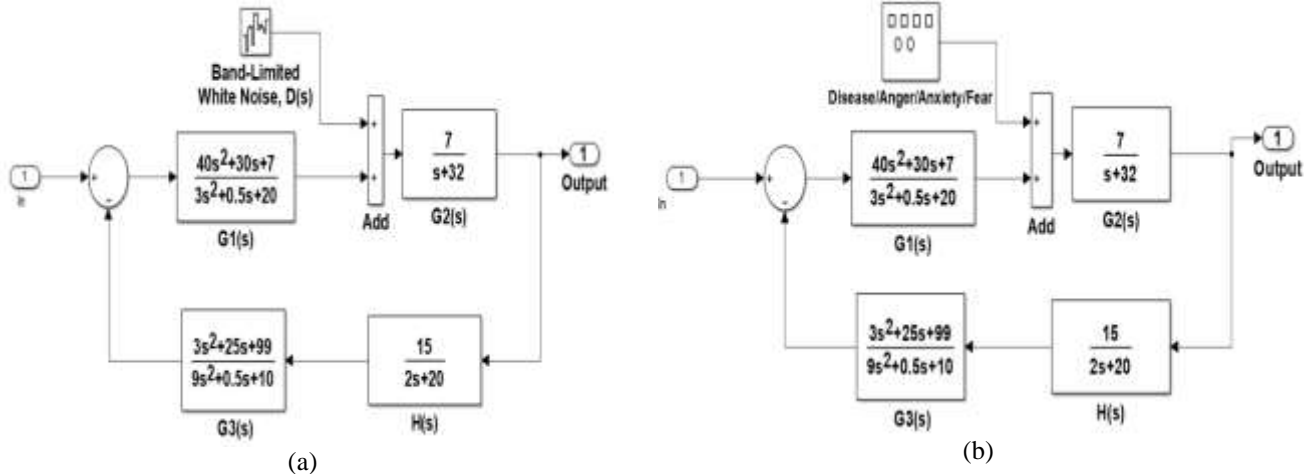


Fig. 5. Simulink model for heart under attack by noise/disease

### 2.3 Two-DOF PID Controller

The human heart stabilization technique implemented in this paper is 2DOF-PID controller. In 2DOF-PID controller, the proportional and derivative terms are multiplied by setpoint weighting. These setpoint weighting parameters (beta and gamma as shown in Figure 6), can be adjusted to determine the degree of proportional (beta) action derivative (gamma) action that can be applied to the changes in setpoint. The controller can offer fast disturbance rejection with no significant overshoot increase in tracking of reference input.

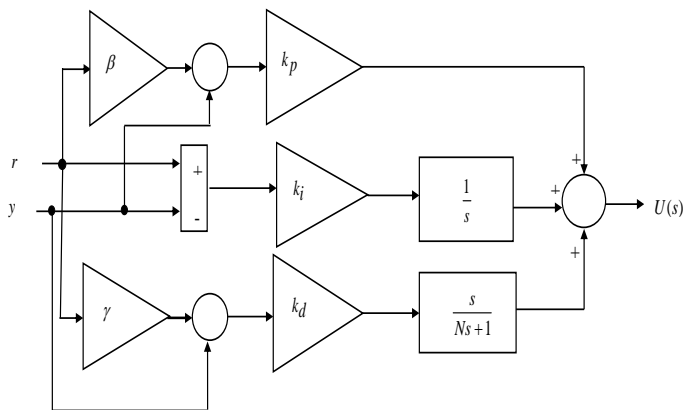


Fig. 6. Block diagram of 2-DOF PID controller

The Simulink models in the situations when the heart is attacked by noise or disease are shown in Fig. 5a and b.

The mathematical definition of the control command of the 2-DOF PID controller is given by:

$$U(s) = k_p (\beta r - y) + k_i \frac{1}{s} + \frac{k_d s}{Ns + 1} (\gamma r - y) \quad (7)$$

where  $\beta \in \{0,1\}$  and  $\gamma \in \{0,1\}$ ,  $k_p, k_i, k_d$  are the proportional, integral and derivative gains of the PID controller,  $r$  and  $y$  are the setpoint (reference input) and the output of the system, and  $N$  is the filter coefficient.

The PID controller is designed using the PID tuner of the MATLAB/Simulink. The tuned parameters, selected setpoint weighting and filter coefficient are shown in Table 1. The Simulink model of the system is shown in Fig. 7.

Table 1 PID parameters

Definition	Symbol	Value
Proportional gain	$k_p$	1.8
Integral gain	$k_i$	1.2
Derivative gain	$k_d$	0.85
Filter coefficient	$N$	20
Setpoint weight	$\beta$	0.25
Setpoint weight	$\gamma$	1

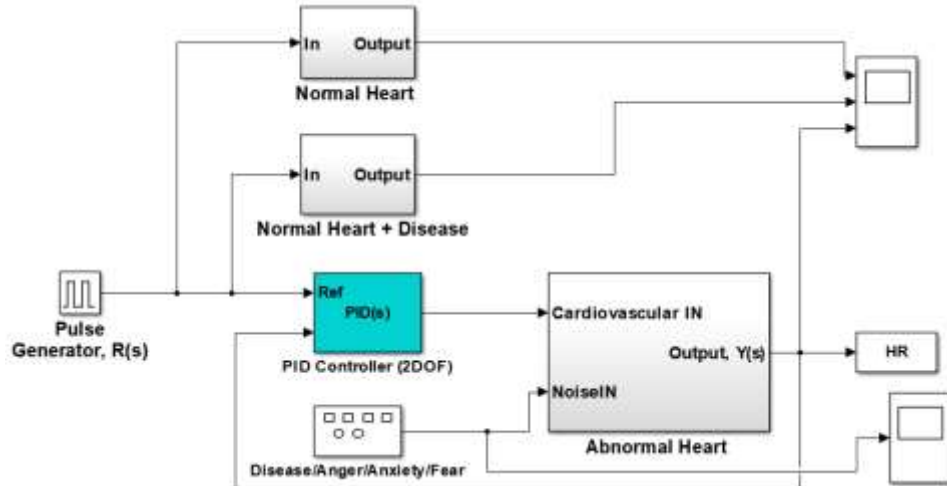


Fig. 7. Simulink model of human heart simulated using 2DOF-PID

### 3. SIMULATION RESULTS

This section presents the simulation results for the condition of human heart at different working conditions. Fig. 8 and 9 are the simulation results of the normal state of the heart and the state of the heart when disturbance in the form noise attack the heart. In Fig. 10, the simulation signal when diseases/anger/anxiety/fear attack the human heartbeat system is shown. Fig. 11 shows the simulation plot of heartbeat system controlled when infected by noise. The plots of normal heart, disease infected heart and controlled disease infected heart are shown in Fig. 12.

For the purpose of simulation of noise and disease infected heart in this paper, the brainwave disturbance parameters have been considered. This is because the rate of heartbeat and the cardiovascular system is largely affected by brainwaves, and can serve as a disturbance to the heart if overdosed [9]. There are four frequency bands in which brainwaves are usually categorized namely, Delta, Theta, Alpha and Beta [10, 2]. The Beta has the highest frequency band but lowest amplitude whereas the Delta frequency band has the lowest frequency but highest amplitude [2]. The category of Brainwave like disturbance used in this paper is the Beta frequency band and the frequency band is 14 – 100 Hz, which comprises arousal, anxiety, disease, and fight [2]. The parameters of the brainwave frequency used are random waveform, amplitude equal to 20, and frequency (anxiety, disease) equal to 100 Hz. The noise disturbance parameters are noise power of 1, sample time of 0.09, and seed 23341. These values were selected for simulation in MATLAB using appropriate Simulink models.

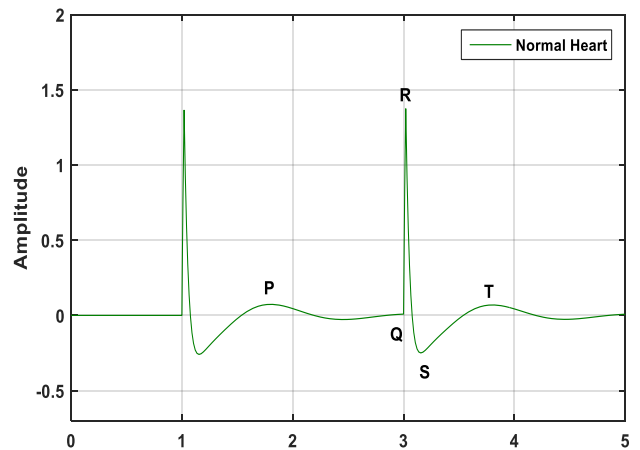


Fig. 8. Normal state of the heart

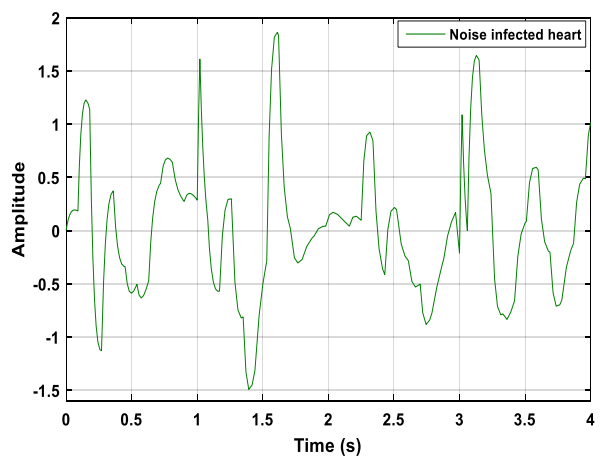


Fig. 9. Human heartbeat with disturbance in form of noise

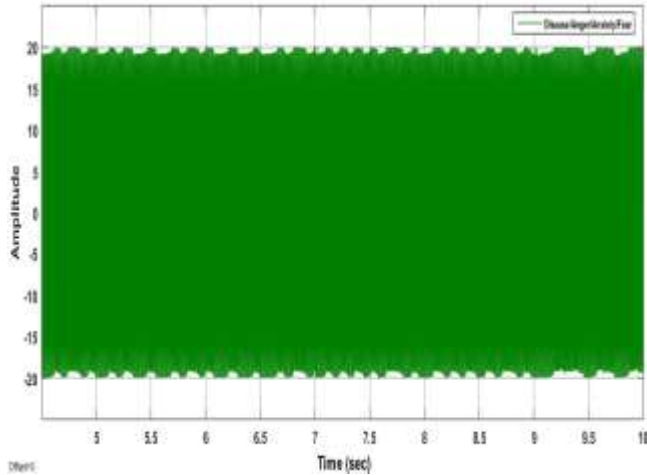


Fig. 10. Disease profile

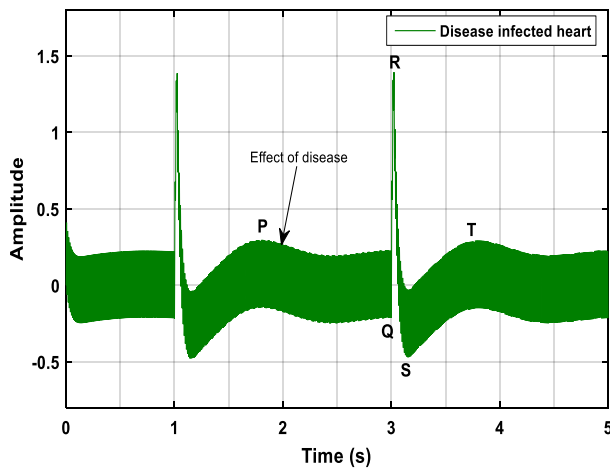


Fig. 11. Disease infected human heart

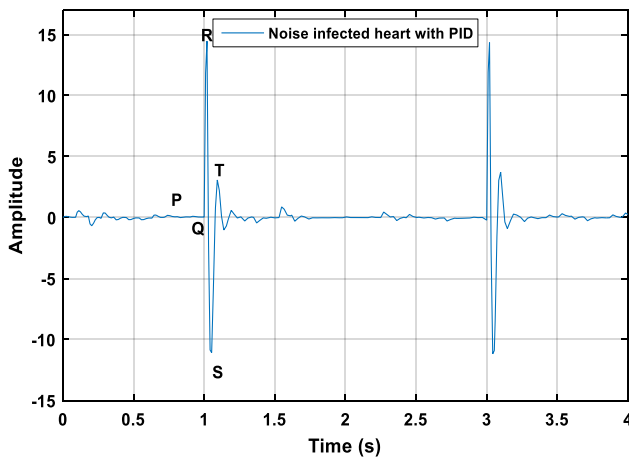


Fig. 12. Controlled noise infected heart

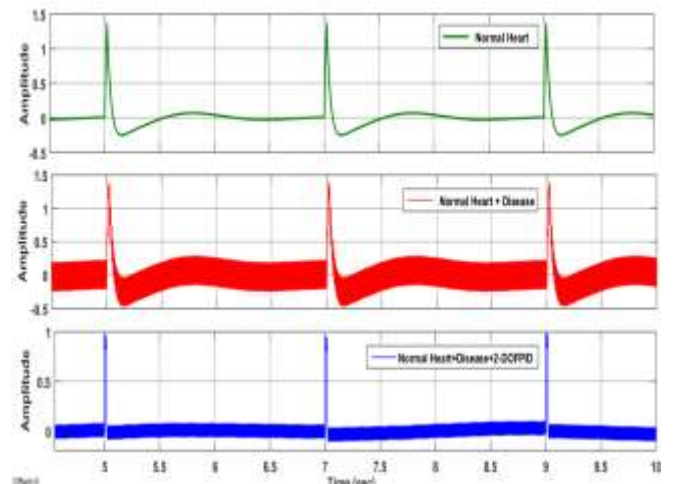


Fig.13. Simulation plots of normal, disease infected and 2-DOF PID control heartbeat system

The frequent potential actions emanating from the peacemaker situated in the senatorial node is largely the cause of the periodic action of the heart (Aabid et al., 2016). Through the myocardium, each periodic heartbeat signal (impulse) will be transmitted to the Atria ventricular node. As shown in Fig. 3, a periodic sequence of three waves namely, P wave, QRS complex wave Which is a combination of Q, R and S waves), and lastly T wave characterised a typical ECG signal. Of these waves, the QRS complex wave posses the most energy with the highest amplitude compared to P and T waves for a given RR interval (i.e. the time between the two adjoining R waves). For precise monitoring of the patients' heartbeat rate, QRS complex wave (or simply the R wave) needs to be detected with high accuracy. The continuous and periodic performance of the natural peacemaker and reliability of neurons in conducting pathways is dependent upon by the normal heartbeat as shown by the simulation result Fig. 8. Hence, the response performance of the human heartbeat in Fig. 8 during normal working condition compares very well with the normal heartbeat scheme from a typical electrocardiogram (ECG) shown in Fig. 3 such that P-wave at 1.80 s, PR-interval at 3.00 s, QT-interval at 3.77 s, and QRS-complex at 3.02 s. The amplitude or strength of the QRS is 1.38 and this remains consistent throughout the working of the heart.

In Fig. 9, the heart is assumed to be infected by noise such that the heartbeat response of the heart is shattered. It can be seen that the strength or amplitude of the heartbeat wave drops to almost zero with the noise or disturbance continuously rising and dropping in amplitude in irregular (or random) manner. This indicates that the noise seriously reduced the working of the cardiovascular system. In Fig. 10, the profile of the cardiovascular disease (such as arousal, anxiety, disease) is presented, which has steady peak to peak amplitude of -20 to 20 and this is above the peak amplitude of the heartbeat during normal condition. When it infects the heart as shown in Fig. 11, it results to increased width of heart pulse and this is

abnormal state compared to the typical human heartbeat from ECG shown in Fig. 3 and as such can lead to heart failure.

In order to stabilize the heart and restore its working integrity, a 2-DOF-PID controller was added to the regulating loop of the heartbeat with noise infection as shown in Fig. 12. It can be seen that the controller ensured that the heart is automatically restored to its appropriate working periodic waveform. That is, the controller was able to stabilize the heart to its right of working state when compared to the heartbeat response during noise infection in Fig. 9. In Fig. 13, the introduction of the controller helps to suppress effect of the disease and stabilized the heart.

#### 4. CONCLUSION

This paper is based on the control of human heartbeat rate as a hydro-electromechanical system (HEMS) using mathematical model. The mathematical model and the response present some of the significant characteristics of cardiovascular system of human when arousal (provocation), anxiety, disease or fight disturbed it and how to heal the effect of the disturbances. Appropriate physical analogy, mathematical descriptions of the system dynamics, and computer based models were formulated to carry out the study. A 2-DOF-PID controller was added to improve the HEMs of human heart model and thus provides control signal to adjust the heartbeat rate if it is disturbed and to command the cardiovascular system. The entire system was built and simulated in MATLAB/Simulink environment. The heartbeat model has been applied for steady state study of cardiovascular system when a disease attacks the heart. The result obtained proves the effectiveness of the developed control strategy. Nevertheless, the heart model can be applied to dynamic processes of the cardiovascular system; the model may be used for cardiac abnormalities detection and analysis; and alternatively, intelligent control strategy such as fuzzy logic can be implemented with the model to improve system performance and robustness.

#### REFERENCES

- [1] Mallick, B., & Patro, A. K. (2016). Heart rate monitoring system using finger tip through arduino and processing software, *International Journal of Science, Engineering and Technology Research*, vol. 5, pp. 84 – 89.
- [2] Aabid, M., Elakkary, A., & Sefiani, N. (2016). Stabilization of human heart using PID controller, *Journal of Theoretical and Applied Information Technology*, vol. 92, no. 1, pp. 162 – 170.
- [3] Thanom, W., & Loh (2011). Nonlinear control of heartbeat models, *Journal of Systematics Cybernetics and Informatics*, vol. 9, no. 1, pp. 21-27.
- [4] Kannathal, N., Acharya, U. R., YK Ng., E., & Lim, C. M. (2006). Cardiac health diagnosis using data fusion of cardiovascular and haemodynamic signals, *Computer Methods and Programs in Biomedicine*, vol. 84, No.2, pp. 87-86.
- [5] Yalcinkaya, F., Kizilkaplan, E., & Erbas, A. (2013). Mathematical modelling of human heart as a hydroelectromechanical system, pp. 362 – 366. <https://www.researchgate.net/publication/261423180>
- [6] Chen, Z., Brown, E. N., & Barbieri, R. (2008). A study of probabilistic models for characterizing human heart beat dynamics in autonomic blockade control, in *proceedings of IEEE International Conference on Acoustic, Speech, and Signal Processing*, March 31, pp. 481-484.
- [7] Brandt, M. E., Wang, G., & Shih, H. T. (2003). Feedback control of nonlinear dual-oscillator heartbeat model, *Lecture Notes in Control and Information Sciences*, vol. 293, pp. 265 – 273.
- [8] Sargolzaei, A., Yen, K. K., & Abdelghani, M. N. (2014). Control of nonlinear heartbeat models under time-delay-switched feedback using emotional learning control. *International Journal on Recent Trends in Engineering And Technology*, 10(2), pp. 85 – 91.
- [9] Lee, W.-N., Fujikura, K., & Konofagou, E. E. (2010). Electromechanical wave imaging of normal and Ischemic hearts, in *Vivo Jean Provost*, 625 – 629.
- [10] Ibrahim, F., Osman, N. A. A., Usman, J., & Kadri, N. A. (2007). Performance evaluation of coifman wavelet for ECG signal denoising, *IFMBE Proceedings*, vol. 15, pp. 419 – 422.