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On Stereographic Semi-circular Shankar Distribution: Properties and Applications

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Abstract: *In this paper, we made an attempt to construct a new Semi-circular model, we call this as "Stereographic Semi-circular Shankar distribution," by applying modified inverse Stereographic projection on Shankar distribution (Rama Shankar (2017)) for modelling semi-circular data. Probability density and cumulative distribution functions of said model are derived and their graphs are plotted for various values of parameters. The first two trigonometric moments are derived and the proposed model is extended for l-axial data also.*

Keywords: The Shankar distribution, moments Semi-circular models, inverse stereographic projection, trigonometric moments.,parameter estimation, maximum likelihood estimation.

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1. Introduction

Angular data/ circular data are very common in the areas of biology, geology, meteorology, earth science, political science, Economics, computer science, etc. Full circular models are prevalent at most of the text books (Fisher, 1993; Jamaal madaka and Sen Gupta, 2001; Mardia and Juppe, 2000). In some of the cases the angular data does not require full circular models for modelling; this fact is noted in Guard Iola (2004), Jones (1968), Byoung et al (2008) and Phani et al (2013). For example, when sea turtles emerge from the ocean in search of a nesting site on dry land, a random variable having values on a semicircle is well sufficient for modelling such data. Similarly, when an aircraft is lost but its departure and its initial headings are known, a semi-circular random variable is sufficient for such angular data. And few more examples of semi-circular data are available in Ugai et al (1977). Guard Iola (2004) obtained the semi-circular normal distribution by using a simple projection and Byoung et al (2008) developed a family of the semi-circular Laplace distributions for modelling semi-circular data by simple projection, Phani et al (2013) constructed some semi-circular distributions by applying modified inverse Stereographic projection, Garcia et al (2014) developed a family of semi-circular Logistic distributions by applying simple projection. In this paper we develop new Semi-circular model coined as Stereographic Semi-circular Shankar distribution by applying modified inverse stereographic projection on Shankar distribution. The graphs of the density function and distribution function for various values of parameters are plotted. We derive the first two trigonometric moments of the proposed model to evaluate population characteristics.

2. Methodology of modified Inverse Stereographic Projection:

Modified Inverse Stereographic Projection is defined by a one to one mapping given by :

$\mathrm{T}(\theta) = x = \mathrm{V}\tan\left(\frac{\theta}{2}\right)$,wher $x \in (-\infty, \infty), \theta \in (-\pi, \pi)$Suppose x is randomly chosen on the interval (–∞ ∞,) .

Let F (x) and f (x) denote the Cumulative distribution and probability density functions of the linear random variable X respectively.

$\mathrm{T}^{-1}(x) = \theta = 2\tan^{-1}\left(\frac{x}{v}\right)$

by Minh and Farnum (2003) is a random point on the unit circle. Let G(θ) and g (θ) denote the Cumulative distribution and probability density functions of this random point θ respectively. Then G(θ) and g (θ) can be written in terms of F (x) and f (x) using the following Theorem.

Theorem 2.1: For v > 0

i. $G(\theta) = \mathrm{F}\left(\mathrm{V}\tan\left(\frac{\theta}{2}\right)\right)$ (1)

ii. $g(\theta) = \mathrm{V}\left(\frac{1+tan^2\left(\frac{\theta}{2}\right)}{2}\right) f\left(\mathrm{V}\tan\left(\frac{\theta}{2}\right)\right)$ (2)

If a linear random variable X has a support on ¡ , then θ has a support on (–π, π) and if X has a support, then θ has a support on (0, π) . These means that, after the Inverse Stereographic Projection is applied, we can deal circular data if the support of X is on ¡ and we can handle semi-circular data if the support of X is on.

3. Stereographic Semi-circular Shankar distribution

Here a linear model Shankar distribution is considered and by inducing modified inverse stereographic projection, a stereographic semi-circular model is defined. Definition: A random variable X on the real line is said to have Shankar distribution with scale parameter $\alpha > 0$ and location parameter α if the probability density and cumulative distribution functions of X are respectively given by

$$f(x,\alpha) = \frac{\alpha^2}{\alpha^2+1}(\alpha+x)e^{-\alpha x} \qquad (3)$$

and

$$F(x,\alpha) = 1 - \left[1 + \frac{\alpha x}{\alpha^2+1}\right]e^{-\alpha x} \qquad (4)$$

Then by applying modified inverse Stereographic projection defined by a one to one mapping

$x = V\tan\left(\frac{\theta}{2}\right), v \in R$which leads to a Semicircular Model on unit semicircle. We call this model as Stereographic Semicircular Shankar distribution. Definition: A random variable XSC on the Semicircle is said to have the Stereographic Semi-circular Shankar distribution with shape parameter $\alpha > 0$, location denoted by Stereographic Semi-circular Shankar distribution if the probability density and the cumulative distribution functions are respectively given by**:**

$$f(\theta,\alpha) = \frac{V}{2} f\left(V\tan\left(\frac{\theta}{2}\right)\right) = \frac{V}{2}\frac{\alpha^2}{\alpha^2+1}\left(\alpha + V\tan\left(\frac{\theta}{2}\right)\right)e^{-\alpha\left(V\tan\left(\frac{\theta}{2}\right)\right)}\sec^2\left(\frac{\theta}{2}\right); \alpha > \mathbf{0}, 0 \le \theta \le \boldsymbol{\pi} \qquad (5)$$

proof the room

$$= \int_0^{\pi} \frac{V}{2}\frac{\alpha^2}{\alpha^2+1}\left(\alpha + V\tan\left(\frac{\theta}{2}\right)\right)e^{-\alpha\left(V\tan\left(\frac{\theta}{2}\right)\right)}\sec^2\left(\frac{\theta}{2}\right)d\theta$$

$$= \frac{\alpha^2}{\alpha^2+1}\left(\frac{V}{2}\right)\int_0^{\pi}\left(\alpha + V\tan\left(\frac{\theta}{2}\right)\right)e^{-\alpha\left(V\tan\left(\frac{\theta}{2}\right)\right)}\sec^2\left(\frac{\theta}{2}\right)d\theta$$

$$= \frac{\alpha^2}{\alpha^2+1}\left(\frac{V}{2}\right)\int_0^{\pi}\left(\alpha + V\tan\left(\frac{\theta}{2}\right)\right)e^{-\alpha\left(V\tan\left(\frac{\theta}{2}\right)\right)}\sec^2\left(\frac{\theta}{2}\right)d\theta$$

let $x = v\tan\left(\frac{\theta}{2}\right), \theta = 2\tan^{-1}\left(\frac{x}{v}\right), d\theta = \frac{2}{\frac{v+x^2}{v}}dx$

$$= \frac{\alpha^2}{\alpha^2+1}\left(\frac{V}{2}\right)\int_0^{\pi}\left(\alpha + V\tan\left(\frac{2\tan^{-1}\left(\frac{x}{v}\right)}{2}\right)\right)e^{-\alpha V\tan\left(\frac{2\tan^{-1}\left(\frac{x}{v}\right)}{2}\right)}\sec^2\left(\frac{2\tan^{-1}\left(\frac{x}{v}\right)}{2}\right)\frac{2}{\frac{v+x^2}{v}}dx$$

$$= \frac{\alpha^2}{\alpha^2+1}\left(\frac{V}{2}\right)\int_0^{\infty}(\alpha+x)e^{-\alpha x}\sec^2\left(\tan^{-1}\left(\frac{x}{v}\right)\right)\frac{2}{v+\frac{x^2}{v}}dx$$

$$\sec^2\tan^{-1}(x) = x^2 + 1 = \sec^2\tan^{-1}\left(\frac{x}{v}\right) = \frac{x^2}{v^2} + 1$$

$$= \frac{\alpha^2}{\alpha^2+1}\left(\frac{V}{2}\right)\int_0^{\infty}(\alpha+x)e^{-\alpha x}\left(\frac{x^2}{v^2}+1\right)\frac{2}{v+\frac{x^2}{v}}dx$$

$$= \frac{\alpha^2}{\alpha^2+1}\left(\frac{V}{2}\right)\int_0^{\infty}(\alpha+x)e^{-\alpha x}\frac{2}{v}dx$$

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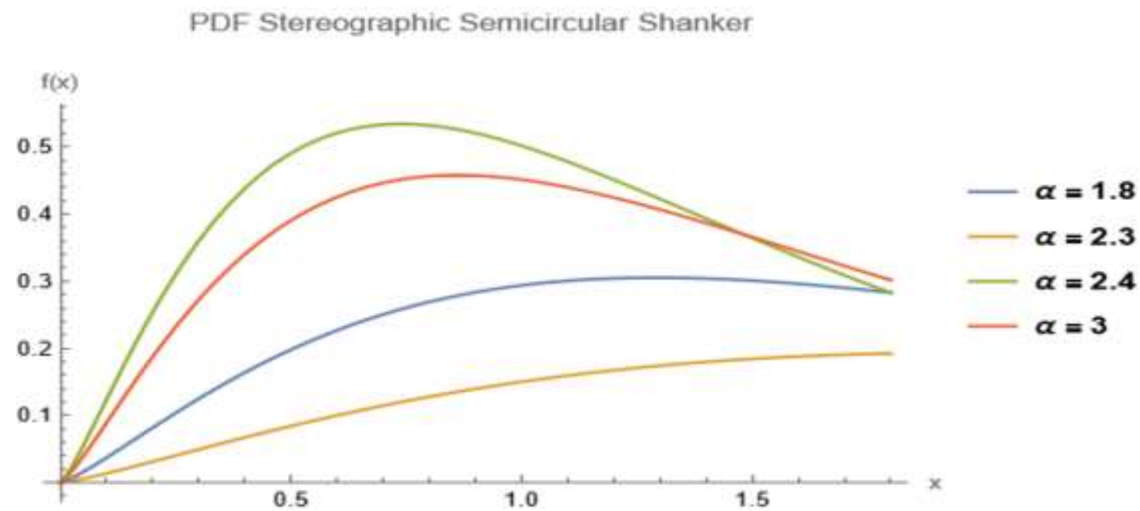
$$=\frac{\alpha^2}{\alpha^2+1}\int_0^{\infty}(\alpha+x)e^{-\alpha x}dx$$

$$=\frac{\alpha^2}{\alpha^2+1}\ \frac{\alpha^2+1}{\alpha^2}$$

$$=1$$

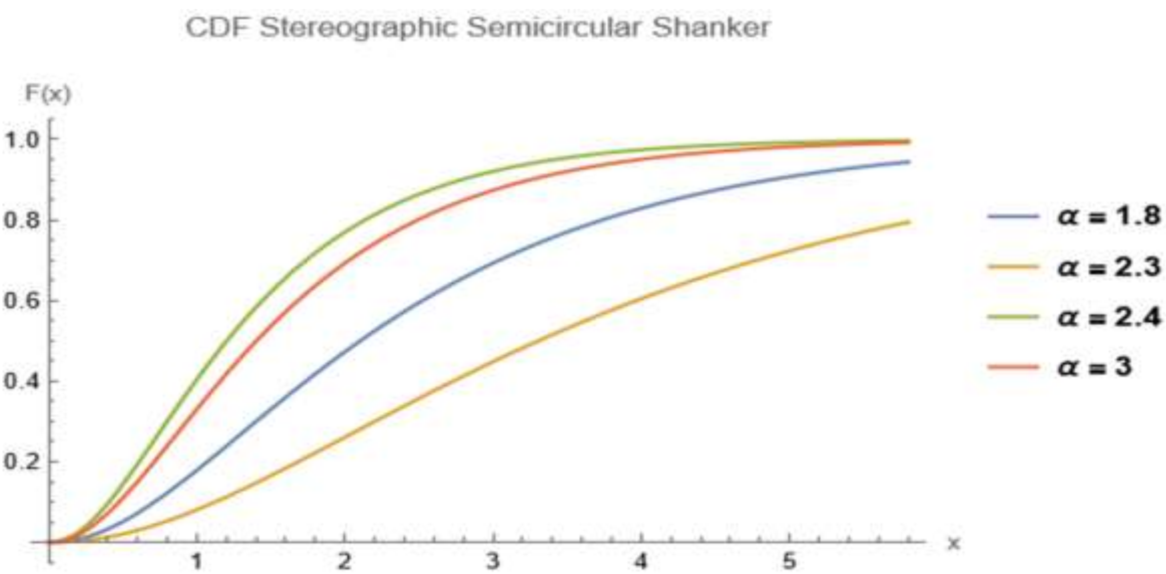
The reasonable shapes of PDF Semi-circular Shankar distribution

The CDF of this distribution is



$$F(\theta,\alpha)=1-\left[1+\frac{\alpha V\tan\left(\frac{\theta}{2}\right)}{\alpha^2+1}\right]e^{-\alpha V\tan\left(\frac{\theta}{2}\right)} \qquad (6)$$

The reasonable shapes of CDF Semi-circular Shankar distribution



The Survival function of the new distribution is:

$R(\theta,\alpha)\ =1-F(\theta,\alpha)$

$$R(\theta,\alpha) = \left(1+\frac{\alpha V\tan\left(\frac{\theta}{2}\right)}{\alpha^2+1}\right)e^{-\alpha V\tan\left(\frac{\theta}{2}\right)} \quad (7)$$

The reasonable shapes Survival Semi-circular Shankar distribution

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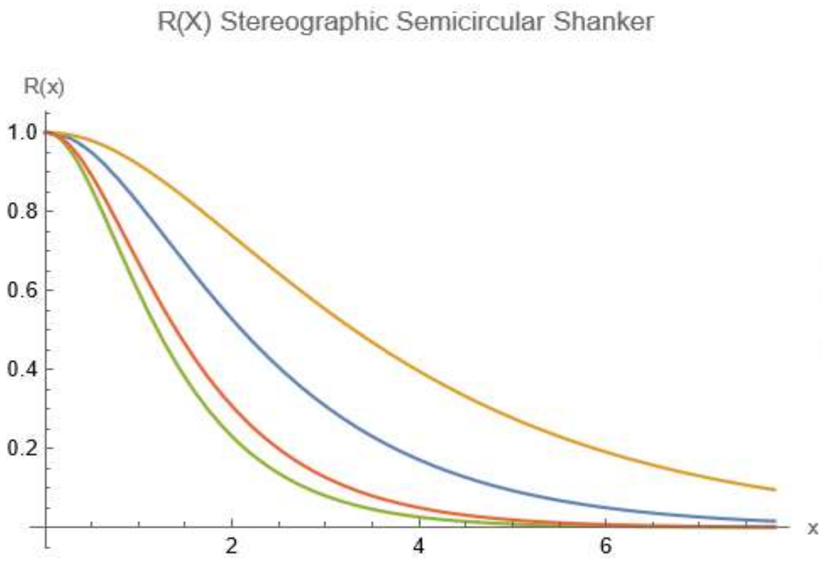

The hazard function of the new distribution is

For any random variable X which follows Semi-circular Shankar distribution, its hazard function is given as:

$$h(\theta,\alpha) = \frac{f(\theta,\alpha)}{s(\theta,\alpha)}$$

$$h(\theta,\alpha) = \frac{\frac{V}{2}f\left(V\tan\left(\frac{\theta}{2}\right)\right) = \frac{V}{2}\frac{\alpha^2}{\alpha^2+1}\left(\alpha+V\tan\left(\frac{\theta}{2}\right)\right)e^{-\alpha\left(V\tan\left(\frac{\theta}{2}\right)\right)}\sec^2\left(\frac{\theta}{2}\right)}{\left(1+\frac{\alpha V\tan\left(\frac{\theta}{2}\right)}{\alpha^2+1}\right)e^{-\alpha V\tan\left(\frac{\theta}{2}\right)}} \quad (8)$$

4.Statistical Properties

In this section, some of the properties of the Semi-circular Shankar distribution are discussed:

4.1Quantile function

The quintile function or inverse cumulative distribution function. returns the value t such that:

$$t = Q(u) = F^{-1}(u), 0 < u < 1$$

$$u = 1-\left(1+\frac{\alpha V\tan\left(\frac{\theta}{2}\right)}{\alpha^2+1}\right)e^{-\alpha V\tan\left(\frac{\theta}{2}\right)}$$

$$\theta = 2\text{ArcTan}[\frac{-\alpha\text{Log}[e]-\alpha^2\text{Log}[e]-\alpha^3\text{Log}[e]-\text{ProductLog}[e^{\alpha(1+\alpha^2)(-1-\frac{\alpha}{1+\alpha^2})}(-1+u)\alpha(1+\alpha^2)\text{Log}[e]]}{v\alpha\text{Log}[e]} \quad (9)$$

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4.2Trigonometric moments of Stereographic Semicircular Shankar distribution

It is customary to derive the trigonometric moments when a new distribution is proposed. Without loss of generality here we assume that $\mu = 0$.The trigonometric moments of the distribution are given by

$\{\varphi_p: p = 0, \pm 1, \pm 2, \pm 3 \ldots\}$ where$\varphi_p = \varphi_p = \alpha_p + i\beta_p$ with $\alpha_p = E(\cos(p\theta))and\beta_p = E(\sin(p\theta))$

being the p^{th} order cosine and sine moments of the random angle θ , respectively.

Theorem 4.2Under the pdf of Stereographic Semicircular Shankar distribution with $\mu = 0$, the first four α θ $\alpha_p = E(\cos(p\theta))$ and $\beta_p = E(\sin(p\theta))$ are given as follows:

$$\alpha_1 = \int_0^\infty \cos(\theta) \frac{\alpha^2}{\alpha^2+1}\left(\alpha + V\tan\left(\frac{\theta}{2}\right)\right) e^{-\alpha\left(V\tan\left(\frac{\theta}{2}\right)\right)} \sec^2\left(\frac{\theta}{2}\right) d\theta$$

Consider the transformation $x = \tan, \cos(\theta) = 1 - \frac{2x^2}{1+x^2}$

$$\alpha_1 = \left(1 - \frac{2\alpha^3}{\alpha^2+1}\left[\frac{2}{2\sqrt{\pi}} G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}-\frac{1}{2}\\ \frac{-1}{2},0,\frac{1}{2}\end{matrix}\right) - \frac{2\alpha^2}{2(\alpha^2+1)\sqrt{\pi}} G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}1\\ -1,0,\frac{1}{2}\end{matrix}\right)\right]\right) \tag{10}$$

$$\beta_1 = \frac{\alpha^2}{\alpha^2+1}\int_0^\infty (\sin(\theta)) \; (\alpha + x)e^{-\alpha x} dx$$

Consider the transformation $x = \tan, \sin(\theta) = \frac{2x}{1+x^2}$

$$\beta_1 = \left(\frac{2\alpha^3}{\alpha^2+1}\left[\frac{1}{2\sqrt{\pi}} G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}0\\ 0,0,\frac{1}{2}\end{matrix}\right) + \frac{2}{2\sqrt{\pi}} G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}-\frac{1}{2}\\ \frac{-1}{2},0,\frac{1}{2}\end{matrix}\right)\right]\right) \tag{11}$$

$$\alpha_2 = \frac{\alpha^2}{\alpha^2+1}\left(\int_0^\infty \cos 2\theta \; (\alpha + x)e^{-\alpha x} dx\right)$$

Consider the transformation $x = \tan; \cos 2\theta = 1 + \frac{8x^4}{(1+x^2)^2} - \frac{8x^2}{(1+x^2)}$

$$\alpha_2 = \left(\begin{matrix}1 + \frac{\alpha^2}{\alpha^2+1}\left[\frac{4\alpha}{\sqrt{\pi}} G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}\frac{-3}{2}\\ \frac{-1}{2},0,\frac{1}{2}\end{matrix}\right) + \frac{8}{2\sqrt{\pi}} G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}-2\\ -1,0,\frac{1}{2}\end{matrix}\right) - \frac{8\alpha}{2\sqrt{\pi}} G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}\frac{-1}{2}\\ \frac{-1}{2},0,\frac{1}{2}\end{matrix}\right)\right.\\ \left. - \frac{4}{\sqrt{\pi}} G_{13}^{31} \frac{\alpha^2}{4}\middle|\begin{matrix}1\\ -1,0,\frac{1}{2}\end{matrix}\right]\end{matrix}\right) \tag{12}$$

Consider the transformation $x = \tan, \sin 2\theta \; = \frac{\mathbf{4x}}{(\mathbf{1+x^2})} - \frac{\mathbf{8x^3}}{(\mathbf{1+x^2})^2}$

$$\beta_2 = \left(\begin{matrix}\frac{\alpha^2}{\alpha^2+1}\frac{2\alpha}{\sqrt{\pi}} G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}0\\ 0,0,\frac{1}{2}\end{matrix}\right) + \frac{4}{\sqrt{\pi}} G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}-\frac{1}{2}\\ \frac{-1}{2},0,\frac{1}{2}\end{matrix}\right) - \frac{4\alpha}{\sqrt{\pi}} G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}1\\ -1,0,\frac{1}{2}\end{matrix}\right)\\ - \frac{4}{\sqrt{\pi}} G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}\frac{-3}{2}\\ \frac{-1}{2},0,\frac{1}{2}\end{matrix}\right)\end{matrix}\right) \tag{13}$$

4.3 the Circular mean

Coefficient of mean for Stereographic Semi-circular Shankar distribution is given by:

$$\mu = \tan^{-1}\left(\frac{\beta_1}{\alpha_1}\right)$$

$$\mu=\tan^{-1}\left(\frac{\left(\frac{2\alpha^3}{\alpha^2+1}\left[\frac{1}{2\sqrt{\pi}}G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}0\\0,0,\frac{1}{2}\end{matrix}\right)+\frac{2}{2\sqrt{\pi}}G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}-\frac{1}{2}\\\frac{-1}{2},0,\frac{1}{2}\end{matrix}\right)\right]\right)}{\left(1-\frac{2\alpha^3}{(\sqrt{\pi})\alpha^2+1}\left[G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}-\frac{1}{2}\\\frac{-1}{2},0,\frac{1}{2}\end{matrix}\right)-\frac{\alpha^2}{(\sqrt{\pi})\alpha^2+1}G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}1\\-1,0,\frac{1}{2}\end{matrix}\right)\right]\right)}\right) \qquad (14)$$

4.4 the Circular median

Coefficient of median for Stereographic Semi-circular Shankar distribution is given by:

$$\rho=\sqrt{{\alpha_1}^2+{\beta_1}^2}$$

$$\rho=\sqrt{\left(\begin{matrix}\left(1-\frac{2\alpha^3}{(\sqrt{\pi})\alpha^2+1}\left[G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}-\frac{1}{2}\\\frac{-1}{2},0,\frac{1}{2}\end{matrix}\right)-\frac{\alpha^2}{(\sqrt{\pi})\alpha^2+1}G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}1\\-1,0,\frac{1}{2}\end{matrix}\right)\right]\right)^2\\+\left(\frac{2\alpha^3}{\alpha^2+1}\left[\frac{1}{2\sqrt{\pi}}G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}0\\0,0,\frac{1}{2}\end{matrix}\right)+\frac{2}{2\sqrt{\pi}}G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}-\frac{1}{2}\\\frac{-1}{2},0,\frac{1}{2}\end{matrix}\right)\right]\right)^2\end{matrix}\right)} \qquad (15)$$

4.5 the circular variance

Coefficient of variance for Stereographic Semi-circular Shankar distribution is given by:

$$v=1-\rho=\sqrt{{\alpha_1}^2+{\beta_1}^2}$$

$$v=1-\sqrt{\left(\begin{matrix}\left(1-\frac{2\alpha^3}{(\sqrt{\pi})\alpha^2+1}\left[G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}-\frac{1}{2}\\\frac{-1}{2},0,\frac{1}{2}\end{matrix}\right)-\frac{\alpha^2}{(\sqrt{\pi})\alpha^2+1}G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}1\\-1,0,\frac{1}{2}\end{matrix}\right)\right]\right)^2\\+\left(\frac{2\alpha^3}{\alpha^2+1}\left[\frac{1}{2\sqrt{\pi}}G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}0\\0,0,\frac{1}{2}\end{matrix}\right)+\frac{2}{2\sqrt{\pi}}G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}-\frac{1}{2}\\\frac{-1}{2},0,\frac{1}{2}\end{matrix}\right)\right]\right)^2\end{matrix}\right)} \qquad (16)$$

4.6the circular Standard deviation

Coefficient of Standard deviation for Stereographic Semi-circular Shankar distribution is given by:

$$\sigma=\sqrt{-\log\left({\alpha_1}^2+{\beta_1}^2\right)}$$

$$\sigma=\sqrt{-\log\left(\begin{matrix}\left(1-\frac{2\alpha^3}{(\sqrt{\pi})\alpha^2+1}\left[G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}-\frac{1}{2}\\\frac{-1}{2},0,\frac{1}{2}\end{matrix}\right)-\frac{\alpha^2}{(\sqrt{\pi})\alpha^2+1}G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}1\\-1,0,\frac{1}{2}\end{matrix}\right)\right]\right)^2\\+\left(\frac{2\alpha^3}{\alpha^2+1}\left[\frac{1}{2\sqrt{\pi}}G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}0\\0,0,\frac{1}{2}\end{matrix}\right)+\frac{2}{2\sqrt{\pi}}G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}-\frac{1}{2}\\\frac{-1}{2},0,\frac{1}{2}\end{matrix}\right)\right]\right)^2\end{matrix}\right)} \qquad (17)$$

4.7 the Circular skewness

Coefficient of Sleekness for Stereographic Semi-circular Shankar distribution is given by:

$$\text{Circular skewness}=\frac{\beta_2}{(1-\rho)^{\frac{2}{3}}}$$

$$S_k=\frac{\left(\begin{matrix}\frac{\alpha^2}{\alpha^2+1}\frac{2\alpha}{\sqrt{\pi}}G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}0\\0,0,\frac{1}{2}\end{matrix}\right)+\frac{4}{\sqrt{\pi}}G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}-\frac{1}{2}\\\frac{-1}{2},0,\frac{1}{2}\end{matrix}\right)-\frac{4\alpha}{\sqrt{\pi}}G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}1\\-1,0,\frac{1}{2}\end{matrix}\right)\\-\frac{4}{\sqrt{\pi}}G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}\frac{-3}{2}\\\frac{-1}{2},0,\frac{1}{2}\end{matrix}\right)\end{matrix}\right)}{\left(1-\sqrt{\left(\begin{matrix}\left(1-\frac{2\alpha^3}{(\sqrt{\pi})\alpha^2+1}\left[G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}-\frac{1}{2}\\\frac{-1}{2},0,\frac{1}{2}\end{matrix}\right)-\frac{\alpha^2}{(\sqrt{\pi})\alpha^2+1}G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}1\\-1,0,\frac{1}{2}\end{matrix}\right)\right]\right)^2\\+\left(\frac{2\alpha^3}{\alpha^2+1}\left[\frac{1}{2\sqrt{\pi}}G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}0\\0,0,\frac{1}{2}\end{matrix}\right)+\frac{2}{2\sqrt{\pi}}G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}-\frac{1}{2}\\\frac{-1}{2},0,\frac{1}{2}\end{matrix}\right)\right]\right)^2\end{matrix}\right)}\right)^{\frac{2}{3}}} \qquad (18)$$

4.8 Coefficient of Kurtosis

The Coefficient of Kurtosis of for Stereographic Semi-circular Shankar distribution is given by:

$$\text{Circular kurtosis} = \frac{\alpha_2 - \rho^4}{(1-\rho)^4}$$

$$= \frac{\left(1+\frac{\alpha^2}{\alpha^2+1}\left[\frac{4\alpha}{\sqrt{\pi}}G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|{\frac{-3}{2} \atop \frac{-1}{2},0,\frac{1}{2}}\right)+\frac{8}{2\sqrt{\pi}}G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|{-2 \atop -1,0,\frac{1}{2}}\right)-\frac{8\alpha}{2\sqrt{\pi}}G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|{\frac{-1}{2} \atop \frac{-1}{2},0,\frac{1}{2}}\right) - \frac{4}{\sqrt{\pi}}G_{13}^{31}\frac{\alpha^2}{4}\middle|{1 \atop -1,0,\frac{1}{2}}\right]\right) - \left(\sqrt{\left(\left(1-\frac{2\alpha^3}{(\sqrt{\pi})\alpha^2+1}\left[G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|{-\frac{1}{2} \atop \frac{-1}{2},0,\frac{1}{2}}\right)-\frac{\alpha^2}{(\sqrt{\pi})\alpha^2+1}G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|{1 \atop -1,0,\frac{1}{2}}\right)\right]\right)^2 + \left(\frac{2\alpha^3}{\alpha^2+1}\left[\frac{1}{2\sqrt{\pi}}G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|{0 \atop 0,0,\frac{1}{2}}\right)+\frac{2}{2\sqrt{\pi}}G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|{-\frac{1}{2} \atop \frac{-1}{2},0,\frac{1}{2}}\right)\right]\right)^2\right)}\right)^4}{\left(1-\sqrt{\left(\left(1-\frac{2\alpha^3}{(\sqrt{\pi})\alpha^2+1}\left[G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|{-\frac{1}{2} \atop \frac{-1}{2},0,\frac{1}{2}}\right)-\frac{\alpha^2}{(\sqrt{\pi})\alpha^2+1}G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|{1 \atop -1,0,\frac{1}{2}}\right)\right]\right)^2 + \left(\frac{2\alpha^3}{\alpha^2+1}\left[\frac{1}{2\sqrt{\pi}}G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|{0 \atop 0,0,\frac{1}{2}}\right)+\frac{2}{2\sqrt{\pi}}G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|{-\frac{1}{2} \atop \frac{-1}{2},0,\frac{1}{2}}\right)\right]\right)^2\right)}\right)^4} \quad (19)$$

5.Parameter estimation:

the Method of Maximum Likelihood Estimate is Used for Estimating The Parameters of The Newly Proposed Distribution Known as of Semi-circular Shankar distribution. Let x1, x2,…,xn be a Random Sample of Ssize n From of The Semi-circular Shankar distribution, Fhen the Corresponding likelihood Function is Given By:

Let x1 ,x2,x3, x4,........xn be a random sample of size n from Semi-circular Shankar distribution The likelihood function, L Semi-circular Shankar distribution is given by:

$$L(\theta_1, \theta_2 \dots \theta_n, \alpha) = f(\theta_1, \alpha).f(\theta_2, \alpha) \dots f(\theta_n, \alpha)$$

$$L(\theta_i;\ \alpha) = \prod_{i=1}^{n} f(\theta_i, \alpha)$$

$$L(\theta_i;\ \alpha) = \prod_{i=1}^{n}\left[\frac{V\alpha^2}{2(\alpha^2+1)}\left(\alpha + V\tan\left(\frac{\theta}{2}\right)\right)e^{-\alpha\left(V\tan\left(\frac{\theta}{2}\right)\right)}\sec^2\left(\frac{\theta}{2}\right)\right]$$

$$L(\theta_i;\ \alpha) = \left(\frac{V\alpha^2}{2(\alpha^2+1)}\right)^n e^{-\alpha V\sum_{i=1}^{n}\tan\left(\frac{\theta_i}{2}\right)}\sum_{i=1}^{n}\sec^2\left(\frac{\theta_i}{2}\right)\prod_{i=1}^{n}\left(\alpha + V\tan\left(\frac{\theta}{2}\right)\right)$$

$$\log L = n\log(V\alpha^2) - n\log\left(2(\alpha^2+1)\right) - \alpha V\sum_{i=1}^{n}\tan\left(\frac{\theta_i}{2}\right) + 2\sum_{i=1}^{n}\log\left(\sec\left(\frac{\theta_i}{2}\right)\right) + \sum_{i=1}^{n}\log\left(\alpha + V\tan\left(\frac{\theta}{2}\right)\right)$$

$$\frac{\partial \log L}{\partial \alpha} = \frac{2n}{\alpha} - \frac{4n\alpha}{2+2\alpha^2} - nv\mathrm{Tan}\left[\frac{\theta i}{2}\right] + \frac{n}{\alpha + v\mathrm{Tan}\left[\frac{\theta i}{2}\right]} = 0 \quad (20)$$

The maximum likelihood estimates ($\hat{\alpha}$) equations $\frac{dLogL}{d\alpha} = 0$, ,The Equation (20), cannot be solved as they both are in closed forms. So we compute the parameters of the Semi-circular Shankar distribution.

Conclusions

In this paper, we discussed circular distribution resulting from extending Stereographic Semi-circular Shankar distribution on unit semicircle which is obtained by inducing Inverse Stereographic Projection on the real line. The density and distribution functions of Journal of New Theory Stereographic Circular Shankar distribution admit explicit forms, as do trigonometric moments and observed that in similar to linear case, Stereographic Circular Exponential and Stereographic Circular Rayleigh distributions are Special cases to proposed Semi-circular Shankar distribution. As this distribution is asymmetric, promising for modelling asymmetrical directional data. Semi-circular Shankar distribution.

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