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Intuitionistic Fuzzy K-ideal in Q-algebra

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Abstract : *We consider the intuitionistic fuzzy K-ideal of Q-algebra and image (preimage) of intuitionistic fuzzy k-ideal in Q-algebra , and investigate some of their properties .Moreover , we introduce definition constant intuitionistic fuzzy set* A_I *, and investigate some related properties .*

1 Introduction

Q-algebra is introduced by H.S. Kim ([2]) in 2001 . The concept of fuzzy set was first initiated by Zadeh ([8]) in 1965 intuitionistic fuzzy set have been defined by G.Takeuti and S.Titanti in ([9]) . H.K Abdullah and H.K.jawad([3]) stud . K-ideal and fuzzy K-ideal in Q-algebra In this paper , we introduce the notion of intuitionistic fuzzy k-ideals in Q-algebra and fuzzy intuitionistic image (inverse) of intuitionistic fuzzy k-ideals in Q-algebra and investigate some result . We also studied the intersection and the union is not necessary to verify the definition of intuitionistic fuzzy k-ideal and showed some important relationships between the intuitionistic fuzzy k-ideal and showed that the converse is not always true . We also added a property to make the converse true , which is the involutory property . We also added a property to make the converse true , which is the involutory property . We also showed the relationship of k-ideal to the lower and upper-cut set in Q-algebra .

2 Background

In this section , we recalled the definitions Q-algebra , bounded Q-algebra , unit , involution intuitionistic fuzzy set , $(\alpha\ ,\ \beta)$ - cut set , homomorphism , epimorphism , monomorphism , isomorphism , intuitionistic fuzzy ideal .

Definition (2.1) [2]
A Q-algebra is a set X with a binary operation $\star$ and constant 0 that fulfilled the following axioms :

1- $x \star x = 0$, $\forall x \in X$.
2- $x \star 0 = x$, $\forall x \in X$.
3- $(x \star y) \star z = (x \star z) \star y$, $\forall x, y, z \in X$.

Remark(2.2) [2]
In a Q-algebra X , we can define a binary relation $\leq$ by putting $x^\star$ represented relation $\leq$ by which $x \leq y$ if and only if $x \star y = 0$, $\forall x, y \in X$.

Definition(2.3) [1]
A Q-algebra $(X, \star, 0)$ is called bounded if there is an element $e \in X$ that satisfies $x \leq e$, $\forall x \in X$. Then e is said to be a unit , we denoted $e \star x$ by $x^\star$, for each $x \in X$ in bounded Q-algebra .

Remark(2.4)
From now on , all Q-algebra will be bounded with only one unit .

Definition(2.5)[1]
For a bounded Q-algebra X , if element x of X satisfies $x^{\star\star} = x$, then x is called an involution if every element of X is an involution , we call X is an involutory Q-algebra .

Proposition (2.6) [3]
In bounded Q-algebra X , for any $x, y \in X$, the following are hold :

1- $e^\star = 0$ $.0^\star = e$
2- $x^\star \star y = y^\star \star x$
3- $0 \star y = 0$
4- $e^\star \star x = 0$
5- $x^{\star\star} \leq x$

Definition(2.7)[3]

Let $(X ,\star , 0)$ be a bounded Q-algebra and I be a nonempty subset of X . Then I is called K-ideal of X satisfies :

1- $0 \in I$
2- $y^{\star} \star x \in I \ , y \in I$ imples $x^{\star} \in I \ , \ \forall x \in X$.

In short we use $(K - ID)$ instead of K-ideal .

Definition(2.8)[5]

An intuitionistic fuzzy set ($I - F - S\ for\ short$) A in a set X is object having the form $A = \{< x , \mu_A(x)\ , \nu_A(x)\ >: x \in X\ \}$, such that $\mu_A : X \to [0 , 1]$ and $\nu_A : X \to [0 , 1]$ denoted the degree of membership (namely $\mu_A(x)$) , and $0 \le \mu_A(x) + \nu_A(x) \le 1$, $\forall x \in X$ for the sake of simplicity , we shall use the notation $A = \{< x , \mu_A(x)\ , \nu_A(x) >\}$ instead of $A = \{< x , \mu\_A\ (x)\ , \nu\_A\ (x)\ >: x \in X\ \}$

Definition(2.9)[4]

Let X be a Q-algebra . An intuitionistic fuzzy set A in X is called an intuitionistic fuzzy Q-subalgebra of X if it satisfies

1- $\mu_A(x \star y) \ge \min\{\mu_A(x\ ) , \mu_A(\ y) \}$, $\forall x \ \in X$.
2- $\nu_A(x \star y) \le \max\{\nu_A(x\ ) , \nu_A(\ y) \}$, $\forall x \in X$.

Definition(2.10)[7]

Let α be a mapping from a set X to a set G if $B = \left\{< x , \mu_\beta(x)\ , \nu_\beta(x) >: x \in X\right\}$ is an I F S in X , then the pre – image of B under α denoted by $\alpha^{-1}\ (B) = \left\{< x , \alpha^{-1}\left(\mu_\beta(x)\right) , \alpha^{-1}\left(\nu_\beta(x)\right)\right\}$ such that $\alpha^{-1}\left(\mu_\beta(x)\right) = \mu_\beta\big(\alpha(x)\big)\quad and \quad \alpha^{-1}\left(\nu_\beta(x)\right) = \ \nu_\beta\big(\alpha(x)\big)$ and if $A = \{< x , \mu_A(x)\ , \nu_A(x) >: x \in X\ \}$ is I F S in X , then the image of A ander α denoted by $\alpha(A) = \left\{<\ g , \alpha_{sup}\big(\mu_A(g)\big) , \alpha_{inf}\big(\nu_A(g)\big)\ >: g \in X\right\}$, where

$$\alpha_{sup}\big(\mu_A(g)\big) = \begin{cases} sup_{x\in\alpha^{-1}(g)}\ \mu_A(x) & if \quad \alpha^{-1}(g) \ne \phi \\ 0 & otherwise \end{cases}$$

$$\alpha_{inf}\big(\nu_A(g)\big) = \begin{cases} inf_{\,x\in\alpha^{-1}(g)}\ \nu_A(x) & if \quad \alpha^{-1}(g) \ne \phi \\ 0 & otherwise \end{cases}$$ for all $g \in G$.

Definition(2.11)[5]

IF $A = \{< x , \mu_A(x)\ , \nu_A(x)\ >: x \in X\ \}$ and $B = \{< x , \mu_B(x)\ , \nu_B(x)\ >: x \in X\ \}$
Be any two IFS of A

1- $A \subseteq B$ if and only if for all $x \in X$, $\mu_A(x) \ge \mu_B(x)$ and $\nu_A(x)\ \le \nu_B(x)$.
2- $A = B$ if and only if for all $x \in X$, $\mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x)$.
3- $A \cap B = \{ < x : (\ \mu_A \cap \mu_B)(x)\ , (\ \nu_A \cup \nu_B)(x)\ \ > : x \in X \}$ Where
   $(\ \mu_A \cap \mu_B)(x) = \min\{\ \mu_A(x)\ , \mu_B(x) \}$ and $(\ \nu_A \cup \nu_B)(x) = \max\{\ \nu_A(x)\ , \nu_B(x) \}$
4- $A \cup B = \{ < x : (\ \mu_A \cup \mu_B)(x)\ , (\ \nu_A \cap \nu_B)(x)\ \ > : x \in X \}$ Where
   $(\ \mu_A \cup \mu_B)(x) = \max\{\mu_A(x)\ , \mu_B(x) \}$ and $(\ \nu_A \cap \nu_B)(x) = \min\{\nu_A(x)\ , \nu_B(x) \}$
   In general , if $\{A_\alpha : \alpha \in \Lambda \}$ is family of I-F-S $\in X$
   $\cap_{\alpha\in\Lambda}\ A_\alpha(x) = \left(\inf\{\mu_{A_\alpha}(x)\ , \alpha \in \Lambda\}, \sup\{\nu_{A_\alpha}(x)\ , \alpha \in \Lambda\}\right)$ and
   $\cup_{\alpha\in\Lambda}\ A_\alpha(x) = \left(\sup\{\mu_{A_\alpha}(x)\ , \alpha \in \Lambda\}, \inf\{\nu_{A_\alpha}(x)\ , \alpha \in \Lambda\}\right)$

Definition(2.12)[6]

For any Intuitionistic fuzzy set $A = \{< x , \mu_A(x)\ , \nu_A(x)\ >: x \in X\ \}$ of a set X we define a $(\alpha , \beta) - cut$ of A as the crisp set $\{\ x \in X\ |\ \mu_A(x) \ge \alpha\ ,\ \nu_A \le \beta\}$ of X and it is denoted by $C_{\alpha,\beta}$ or $A_{\alpha,\beta}$.

Definition(2.13)[4]

An intuitionistic fuzzy set $A = \{< x , \mu_A(x)\ , \nu_A(x)\ >: x \in X\ \}$ In Q-algebra X is called an Intuitionistic fuzzy ideal (In short I-F-ID) if

1- $\mu_A(0) \ge \mu_A(x)$ $\forall x \in X$.
2- $\nu_A(0) \le \nu_A(x)$ $\forall x \in X$.
3- $\mu_A(x) \ge \min\{\mu_A(x \star\ y)\ \ , \mu_A(y) \}$ $\forall x , y \ \in\ X$.
4- $\nu_A(x) \le \max\{\nu_A(x \star\ y)\ \ , \nu_A(y) \}$ $\forall x , y \ \in\ X$.

Definition(2.14)

Let I be a subset of A Q-algebra X and let $\alpha , \beta \in [0 ,1]$ s.t $0 \leq \alpha + \beta \leq 1$ .Define the $I - f - S$ $A_I : X \to [0 , 1]$ by :

$$\mu_{A_I} = \begin{cases} \alpha & : if \ x \in X \\ 0 & otherwise \end{cases} \qquad \nu_{A_I} = \begin{cases} \beta & : if \ x \in X \\ 0 & otherwise \end{cases}$$

Then A_I is called constant intuitionistic fuzzy set.

3 Intuitionistic fuzzy k-ideal

We define intuitionistic fuzzy k-ideal and give some example and describe its relationship with an intuitionistic fuzzy ideal and give it some results .

Definition(3.1)

An intuitionistic fuzzy set A In bounded Q-algebra $(X ,\star , 0)$ is called an Intuitionistic fuzzy k-ideal (In short($I - F - k - ID$) if

1- $\mu_A(0) \geq \mu_A(x)$ & $\nu_A(0) \leq \nu_A(x)$ $\forall x \in X$.
2- $\mu_A(x^\star) \geq \min\{\mu_A(y^\star \star x) , \mu_A(y) \}$ $\forall x , y \in X$.
3- $\nu_A(x) \leq \max\{\nu_A(y^\star \star x) , \nu_A(y) \}$ $\forall x , y \in X$.

Example (3.2)

Let $X = \{ 0 , a, b, c , d \}$ and let $\star$ ba a binary operation on X which is defined by

$\star$	0	a	b	c	d
0	0	0	0	0	0
a	*a*	0	*a*	0	0
b	*b*	*b*	0	*b*	0
c	*c*	0	*c*	0	0
d	*d*	*d*	0	*b*	0

Its clear that $(X ,\star , 0)$ is a bounded Q-algebra with unit d .

Define the I-F-S A and B in X by :

$$\mu_A(x) = \begin{cases} 0.5 & : \ if \ x = 0 , b , d \\ 0.2 & : \ if \ x = a , c \end{cases} \quad \& \quad \nu_A(x) = \begin{cases} 0.5 & : \ if \ x = 0 , b , d \\ 0.8 & : \ if \ x = a , c \end{cases}$$

$$\alpha_B(x) = \begin{cases} 0.4 & : \ if \ x = 0 , a , d \\ 0.3 & : \ if \ x = b , c \end{cases} \quad \& \quad \beta_B(x) = \begin{cases} 0.6 & : \ if \ x = 0 , a , d \\ 0.7 & : \ if \ x = b , c \end{cases}$$

Then A is a $I - F - k - ID$ in X since

$\mu_A(x^\star) \geq \mu_A(y)$ $and \ \nu_A(x) \leq \nu_A(y)$, $\forall x , y \in X$.

While B is not $-F - k - ID$, because $\alpha_\beta(c^\star) = 0.3 \ngeq \min\{\alpha_B(d^\star \star c) , \alpha_B(d)\} = 0.4$.

Proposition(3.3)

Every $I - F - ID$ of bounded Q-algebra is $I - F - K - ID$.

Prooof :

Assume that A be an I-F ideal of a bounded Q-algebra X then by Definition(2.16) we have

1- $\mu_A(0) \geq \mu_A(x)$ & $\nu_A(0) \leq \nu_A(x)$ $\forall x \in X$.

2- $\mu_A(x) \geq \min\{\mu_A(x \star y) , \mu_A(y) \}$ $\forall x , y \in X$ $.then$

$\mu_A(x^\star) \geq \min\{\mu_A(x^\star \star y) , \mu_A(y) \}$, $\forall x , y \in X$

$= \min\{\mu_A(y^\star \star x) , \mu_A(y) \}$ (by Proposition(2.6))

3- $\nu_A(x) \leq \max\{\nu_A(x \star y) , \nu_A(y) \}$ $\forall x , y \in X$ $.then$

$\nu_A(x^\star) \leq \max\{\nu_A(x^\star \star y) , \nu_A(y) \}$, $\forall x , y \in X$

$= \max\{\nu_A(y^\star \star x) , \nu_A(y) \}$ (by Proposition(2.6))

Thus A is is $I - F - K - ID$ of X .

Remark (3.4)

The converse of Proposition (3.3) needs to be not true in general and in the following example , we can show that

Example (3.5)

In Example (3.2) , A is $I - F - K - ID$ in X but it is not $I - F - ID$, because

$\mu_A(c) = 0.2 \ngeq \min\{\mu_A(c \star d) , \mu_A(d) \} = 0.5$

Proposition(3.6)

Every $I-F-k-ID$ in a involutory Q-algebra is $I-F-k-ID$.

Proof :

Suppose that A be a $I-F-k-ID$ of X then by Definition (3.1) we have

1- $\mu_A(0) \geq \mu_A(x)$ & $\nu_A(0) \leq \nu_A(x)$ $\forall\, x \in X$.

2- $\mu_A(x) = \mu_A(x^{\star^\star}) \geq \min\{\mu_A(y^\star \star x^\star),\ \mu_A(y)\}$
$= \min\{\mu_A(x^{\star\star} \star y),\ \mu_A(y)\}$
$= \min\{\mu_A(x \star y),\ \mu_A(y)\}$ $\forall\, x, y \in X$.

3- $\nu_A(x) = \nu_A(x^{\star^\star}) \leq \max\{\nu_A(y^\star \star x^\star),\ \nu_A(y)\}$
$= \max\{\mu_A(x^{\star\star} \star y),\ \mu_A(y)\}$
$= \max\{\mu_A(x \star y),\ \mu_A(y)\}$ $\forall\, x, y \in X$.

Proposition(3.7)

Let A be an $I-F-k-ID$ of a bounded Q-algebra X , then

1- $\mu_A(x^\star) \geq \mu_A(e)$ and $\nu_A(x^\star) \leq \nu_A(e)$, $\forall x \in X$.
2- $\mu_A(x^{\star\star}) \geq \mu_A(x)$ and $\nu_A(x^{\star\star}) \geq \nu_A(x)$, $\forall x \in X$.
3- $if \quad x^\star \leq y$, then $\mu_A(y) \leq \mu_A(x^\star)$ and $\nu_A(y) \geq \nu(x^\star)$, $\forall\, x, y \in X$.

Proof :

1. Since A is $I-F-k-ID$, then
$\mu_A(x^\star) \geq \min\{\mu_A(e^\star \star x), \mu_A(e)\}$
$= \min\{\mu_A(0 \star x), \mu_A(e)\}$
$= \min\{\mu_A(0), \mu_A(e)\}$
$= \mu_A(e)$

2 – Since A is $-F-K-ID$, then for any $x \in X$.
$\mu_A(x) = \min\{\ \mu_A(0)\ ,\ \mu_A(x)\})$
$= \min\{\ \mu_A(x^\star \star x^\star)\ ,\ \mu_A(x)\})$
$\leq \mu_A(x^{\star\star})$,

And

$\nu_A(x) = \max\{\ \nu_A(0)\ ,\ \nu_A(x)\})$
$= \max\{\ \nu_A(x^\star \star x^\star)\ ,\ \nu_A(x)\})$
$\leq \nu_A(x^{\star\star})$,

3 – if $x^\star \leq y$, i.e $x^\star \star y = 0$, then
$\mu_A(x^\star) \geq \min\{\ \mu_A(\ y^\star \star x\ ), \mu_A(y)\ \}$, $(since\ A\ is\ I-F-K-ID)$
$= \min\{\ \mu_A(\ x^\star \star y\ ), \mu_A(y)\ \}$ (by Proposition (2.6) ,2)
$= \min\{\ \mu_A(\ 0\ ), \mu_A(y)\ \}$
$= \mu_A(y)$

And

$\nu_A(x^\star) \leq \max\{\ \nu_A(\ y^\star \star x\ ), \nu_A(y)\ \}$, $(since\ A\ is\ I-F-K-ID)$
$= \max\{\ \nu_A(\ x^\star \star y\ ), \nu_A(y)\ \}$ (by Proposition (2.6) ,2)
$= \max\{\ \nu_A(\ 0\ ), \nu_A(y)\ \}$
$= \nu_A(y)$

Theorem(3.8)

Let I be a subset of a bounded Q-algebra X . Then I is $K-ID$ of X if and only if A_I is an $I-F-K-ID$ of X .

Proof .

$\Longrightarrow$

Suppose that A_I is not $I-F-K-ID$, then there exists $x\ , y \in X$ such that
$\mu_{A_I}(x^\star) = \beta \quad and \quad \min\{\mu_{A_I}(y^\star), \mu_{A_I}(y)\} = 1$
$\nu_{A_I}(x^\star) = \beta \quad and \quad \max\{\nu_{A_I}(y^\star), \nu_{A_I}(y)\} = 0$
Thus $\mu_{A_I}(y^\star \star x) = 1$, $\mu_{A_I}(y) = 1$
And $\nu_{A_I}(y^\star \star x) = 0$, $\nu_{A_I}(y) = 0$
Hence $y^\star \star x \in I$, $y \in I$
Thus $x^\star \in I$ (since I is $K-ID$))
Thus implies $\mu_{A_I}(x^\star) = 1$ and $\nu_{A_I}(x^\star) = 0$ It leads to contradiction
Thus A_I is an $I-F-K-ID$ of X .

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$\Longleftarrow$

Let A_I be an $I-F-K-ID$ of X and $y^\star \star x$, $y \in I$

Then $\mu_{A_I}(y^\star \star x) = 1$, $\mu_{A_I}(y) = 1$

And $\nu_{A_I}(y^\star \star x) = 0$, $\nu_{A_I}(y) = 0$

But , $\mu_{A_I}(x^\star) \geq \min\{\mu_{A_I}(y^\star \star x) , \mu_{A_I}(y)\} = 1$ and

$\nu_{A_I}(x^\star) \leq \max\{\nu_{A_I}(y^\star \star x) , \nu_{A_I}(y)\} = 0$ (since A_I is $I-F-K-ID$)

Thus $\mu_{A_I}(x^\star) = 1$ and $\nu_{A_I}(x^\star) = 0$, i.e $x^\star \in I$.

Hence I is $K-ID$ of X .

Proposition(3.9)

Let $\{A_\alpha : \alpha \in \Delta\}$ be a family of $I-F-K-ID$ in bounded Q-algebra X .

Then $\cap_{\alpha\in\Delta} (\mu_\alpha , \nu_\alpha)$ is an $I-F-K-ID$ of X .

Proof :

1- Since $\mu_{A_\alpha}(0) \geq \mu_{A_\alpha}(x)$ $\forall\alpha \in \Delta$, $\forall x \in X$,

$then$ $\inf\{\mu_{A_\alpha}(0)\} \geq \inf\{\mu_{A_\alpha}(x)\}$.

So $\cap_{\alpha\in\Delta} \mu_{A_\alpha}(0) \geq \cap_{\alpha\in\Delta} \mu_{A_\alpha}(x)$ and $\cap_{\alpha\in\Delta} \nu_{A_\alpha}(0) \leq \cap_{\alpha\in\Delta} \nu_{A_\alpha}(x)$,

Then $\sup\{\mu_{A_\alpha}(0)\} \geq \sup\{\mu_{A_\alpha}(x)\}$

So $\cup_{\alpha\in\Delta} \mu_{A_\alpha}(0) \geq \cup_{\alpha\in\Delta} \mu_{A_\alpha}(x)$

2- Let $x , y \in X$, Since $\mu_{A_\alpha}(x^\star) \geq \min\{\mu_{A_\alpha}(y^\star \star x) , \mu_{A_\alpha}(y)\}$

Then $\inf\{\mu_{A_\alpha}(x^\star)\} \geq \inf\{\min\{\mu_{A_\alpha}(y^\star \star x) , \mu_{A_\alpha}(y)\}\}$,

$\geq \min\{\inf\{\mu_{A_\alpha}(y^\star \star x) , \mu_{A_\alpha}(y)\}\}$,

So $\mu_{A_\alpha}(x^\star) \geq \min\{\cap_{\alpha\in\Delta} \mu_{A_\alpha}(y^\star \star x) , \cap_{\alpha\in\Delta} \mu_{A_\alpha}(y)\}$, and

$\nu_{A_\alpha}(x^\star) \leq \max\{\nu_{A_\alpha}(y^\star \star x) , \nu_{A_\alpha}(y)\}$

Then $\sup\{\nu_{A_\alpha}(x^\star)\} \leq \sup\{\max\{\nu_{A_\alpha}(y^\star \star x) , \nu_{A_\alpha}(y)\}\}$,

$\leq \max\{\sup\{\nu_{A_\alpha}(y^\star \star x) , \nu_{A_\alpha}(y)\}\}$,

So $\nu_{A_\alpha}(x^\star) \leq \max\{\cup_{\alpha\in\Delta} \nu_{A_\alpha}(y^\star \star x) , \cup_{\alpha\in\Delta} \nu_{A_\alpha}(y)\}$,

Hence $\cap_{\alpha\in\Delta} (\mu_\alpha , \nu_\alpha)$ is an $I-F-K-ID$ of X .

Remark (3.10)

Note that union of $I-F-K-ID$ in general is not necessarily $-F-K-ID$, as shown in the following example .

Example (3.11)

Let $X = \{0 , a, b, c , d\}$ and let $\star$ ba a binary operation on X which is defined by

$\star$	**0**	***a***	***b***	***c***	***d***	**q**
0	**0**	**0**	**0**	**0**	**0**	**0**
a	***a***	**0**	***a***	**0**	**0**	***a***
b	***b***	***b***	**0**	***b***	**0**	**0**
c	***c***	***c***	***c***	**0**	**0**	**0**
d	***d***	***d***	***c***	***b***	**0**	**0**
q	***q***	***q***	***a***	**0**	**0**	**0**

it is clear that $(X , \star , 0)$ $\boldsymbol{is\ bounded\ Q-algebra\ with\ unit\ d}$.

Define two $\boldsymbol{I-F-S}$ **A and B in X by :**

$$\mu_A(x) = \begin{cases} 0.7 & : \quad if\ x = 0 , c \\ 0.3 & : \quad if\ x = a , b , d , q \end{cases} \quad \& \quad \nu_A(x) = \begin{cases} 0.3 & : \quad if\ x = 0 , c \\ 0.7 & : \quad if\ x = a , b , d , q \end{cases}$$

$$\alpha_B(x) = \begin{cases} 0.8 & : \quad if\ x = 0 , b \\ 0.4 & : \quad if\ x = a , c , d, q \end{cases} \quad \& \quad \beta_B(x) = \begin{cases} 0.2 & : \quad if\ x = 0 , b \\ 0.6 & : \quad if\ x = a, c, d, q, \end{cases}$$

Notice that A and B are $I-F-K-ID$, since

$\mu_A(0) \geq \mu_A(x)$ and $\nu_A(0) \leq \nu_A(x)$, $\forall\, x \in X$ and

$\mu_A(x^\star) = 0.3 \ \geq \min\{\mu_A(y^\star \star x) \ , \mu_A(x)\} = \ 0.3 \quad \forall\, y \in x \ , \ x = 0, a, c$
$\nu_A(x^\star) = 0.7 \ \leq \max\{\nu_A(y^\star \star x) \ , \nu_A(x)\} = \ 0.7 \quad \forall\, y \in x \ , \ x = 0, a, c$
and
$\alpha_A(x^\star) = 0.4 \ \geq \min\{\alpha_A(y^\star \star x), \alpha_A(y)\} = 0.4 \quad \forall\, y \in X \ , x = 0, a, b$
$\beta_A(x^\star) = 0.6 \leq \max\{\beta_A(y^\star \star x), \beta_A(y)\} = 0.6 \quad \forall\, y \in X \ , x = 0, a, b$
$A \cup B$ is not $I - F - K - ID$, where

$$\mu_A(x) = \begin{cases} 0.5 & : \ if\ x = 0, c \\ 0.4 & : \ if\ x = a, d, q \\ 0.2 & : \ if\ x = c \end{cases} \qquad \nu_A(x) = \begin{cases} 0.5 & : \ if\ x = 0, c \\ 0.6 & : \ if\ x = a, d, q \\ 0.8 & : \ if\ x = c \end{cases}$$

Because

$$(\mu_A \cup \alpha_A)(0^\star) = 0.4 \ \ngeq \min\{(\mu_A \cup \alpha_A)(y^\star \star 0), (\mu_A \cup \alpha_A)(y)\} = 0.7 \ .$$

Proposition(3.12)
let g be an isomorphism from bounded Q-algebra X into bounded Q-algebra Y .
then

1- The image of $I - F - K - ID$ is an $-F - K - ID$.
2- The inverse of $I - F - K - ID$ is an $I - F - K - ID$.

Proof:
1-Let A be an $I - F - K - ID$ in X
Now , $g(\mu_A(0\acute{})) = \ \mu_A(g^{-1}(0)) = \mu_A(0)$ (since g is one to one) , then
$\mu_A(0) \geq \mu_A(x) \quad \& \ \nu_A(0) \leq \nu_A(x)$,
Since g is isomorphism ,
Then $\mu_A(0) \geq \mu_A(g^{-1}(y))$ and $\nu_A(0) \leq \nu_A(g^{-1}(y)) \quad \forall y \in Y$.
Thus $\mu_A(0) \geq \mu_A(g^{-1}(y)) = g(\mu_A(y)) \quad \forall y \in Y$.
Now , $g(\mu_A(y^\star)) = \mu_A(g^{-1}(y^\star)) = \ \mu_A(g^{-1}(y))^\star$ and
$g(\nu_A(y^\star)) = \nu_A(g^{-1}(y^\star)) = \ \nu_A(g^{-1}(y))^\star$
Since A is $I - F - K - ID$, then
$\mu_A(g^{-1}(y))^\star \geq \min\{\mu_A(x_1^\star \star g^{-1}(y)), \mu_A(x_1)\}$
$= \ \min\{\mu_A(x_1^\star \star x), \mu_A(x_1)\}$
$= \min\{\mu_A(g^{-1}(y_1^\star \star y)) \ , \mu_A(g^{-1}(y_1)\}$
Since g is isomorphism
$= \min\{\mu_A(y_1^\star \star y) \ , \mu_A(y_1)\}$
Thus
$\mu_A(g^{-1}(y))^\star \ \geq \min\{g(\mu_A(y_1^\star \star y)) \ , g(\mu_A(y_1)\}$ and
$\nu_A(g^{-1}(y))^\star \leq \max\{\nu_A(x_1^\star \star g^{-1}(y)), \nu_A(x_1)\}$
$= \ \max\{\nu_A(x_1^\star \star x), \nu_A(x_1)\}$
$= \max\{\nu_A(g^{-1}(y_1^\star \star y)) \ , \nu_A(g^{-1}(y_1)\}$
Since g is isomorphism
$= \max\{\nu_A(y_1^\star \star y) \ , \nu_A(y_1)\}$
Thus
$$\nu_A(g^{-1}(y))^\star \ \leq \max\{g(\nu_A(y_1^\star \star y)) \ , g(\nu_A(y_1)\}$$
So $g(A)$ is an $I - F - K - ID$

2.Let B be an $I - F - K - ID$ in Y
Now , $g^{-1}(\delta_B(0)) = \ \delta_B(g(0)) = \delta_B(g(0'))$
And $g^{-1}(\beta_B(0)) = \ \beta_B(g(0)) = \beta_B(g(0'))$
Since B is an $I - F - K - ID,$ then
$\delta_B(0') \ \geq \ \delta_B(y)$ and $\beta_B(0') \ \leq \ \beta_B(y)$
Since F is epimorphism , then

$\delta_B(0') \ \geq \ \delta_B(g(x))$ and $\beta_B(0') \ \leq \ \beta_B(g(x))$

Thus , $g^{-1}(\delta_B(0)) \geq \delta_B(g(x)) = g^{-1}(\delta_B(x))$ and
$g^{-1}(\beta_B(0)) \geq \beta_B(g(x)) = g^{-1}(\beta_B(x))$
Now , $g^{-1}(\delta_B(x^\star)) = \delta_B(g(x^\star)) = \delta_B(g(x)^\star)$ and
, $g^{-1}(\beta_B(x^\star)) = \beta_B(g(x^\star)) = \beta_B(g(x)^\star)$
Since B is an I-F-S then
$\delta_B(g(x)^\star) \geq \min\{ \delta_B(x_1^\star \star g(x), \delta_B(x_1)$ and
$\beta_B(g(x)^\star) \leq \max\{ \beta_B(x_1^\star \star g(x), \beta_B(x_1)$

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