

Metacognitive Skills and Achievement of 10th Graders in Problem-Solving using Polya's Heuristics

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Abstract: *This study investigated the metacognitive skills of tenth-grade learners and tested the association between their metacognitive skills and achievement in problem-solving heuristics in mathematics. It employed descriptive and correlational research designs utilizing the metacognitive awareness inventory and achievement test in problem-solving heuristics. The randomly chosen respondents included 580 male and female learners in mainstream classes among 18 public and private secondary schools in Tacloban City, Leyte. Findings revealed that students' level of metacognitive skill in knowledge about cognition is satisfactory while very satisfactory in regulating cognition. Likewise, students generally struggle with certain aspects of problem-solving heuristics, particularly in understanding the problem, locating necessary information, and devising a plan, where their achievements were generally poor or very poor. These difficulties may stem from a fundamental deficiency in problem-solving skills. Meanwhile, students' metacognitive skills in knowledge about cognition and regulation of cognition were significantly related to students' achievement in problem-solving heuristics in mathematics. While students demonstrated varying levels of metacognitive skills, it was evident that the degree of metacognitive proficiency significantly affected their problem-solving achievements. Given the results, it is recommended that educators and policymakers prioritize the development of metacognitive skills as an integral part of mathematics education. To empower teachers in facilitating the growth of students' metacognitive skills and fostering their problem-solving abilities, it is imperative to focus on metacognitive skill development, the seamless integration of problem-solving practices, the implementation of gender-sensitive teaching approaches, and continuous teacher professional development.*

Keywords: *Achievement, Mathematics, Metacognitive Skills, Problem Solving, Polya's Heuristics*

1. INTRODUCTION

Metacognition, a higher-order cognitive function, encompasses the deliberate regulation of mental processes while engaged in learning. Within this cognitive domain lie essential tasks, including strategic planning, ongoing comprehension monitoring, and systematically evaluating one's progress. These metacognitive functions are integral to successful learning outcomes, rendering pivotal components in the educational landscape. It is an awareness and regulation of one's cognitive processing, enabling individuals to manage their cognitive skills and address weaknesses by constructing new cognitive strategies. Metacognition extends beyond cognitive processes, encompassing emotions and motivations (Acosta-Gonzaga & Ramirez-Arellano, 2021; Puente-Diaz, 2023). Teachers can foster children's metacognitive development by using strategies that promote monitoring and regulation of cognitive endeavors. Applying metacognitive techniques like self-awareness and self-monitoring benefits learners, leading to independent and lifelong learners (Jaleel, 2016; Cabuquin, 2022b; Conley, 2014).

Metacognitive and motivational skills based on interest, self-efficacy, and attribution are crucial for successful problem-solving in academic settings. Some studies highlight two aspects of metacognition: knowledge of cognition and regulation of cognition - and their relation to domain-specific knowledge and cognitive abilities (Lai, 2011; Moshman, 2018; Tay et al., 2020). Meanwhile, other studies also reported instructional strategies to promote metacognitive awareness,

such as encouraging general awareness, enhancing self-knowledge and regulatory skills, and creating conducive learning environments for metacognition (Schraw, 2001; Hooshyar et al., 2019; Ingole & Pandya, 2016). Assessing off-line metacognition, including prediction and evaluation, is crucial to differentiate between average and above-average mathematical problem solvers and students with specific mathematics learning disabilities.

Furthermore, problem-solving plays a crucial role in the process of learning mathematics. Beyond its application in everyday life and the workplace, the ability to solve problems is not merely a goal in math education but also an effective method of engagement (Nursyahidah et al., 2018; Klang et al., 2021; Cabuquin & Abocejo, 2023; Liljedahl et al., 2016). It involves tackling tasks that demand innovative solutions. A key characteristic of adept problem solvers is their mathematical disposition - the skill to analyze situations from a mathematical standpoint and instinctively generate problems based on observed conditions (Huda & Syafmen, 2021; Mwei, 2017). Practical problem-solving exercises enable students to fortify and expand their knowledge while stimulating new avenues of learning. Most mathematical concepts can be introduced through problems that draw upon familiar experiences from students' lives or within mathematical contexts. These can include employing diagrams, identifying patterns, and experimenting with specific values or cases.

Instructional attention is imperative to ensure students acquire these essential problem-solving skills. Jie (2007) and Franklin and Harrington (2019) noted that teachers are pivotal in guiding students through various strategies and

methodologies. By creating a supportive and encouraging learning environment, educators can empower students to become proficient problem solvers equipped to tackle mathematical and real-world challenges confidently. Exposure to problem-solving strategies should be integrated throughout the curriculum (Azer et al., 2013; Cabuquin, 2023; Atnafu, 2017), as it is a crucial skill for today's learners. Educators and trainers, guided by recent research, changing professional standards, workplace demands, and learning theory advancements, are now reshaping curricula to foster integrated learning environments prioritizing higher-order thinking skills, especially problem-solving abilities.

Problem-solving is not a standalone topic but rather a pervasive process that should infuse the entire educational program, providing context for grasping concepts and acquiring skills. Students can approach math problems and any challenges they encounter with logic and proficiency to develop problem-solving skills. The heart of mathematics learning lies in problem-solving, which assists in comprehending the subject and fostering critical thinking skills. The problem-solving processes in mathematics parallel those of general problem-solving, allowing students to apply their knowledge and skills to real-life situations, making their learning relevant and practical. As such, problem-solving is invaluable for empowering students to navigate and conquer challenges inside and outside the classroom.

Based on the above discussion, this study is urgently needed to assess the link between metacognition and problem-solving skills in mathematics, especially in an educational landscape that increasingly values independent, critical thinkers. Metacognitive skills play a pivotal role in shaping students' problem-solving abilities and entire learning approach. Enhancing metacognitive skills can pave the way for more effective teaching methodologies, personalized learning approaches, and tailored interventions to bridge the gap in problem-solving capabilities. This study aims to address this critical need by shedding light on students' difficulty in problem-solving heuristics and their correlation with metacognitive skills. Further, the study stands as a catalyst for change, advocating for integrating metacognitive skill development into educational practices, thereby empowering both students and educators to navigate the complexities of problem-solving in mathematics more effectively.

2. METHODOLOGY

This study employed descriptive and correlational research designs to investigate the metacognitive skills and problem-solving heuristics of the 580 grade 10 students from 18 public and private secondary schools in the Eastern Visayas region. Of these respondents, 367 were drawn from public schools, while 213 were from private schools, selected through the convenience sampling technique.

The study utilized two primary research instruments to collect data. The first instrument, a metacognitive awareness inventory adapted from Schraw and Dennison's (1994) work,

consisted of 52 items rated on a 5-point Likert scale (5 - high, 4 - very satisfactory, 3 - satisfactory, 2 - average, 1 - poor). This inventory was categorized into two main sections: knowledge about cognition, encompassing 17 statements, and regulation of cognition, comprising 34 statements. Knowledge about cognition was further divided into procedural knowledge (4 statements), declarative knowledge (8 statements), and conditional knowledge (5 statements). Regulation of cognition included planning (7 statements), information management strategies (10 statements), comprehension monitoring (7 statements), debugging strategies (5 statements), and evaluation (5 statements).

In addition to the metacognitive inventory, a 20-item researcher-designed achievement test in problem-solving heuristics was created and subjected to validity and reliability assessments, with a Cronbach's alpha level of 0.80, indicating good and acceptable internal consistency. This test evaluated how students applied their problem-solving skills following Polya's five-step heuristic process: understanding the problem, gathering necessary information, devising a plan, executing the plan, and reviewing the solution. Students' scores for each heuristic in Polya's method were recorded separately, with mean scores categorized as follows: outstanding (17-20), very satisfactory (13-16.99), satisfactory (9-12.99), poor (5-8.99), and very poor (0-4.99) achievement.

The researcher diligently maintained the privacy of the collected data, handling it with the utmost confidentiality. The gathered data were visually presented using tables, and the analysis included frequency counting, percentages, and weighted means to elucidate the levels of metacognitive skills and problem-solving heuristics among the respondents. Additionally, the Pearson Product Moment Correlation coefficient was employed to investigate the potential relationship between students' achievement in problem-solving heuristics and their levels of metacognitive skills. All data analysis was performed using the Statistical Package for the Social Sciences, with a significance level set at 0.05.

3. RESULTS AND DISCUSSION

Respondents' level of metacognitive skills in knowledge about cognition

In terms of procedural knowledge among the respondents, 185 females (31.9%) and 134 males (23.1%) exhibited skills in procedural knowledge that were considered satisfactory. Some respondents fell into the "very satisfactory" category, with 98 female students (16.9%) and 96 male students (16.55%) achieving this level of proficiency. For declarative knowledge, the collective cohort comprising both public and private school students showed the distribution of declarative knowledge levels, which were as follows: 11 respondents (1.89%) demonstrated a high level of declarative knowledge. In comparison, 239 respondents (41.21%) achieved a very satisfactory level. A substantial majority, encompassing 313 students (53.96%), possessed satisfactory declarative knowledge. Additionally, 16 respondents (2.76%) were found

to have an average level of declarative knowledge, while only one respondent exhibited low declarative knowledge. In summary, the majority of respondents exhibited satisfactory levels of declarative knowledge. These findings underscore the importance of addressing procedural and declarative knowledge development in educational strategies to enhance learning outcomes.

Table 1: Students' level of metacognitive skill in terms of knowledge about cognition

Students' Level of Metacognitive Skill	Sex	Knowledge about Cognition					
		Procedural		Declarative		Conditional	
		f	%	f	%	f	%
High	Female	5	0.86	2	0.34	13	2.24
	Male	10	1.73	9	1.55	18	3.10
	Total	15	2.59	11	1.89	31	5.34
Very satisfactory	Female	98	16.9	137	23.62	145	25.0
	Male	96	16.55	102	17.59	94	16.21
	Total	194	33.45	239	41.21	239	41.21
Satisfactory	Female	185	31.9	179	30.86	155	26.72
	Male	134	23.1	134	23.10	131	22.59
	Total	319	55.0	313	53.96	286	49.31
Average	Female	38	6.55	9	1.55	14	2.41
	Male	12	2.07	7	1.21	10	1.72
	Total	50	8.62	16	2.76	24	4.14
Low	Female	1	0.17	0	-	-	-
	Male	1	0.17	1	0.17	-	-
	Total	2	0.34	1	0.17	-	-

Note: N=580; Procedural Mean Score= 3.42 (Satisfactory); Declarative Mean Score= 3.48 (Satisfactory); Conditional Mean Score= 3.48 (Satisfactory)

The collective distribution of respondents' conditional knowledge indicated that 31 respondents (5.34%) demonstrated a high level of conditional knowledge, 239 students (41.21%) exhibited very satisfactory conditional knowledge, 286 students (49.31%) possessed satisfactory conditional knowledge, and 24 students (4.14%) displayed an average level of conditional knowledge. Some respondents showed a very satisfactory level of conditional knowledge, whereas others demonstrated satisfactory levels. These findings emphasize the potential for tailored educational interventions to further enhance conditional knowledge among students and promote more consistent proficiency across the entire group.

Respondents' level of metacognitive skills in terms of regulation of cognition

A proportion of male and female respondents exhibited a very satisfactory level of metacognitive skill in planning, with 117 males (20.17%) and 180 females (31.03%) falling into this category. The satisfactory category encompassed 109 male students (18.79%) and 119 female students (20.52%).

Meanwhile, the respondents demonstrated satisfactory skills in information management strategies, which constituted the most frequent category. Specifically, 15 respondents (2.59%) displayed a high level of proficiency in these strategies, 204 respondents (35.17%) exhibited very satisfactory skills, 338 respondents (58.28%) possessed satisfactory skills, and 23 respondents (3.97%) demonstrated an average level of competency in information management strategies. Most respondents showcased satisfactory abilities in this domain. In terms of gender, a substantial proportion of

both male and female respondents exhibited satisfactory skills in information management strategies, with 145 male respondents (25.0%) and 193 female respondents (33.28%) falling into this category.

Many students exhibited satisfactory skills in comprehension monitoring. Specifically, 22 students (3.79%) displayed high-level skills, 233 students (40.17%) demonstrated very satisfactory skills, 314 students (54.14%) possessed average skills, and 11 students (1.90%) showed only average proficiency in comprehension monitoring. Notably, the most prevalent category among students was the satisfactory level of comprehension monitoring.

Table 2: Students' level of metacognitive skill in terms of Regulation of Cognition

Students' Level of Metacognitive Skill	Sex	Regulation of Cognition									
		Planning		Information management		Comprehension monitoring		Debugging strategies		Evaluation	
		f	%	f	%	f	%	f	%	f	%
High	Female	22	3.79	5	.86	7	1.21	41	7.07	13	2.24
	Male	19	3.28	10	1.72	15	2.59	23	3.97	13	2.24
	Total	41	7.07	15	2.59	22	3.79	64	11.04	26	4.48
Very satisfactory	Female	180	31.03	113	19.48	134	23.10	141	24.31	129	22.24
	Male	117	20.17	91	15.69	99	17.07	102	17.59	87	15.0
	Total	297	51.21	204	35.17	233	40.17	243	41.90	216	37.24
Satisfactory	Female	119	20.52	193	33.28	180	31.03	130	22.41	162	27.93
	Male	109	18.79	145	25.0	134	23.10	119	20.52	138	23.79
	Total	228	39.31	338	58.28	314	54.14	249	42.93	300	51.72
Average	Female	6	1.03	16	2.76	6	1.03	15	2.59	22	3.79
	Male	8	1.38	7	1.21	5	.86	7	1.21	15	2.57
	Total	14	2.41	23	3.97	11	1.90	22	3.79	37	6.38
Low	Female	-	-	-	-	-	-	0	-	1	.17
	Male	-	-	-	-	-	-	2	.34	0	-
	Total	-	-	-	-	-	-	2	.34	-	-

Note: N=580; P Mean Score= 3.64 (Satisfactory); IM Mean Score= 3.42 (Satisfactory); CM Mean Score= 3.47 (Satisfactory); DS Mean Score= 3.47 (Satisfactory); E Mean Score= 3.49 (Satisfactory)

When examining the data by gender, a substantial proportion of male and female students demonstrated satisfactory skills in comprehension monitoring, comprising 134 male students (23.10%) and 180 female students (31.03%). Furthermore, a notable number of students from both genders exhibited very satisfactory skills, with 99 male students (17.07%) and 134 female students (23.10%) attaining this higher level of proficiency.

Regarding debugging strategies, 42.93% of students exhibited satisfactory proficiency in this area. Additionally, a significant proportion of students, 243 (41.90%), displayed very satisfactory skills in debugging strategies. For evaluation, the predominant level of evaluation skills among students, encompassing 300 students (51.72%), was satisfactory. A significant portion of 216 students (37.24%) exhibited very satisfactory evaluation skills, further emphasizing the prevalence of satisfactory proficiency in this domain. To summarize, most students possessed satisfactory evaluation skills, with only a minority displaying skills at higher or lower levels. Specifically, 26 students (4.48%) showcased high-level evaluation skills, 37 students (6.38%) demonstrated an average level of proficiency, and only one student (.17%) exhibited low-level evaluation skills.

Level of Students' Achievement in Problem-Solving Heuristics

The mean score of 7.9 suggests that students needed better achievement in accurately identifying the critical aspects of the problem. When dissected by school type, students in private schools achieved a mean score of 9.96, signifying a satisfactory level of performance in this heuristic. In contrast, students in public schools achieved a mean score of 5.83, indicating a lower, poor level of achievement in understanding the problem. Regarding gender, female students attained a mean score of 9.07, reflecting a satisfactory level of achievement. In contrast, male students scored lower, with a mean of 6.73, suggesting a poor level of achievement in this aspect. Further, the mean score of 4.66 suggests that students generally exhibited poor performance in locating the necessary information within the problem. This was held across both genders, with male students achieving an average mean score of 4.54 and female students scoring slightly higher with a mean of 5.27, indicating poor achievements for both groups in this particular heuristic.

Table 3: Students' achievement in problem-solving heuristics

Problem-solving Heuristics (Polya's Steps to Problem Solving)	Sex	Private		Public		Grand Mean	QD
		WM	QD	WM	QD		
Understanding the problem	Male	9.21	S	4.24	VP	6.73	P
	Female	10.71	S	7.42	P	9.07	S
	Mean	9.96	S	5.83	P	7.90	P
Finding the needed information	Male	6.76	P	2.31	VP	4.54	P
	Female	6.52	P	4.02	VP	5.27	P
	Mean	6.64	P	3.17	VP	4.66	P
Devising a plan	Male	1.4	VP	0.26	VP	0.83	VP
	Female	1.10	VP	0.57	VP	0.84	VP
	Mean	1.25	VP	0.42	VP	0.84	VP
Carrying out the plan	Male	1.43	VP	0.37	VP	0.90	VP
	Female	1.32	VP	0.73	VP	1.03	VP
	Mean	1.38	VP	0.55	VP	0.97	VP
Looking back	Male	1.05	VP	0.33	VP	0.69	VP
	Female	0.96	VP	0.59	VP	0.78	VP
	Mean	1.01	VP	0.46	VP	0.74	VP

Note: Outstanding (17-20); very satisfactory (13-16.99); satisfactory (9-12.99); poor (5-8.99); and very poor (0-4.99) achievement; Mean Score = 0.67 (Very Poor)

The achievement level in devising a plan was deficient, with a mean score of 0.84, indicating a very poor achievement. Similarly, students in public schools exhibited a mean score of 0.42, reflecting a corresponding very poor level of achievement. When broken down by gender, male and female students demonstrated very poor achievement in this heuristic, with mean scores of 0.83 and 0.84, respectively. Meanwhile, the achievement level for carrying out the plan could have been much higher, as evidenced by its mean score of 0.97, indicating very poor performance. This low achievement was consistent across both genders. Likewise, the mean score of 0.74 underscored that all students did poorly in the "looking back" heuristic. This suggests that students often struggled with this aspect, typically only attempting it when they felt sufficiently confident in addressing the problem. The

persistently low achievement levels in problem-solving heuristics may be attributed to a fundamental deficiency in problem-solving skills (Sachakett & Art-in, 2014; Gordon, 2021; Duque & Tan, 2018). As problem-solving is inherently procedural, students may encounter difficulties in executing specific heuristics when they lack the prerequisite problem-solving skills needed for success.

Mathematical problems are valuable tools to enhance students' thinking abilities and equip them with essential skills for tackling real-life challenges. When teaching mathematics through problem-solving, teachers utilize problems as contexts to foster the development of concepts, principles, procedures, higher-order thinking skills, and positive dispositions. These problems present unfamiliar situations, demanding solutions not obtained through readily available procedures or algorithms. There could be multiple solutions and occasionally different correct answers in solving mathematics problems (Große, 2014; Leavy & Hourigan, 2020; Cabuquin, 2022a). The problem is introduced at the beginning of the lesson, prompting learners to devise their approaches to reach the answers. In this process, they engage various thinking skills and dispositions, connecting prior learning and the current task. They provide justifications for mathematical relationships; they uncover and articulate their reasoning using diverse representations. Through problem-solving, teachers can effectively assess learners' mathematical thinking and gauge their comprehension and application of concepts. As a result, students strengthen their mathematical skills and develop a problem-solving mindset valuable in their everyday lives.

Relationship between Students' Achievement in Problem-Solving Heuristics and Level of Metacognitive Skills

Regarding the student's knowledge about cognition, only one heuristic demonstrated a significant relationship with students' procedural knowledge, specifically the "devising a plan," with an r-value of .096 and a p-value of 0.021. This indicates that students' achievement in problem-solving within this heuristic is notably associated with their proficiency in procedural knowledge. Conversely, the remaining heuristics exhibited p-values greater than .05, implying that achievement in these aspects is not significantly correlated with procedural knowledge. This highlights the importance of understanding how to develop a plan for effective problem-solving (Hogan & Clawson, 2009; Desha et al., 2021; Belland, 2011), as those who possess the skill tend to perform better. This corroborates with the study of Webb et al. (2019), who emphasized the importance of problem-solving strategies and developing effective risk-management strategies for effective practice in the field. In the declarative knowledge, the findings unveiled that the students' problem-solving heuristics displayed significant relationships with declarative knowledge except for the "understanding the problem." Consequently, it can be concluded that achievement in problem-solving within these heuristics is significantly linked to students' declarative knowledge skills. Hence, students with a strong grasp of

declarative knowledge tend to excel in the identified problem-solving heuristics. Regarding conditional knowledge, the findings indicated that all heuristics were significantly related, suggesting that achievement in problem-solving positively correlates with students' level of conditional knowledge. Consequently, students with a strong foundation in conditional knowledge tend to excel in problem-solving heuristics.

In addition, for the students' regulation of cognition, two heuristics exhibited significant relationships with students' planning skills, specifically "what is asked in the problem" and "finding needed information," with corresponding r-values of .087 and .110, and p-levels < .05 (.037 and .008), respectively. These findings indicate a positive correlation, suggesting that students who engage in thoughtful planning before tackling a problem tend to excel in these problem-solving heuristics. In terms of students' information management strategies, two problem-solving heuristics, "finding needed information" and "devising a plan," were found to have significant connections with students' metacognitive skills. These relationships were evident through the r-values of .106 and .120 and p-levels < .05 (.011 and .004), respectively. These results underscore a relationship between students' proficiency in information management strategies and their achievements in the identified heuristics within word problem-solving. This implies that students with strong information management skills tend to perform better in tasks related to finding necessary information and identifying potential solutions within word problem-solving heuristics. Furthermore, students who effectively utilize skills and strategic sequences to process information online tend to excel in problem-solving heuristics.

Table 3: Relationship between students' achievement in problem-solving heuristics and level of metacognitive skill

Teacher Profile Variables	Understanding the problem		Finding needed information		Devising a plan		Carrying out the plan		Looking back	
	r-value	p-value	r-value	p-value	r-value	p-value	r-value	p-value	r-value	p-value
Knowledge about cognition										
Procedural knowledge	.010	.810	.073	.078	.096	.021*	.062	.138	.055	.187
Declarative knowledge	.080	.053	.147	.000*	.118	.005*	.090	.030*	.086	.038*
Conditional knowledge	.129	.002*	.186	.000*	.161	.000*	.147	.000*	.127	.000*
Regulation of cognition										
Planning	.087	.037*	.110	.008*	.012	.773	.024	.566	.031	.460
Information management strategies	.044	.287	.106	.011*	.120	.004*	.070	.094	.074	.076
Comprehension monitoring	.028	.496	.088	.039*	.073	.079	.055	.187	.050	.223
Debugging strategies	.210	.000*	.272	.000*	.227	.000*	.205	.000*	.169	.000*
Evaluation	.054	.193	.106	.011*	.083	.046*	.083	.047*	.084	.043*

*Significant at 0.05 level

In terms of comprehension monitoring, the results reveal that only one heuristic exhibits a significant relationship with comprehension monitoring skills, which is the "finding needed information," with an r-value of .088 and a p-value of 0.05. In contrast, all other heuristics are not significantly correlated with students' comprehension skills. Students who assess their

learning or strategies tend to perform well in problem-solving heuristics. In debugging strategies, students' achievements in all of Polya's problem-solving heuristics are significantly related to their level of debugging strategies skills. Students with strong debugging strategy skills excel in various aspects of word problem-solving. It underscores the value of using strategies to correct comprehension and performance errors during problem-solving (Huang & Cheng, 2010; Abdullah et al., 2015), which may be linked to improved performance. Meanwhile, for evaluation, four of Polya's problem-solving heuristics exhibit significant relationships with students' evaluation skills, suggesting that students' evaluation skills are closely associated with their achievements in these four problem-solving heuristics. Consequently, students with strong evaluation skills are more likely to perform better in the identified problem-solving aspects. However, it is worth noting that "what is asked in the problem" is not significantly related to debugging strategies skills, as indicated by its r-value of .054 and a p-value of 0.05. Students who analyze their performance and assess strategy effectiveness after a learning episode tend to excel in problem-solving.

4. CONCLUSION AND RECOMMENDATIONS

This study has shed light on the metacognitive skills of tenth-grade learners and their relationship with achievement in problem-solving heuristics in mathematics, explicitly following Polya's heuristic process. The findings have highlighted that students exhibit varying levels of metacognitive skills, with satisfactory levels observed in knowledge about cognition, including procedural, declarative, and conditional knowledge. Additionally, the study revealed that students generally struggle with certain aspects of problem-solving heuristics, particularly in understanding the problem, locating necessary information, and devising a plan, where their achievements were generally poor or very poor. These difficulties may stem from a fundamental need for problem-solving skills. It was likewise evident that the degree of metacognitive proficiency affected their problem-solving achievements.

Based on the findings, mathematics teachers should prioritize strengthening students' metacognitive skills in the classroom by engaging them in challenging mathematics problems. Furthermore, mathematics teachers should incorporate problem-solving into the curriculum to allow students to apply their knowledge and skills. Schools should create and implement programs that educate students on how to use their metacognitive and problem-solving skills. Students should be taught to focus on understanding the problem, creating problem-solving strategies, and locating the essential information. Similarly, teachers should be trained and supported in metacognitive education and problem-solving pedagogy. Future research should look into other aspects influencing students' problem-solving abilities, such as motivation and socioeconomic background.

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