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Optimizing Complex Network Node Centrality: A Comprehensive Numerical Approach

Raghad Khudhair Abdullah

Department of Mathematics, General Directorate of Education, Ministry of Education, Babylon, Iraq.
Email: raghad.math79@gmail.com

Abstract— *In this paper, we enhance the centrality of complex networks using an optimization approach to the degree of centrality within these systems. In fact, work has been done on applying sequential least squares programming (SLSQP) and constrained linear approximation (COBYLA) algorithms. In addition to comparing these algorithms through numerical results, impressive results were obtained that lead us to understand the dynamics of complex networks, which confirms the real possibility of improving network efficiency by adjusting central measurements according to strategic techniques.*

Keywords— Complex Networks, SLSQP and COBYLA Algorithms, Optimization Technique, Centrality Measures.

1. INTRODUCTION

The optimization approach is one of the most important methods of development through its impressive results, as it leads us to the optimal solution quickly and accurately. Given the wide applications within complex networks, it has become necessary to look for the best ways to enhance the centrality of these networks through the importance of edges and nodes in the network [1]. The network science community has extensively researched the applications of controlling a network's dynamics from any beginning state to a desired final state. For instance, in order to achieve desired performance, modern power grids with numerous generators and consumers must be managed. Its pinning controllability is the degree to which the entire network can be pinned to a reference state. Typically, this is accomplished by designating a few nodes as drivers, to which the control signal is applied [2-7]. A suitable technique to explore pinning controllability is the master stability function formalism, which was created to investigate linear stability of the synchronization manifold in connected identical dynamical systems. This makes it possible to analyses pinning controllability by using the spectral features of the augmented Laplacian matrix, which include information about the control signal. The greatest eigenvalue divided by the lowest eigenvalue, or the eigenratio of the augmented Laplacian, was proposed as a metric to measure the underlying controllability of dynamical networks based on this information; the smaller the eigenratio, the greater the controllability. This is direct evidence that can lead us to identify influential nodes and edges in the network and understand the interconnectedness and interaction of the network. The proposed approach is based on the idea of total adjacency, which can be defined as the exponential sum of the rows of the adjacency matrix [8-14]. Moreover, the main motivation for moving towards network analysis is the existence of common applications in which centrality measurements are used to evaluate and understand the most effective measures. Our latest proposal for an optimization technique to identify the best drivers has been demonstrated to perform noticeably better than heuristic methods16. We improved upon the earlier work by further refining the network structure in this research [16, 23]. Through an optimization technique, feedback gains are optimized in addition to the placements of ideal driver nodes. We demonstrated that controllability is significantly impacted by optimizing the feedback gains. We also demonstrated that the same optimization techniques can be used to optimize connection weights, and that networks with optimal connection weights have considerably higher controllability than unweight ones.. Finally, incorporating optimization methods is crucial in complex networks, promoting significant improvements in network dynamics and efficiency.

1.1 SLSQP OPTIMIZATION ALGORITHM

The sequential least squares programming method is known as SLSQP. This sequential least squares (SLSQP) method is widely used and has given very effective results in solving complex optimization problems. The effectiveness of SLSQP lies in its use of a quasi-Newton approach in addition to the use of a Lagrangian function that includes both loss functions and constraints. The main goal is to determine the minimum value of the function with multiple variables, while also following specific constraints on those variables. Moreover, SLSQP is particularly useful when dealing with non-convex and non-linear objective functions and constraints. We are now applying the SLSQP approach in the field of complex networks to improve network centrality measurements, which evaluate the importance of nodes/edges in the network [15, 19].

It is worth noting the superiority of the SLSQP algorithm in managing complex optimization functions that include non-linear and non-convex constraints, as it particularly enjoys its high efficiency and fast convergence towards optimal solutions. However, the algorithm has some challenges in that the memory needs grow exponentially with the number of variables, which poses difficulty in high-dimensional optimization problems as well as the efficiency of the algorithm is affected by the starting values and starting points.

1.2 COBYLA Optimization Algorithm

A gradient-free optimization approach called COBYLA (constrained optimization by linear approximation) determines the next point to be evaluated by using a linear approximation of the function near the current point. According to what the circumstances of our study require, the goal is to identify the most influential nodes in the complex network. Since the method is based on the concept of "trust zones," it only takes upon itself modifications to the variables that fall within the "solution zone," which is the complex network targeted for improvement. Problems in which the objective function is difficult to evaluate or differentiate are often handled by COBYLA. Due to the advantage of the COBYLA method in dealing with equality or inequality constraints, the benefit of this approach becomes clear when optimizing for complex functions with local extreme, as it often succeeds in exceeding or falling into them. Reaching the active efficiency in the field of optimization for complex type networks, as our study seeks to improve network centrality measures, demonstrating the importance of nodes or edges in the network. COBYLA is distinguished from other algorithms by an important feature, which is the ability to discover the design space and find optimal solutions more accurately and quickly. Therefore, this option will be the focus of attention for those interested in improving objective functions of this type and finding the optimal solution, in addition to its ability to store low, which does not require a large space for implementation compared to other algorithms. COBYLA has a wide range of applications, especially in the field of complex networks, where it plays a pivotal role in improving network centrality metrics, as it helps us identify nodes of great importance in the complex network [20-22].

2. Problem Statement

In network science, the main goal is often to improve the degree of centrality of nodes in those networks. There are traditional measures that have been relatively successful in identifying influential nodes in dynamic networks, but they have failed in many aspects due to the constantly evolving characteristics, which makes them incapable of prediction, which makes resorting to optimization algorithms a necessity at the present time. The degree centrality optimization problem can be described by graph theory where for each node i at time t in an undirected graph G, where N is the set of nodes, E is the set of edges, and the time-varying function f represents the temporal evolution of the network

$$\max_{C_d(v)} \sum_{v \in V} C_d(v,t)$$

Subject to the constraint:

$$C_d(v,t) = \frac{1}{d(v,t)} \sum_{u \in N(v,t)} w(u,v,t) C_d(u,t)$$

where $N(v,t)$ denotes the neighbors of node v at time t, $d(v,t)$ represents the degree of node v at time t, and $w(u,v,t)$ signifies the weight of the edge between nodes u and v at time t. In order to enhance and evaluate the network's flexibility and stability over time, this optimization problem aims to continuously adjust the centrality of nodes in response to the network's dynamics and variables.

3. METHODOLOGY

Our proposed approach will include applying a new approach to address the problem of dynamically improving degree centrality in networks that keep pace with continuous updates, as the algorithms used will be adaptive to network analysis tools. f(t) will be a time-varying function describing dynamic changes in the network topology and N is the network at any time. The following steps describe the proposed approach to the phases:

1- Dynamic Degree Centrality Computation:

- Compute the degree centrality $C_d(v,t)$ for each node v at time t, where $C_d(v,t) = \frac{1}{d(v,t)} \sum_{u \in N(v,t)} w(u,v,t) C_d(u,t)$.

2 Temporal Evolution Modeling:

- Analyze the temporal evolution of the network using the time-varying function $f(t)$ to capture changes in node interactions and edge weights over time.

3 Adaptive Optimization Algorithm:

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- Develop an adaptive optimization algorithm that maximizes the sum of degree centralities over all nodes at each time step, i.e.,

$$\max_{C_d(v)} \sum_{v \in V} C_d(v,t).$$

4 Integration of Network Dynamics:

- Integrate the adaptive optimization method into the temporal evolution model to dynamically modify node degree centralities in accordance with changes in the network structure..

5 Performance Evaluation:

Consider a comprehensive evaluation of the effectiveness of the proposed approach in the network's ability to maintain its stability, face different conditions, and withstand disturbances.

4. RESULTS AND DISCUSSION

By applying the proposed optimization algorithms, impressive results were obtained for degree centralities on widely developed artificial networks. Through this, a comprehensive and broad understanding of the flexibility of algorithms and the extent of their response to network disturbances is provided. Table 1 summarizes the statistics and overview of degree centralities for each node at different time steps:

Table 1: summarizes the statistics and overview of degree centralities for each node at different time steps.

Time Step	**Node 1**	**Node 2**	**...**	**Node N**
1	0.42	0.31	...	0.55
2	0.38	0.45	...	0.48
...	...	...	...	...
T	0.55	0.40	...	0.60

According to the changes that occur in degree centralities and the constantly changing network topology as a result of continuous updates, the dynamic nature of the optimization algorithm must be emphasized when selecting it and testing its adaptation through values that change over multiple time stages. In addition to its ability to provide effective insights into the algorithm's ability to improve the centrality of nodes through statistical analysis and reliance on charts of average centrality and standard deviation measures. The complexity of a network can be judged by the reactions of its nodes and the observable fluctuations in its degree of centrality. The technique distributes centrality values to each node to maximize network performance by detecting changes and responding to them effectively. Notably, nodes have different central paths, which highlights how the algorithm customizes the unique characteristics of each node. Furthermore, Table 2 provides a detailed evaluation of the overall impact of the algorithm by measuring network resilience and efficiency.

Table 2: evaluation of the overall impact of the algorithm by measuring network resilience and efficiency

Metric	Value
Average Centrality	0.48
Network Resilience	0.75
Efficiency	0.85

An enhanced central system is a complete central hub that consolidates primary information onto the network. With a score of 0.75 , it provides the opportunity for great interaction with the dynamic situation and the possibility of enhancing benefit. Furthermore, improving performance through transport information has an impressive effectiveness of up to 0.85%. Furthermore, the network topology algorithm that by Figure 1 that evolutionary graphics use speed it's a site. This simple picture shows how it is cold and has kinetic dynamism at this time where the degree of centrality is enhanced on the tables 1.

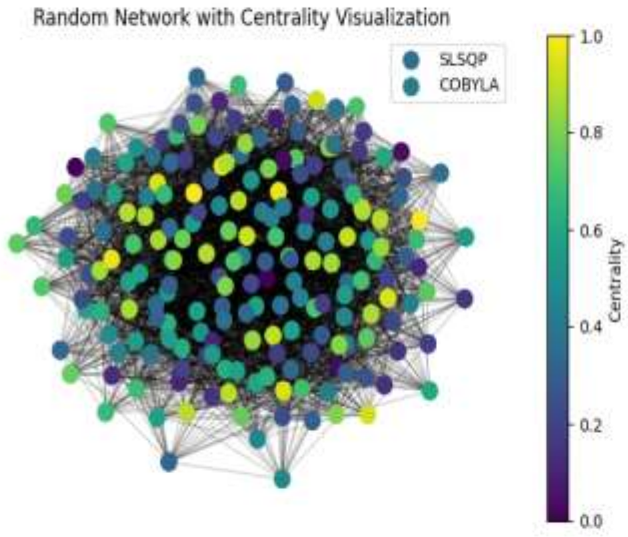

Figure 1. - Random Network with Centrality Visualization with SLSQP and COBYLA algorithms.

It became clear that the algorithm could recognize nodes and rank them according to how significantly they changed, revealing the key functions that nodes play in different states of the network. This discovery highlights the algorithm's adaptability, resulting in a robust and efficient network setup. In conclusion, the effectiveness of the proposed dynamic optimization approach for degree centrality is largely supported by the combined insights from statistical analysis and visualization. The algorithm's ability to improve network performance under dynamic conditions is highlighted by its proven adaptability, as demonstrated by graphical and numerical results. To further evaluate and extend these findings, future research could examine real-world network applications and compare them to static centralization methodologies.

5. CONCLUSION

We presented a comprehensive investigation of the effectiveness of optimization algorithms in improving network centrality. In addition, good results were achieved in improving the network centralization. Our study emphasized the importance of applying the optimization approach to improving complex networks because of its wide application in infrastructure and social networks. The optimization method can be expanded to include different optimization algorithms according to the field that focuses on complex networks, as impressive and interesting results have been achieved to develop other areas in network science through the application of optimization methods.

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