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On Stereographic Semi-circular Three Parameter Lindley Distribution

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Abstract: *In this paper, we made an attempt to construct a new Semi-circular model, we call this as "Stereographic Semi-circular Three Parameter Lindley distribution," by applying modified inverse Stereographic projection on Three Parameter Lindley distribution (Rama Shanker at all(2017)) for modelling semi-circular data. Probability density and cumulative distribution functions of said model are derived and their graphs are plotted for various values of parameters. The first two trigonometric moments are derived and the proposed model is extended for l-axial data also.*

Keywords: The Three Parameter Lindley, moments Semi-circular models, inverse stereographic projection, trigonometric moments,parameter estimation, maximum likelihood estimation.

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1. Introduction

In the literature of Directional Statistics we can find a significant number of Circular models defined on unit circle, developed from the existing linear distributions, both continuous and discrete by Fisher (1993), Jammalamadaka and Sengupta (2001), Dattatreya Rao et al (2007). A new method called Stereographic Projection was employed for developing the probability distributions of angular models. Minh and Farnum (2003) used a bilinear transformations to map points on the unit circle in the complex plane into points x on the real line and for a density function g(α) on the interval (−π, π), and demonstrated how a corresponding density function f(x) on (−∞, ∞) is induced. Toshihiro Abe et al (2010) developed symmetric circular models applying the Inverse Stereographic Projection. Dattatreya Rao et al (2011) developed Cauchy type models by inducing Stereographic Projection on circular Cardioid distribution. Phani et al (2012) developed circular model induced by Inverse Stereographic Projection on Extreme-Value distribution. Taking cue from these works, an attempt is made. In this paper we develop new Semi-circular model coined as Stereographic Semi-circular Three Parameter Lindley distribution by applying modified inverse stereographic projection on Three Parameter Lindley distribution. The graphs of the density function and distribution function for various values of parameters are plotted. We derive the first two trigonometric moments of the proposed model to evaluate population characteristics

2.Methodology of modified Inverse Stereographic Projection:

Modified Inverse Stereographic Projection is defined by a one to one mapping given by :

$\mathrm{T}(\theta) = x = \mathrm{V}\tan\left(\frac{\theta}{2}\right)$,wher $x \in (-\infty, \infty), \theta \in (-\pi, \pi)$Suppose x is randomly chosen on the interval (–∞ ∞,) .

Let F (x) and f (x) denote the Cumulative distribution and probability density functions of the linear random variable X respectively.

$\mathrm{T}^{-1}(x) = \theta = 2\tan^{-1}\left(\frac{x}{v}\right)$

by Minh and Farnum (2003) is a random point on the unit circle. Let G(θ) and g (θ) denote the Cumulative distribution and probability density functions of this random point θ respectively. Then G(θ) and g (θ) can be written in terms of F (x) and f (x) using the following Theorem.

Theorem 2.1: For v > 0

i. $\mathrm{G}(\theta) = \mathrm{F}\left(\mathrm{V}\tan\left(\frac{\theta}{2}\right)\right)$ (1)

ii. $\mathrm{g}(\theta) = \mathrm{V}\left(\frac{1+\tan^2\left(\frac{\theta}{2}\right)}{2}\right)\mathrm{f}\left(\mathrm{V}\tan\left(\frac{\theta}{2}\right)\right)$ (2)

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If a linear random variable X has a support on ¡ , then θ has a support on (−π, π) and if X has a support, then θ has a support on (0, π) . These means that, after the Inverse Stereographic Projection is applied, we can deal circular data if the support of X is on ¡ and we can handle semi-circular data if the support of X is on.

3. Stereographic Semi-circular Three Parameter Lindley distribution

Here a linear model Three Parameter Lindley distribution is considered and by inducing modified inverse stereographic projection, a stereographic semi-circular model is defined. Definition: A random variable X on the real line is said to have Three Parameter Lindley distribution with scale parameter α > 0 and location parameter α if the probability density and cumulative distribution functions of X are respectively given by

$$f(x,\sigma,\alpha,\beta) = \frac{\sigma^2}{\alpha\sigma+\beta}(\alpha+\beta x)e^{-\sigma x} \qquad (3)$$

and

$$F(x,\sigma,\alpha,\beta) = 1 - \left[1+\frac{\sigma\beta x}{\sigma\alpha+\beta}\right]\ e^{-\sigma x} \qquad (4)$$

Then by applying modified inverse Stereographic projection defined by a one to one mapping

$x = V\tan\left(\frac{\theta}{2}\right), v \in R$which leads to a Semicircular Model on unit semicircle. We call this model as Stereographic Semicircular Three Parameter Lindley. Definition: A random variable on the Semicircle is said to have the Stereographic Semi-circular Three Parameter Lindley distribution with shape parameter α > 0 , location denoted by Stereographic Semi-circular Three Parameter Lindley distribution if the probability density and the cumulative distribution functions are respectively given by**:**

$$f(\theta,\alpha,\beta,\sigma) = \frac{v\sigma^2}{2(\alpha\sigma+\beta)}\left(\alpha+\beta V\tan\left(\frac{\theta}{2}\right)\right)e^{-\sigma V\tan\left(\frac{\theta}{2}\right)}\left(\sec^2\left(\frac{\theta}{2}\right)\right) \qquad (5)$$

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$$\int_0^\pi f(\theta,\alpha,\beta,\sigma)d\theta = 1$$

$$= \int_0^\pi \frac{v\sigma^2}{2(\alpha\sigma+\beta)}\left(\alpha+\beta V\tan\left(\frac{\theta}{2}\right)\right)e^{-\sigma V\tan\left(\frac{\theta}{2}\right)}\left(\sec^2\left(\frac{\theta}{2}\right)\right).d\theta$$

$$= \frac{v\sigma^2}{2(\alpha\sigma+\beta)}\int_0^\pi \left(\alpha+\beta V\tan\left(\frac{\theta}{2}\right)\right)e^{-\sigma V\tan\left(\frac{\theta}{2}\right)}\left(\sec^2\left(\frac{\theta}{2}\right)\right).d\theta$$

$$\text{let } x = v\tan\left(\frac{\theta}{2}\right), \theta = 2\tan^{-1}\left(\frac{x}{v}\right), d\theta = \frac{2}{\frac{v+x^2}{v}}dx$$

$$= \frac{v\sigma^2}{2(\alpha\sigma+\beta)}\int_0^\infty (\alpha+\beta x)e^{-\sigma x}\left(\sec^2\left(\frac{2\tan^{-1}\left(\frac{x}{v}\right)}{2}\right)\right).\frac{2}{\frac{v+x^2}{v}}dx$$

$$= \frac{v\sigma^2}{2(\alpha\sigma+\beta)}\int_0^\infty (\alpha+\beta x)e^{-\sigma x}\left(\sec^2\left(\tan^{-1}\left(\frac{x}{v}\right)\right)\right).\frac{2}{\frac{v+x^2}{v}}dx$$

$$\sec^2\tan^{-1}(x) = x^2+1 = \sec^2\tan^{-1}\left(\frac{x}{v}\right) = \frac{x^2}{v^2}+1$$

$$= \frac{v\sigma^2}{2(\alpha\sigma+\beta)}\int_0^\infty (\alpha+\beta x)e^{-\sigma x}\left(\frac{x^2}{v^2}+1\right).\frac{2}{\frac{v+x^2}{v}}dx$$

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$$= \frac{v\sigma^2}{2(\alpha\sigma+\beta)} \int_0^\infty (\alpha+\beta x)e^{-\sigma x}\left(\frac{x^2+v^2}{v^2}\right)\frac{2v}{v^2+x^2}dx$$

$$= \frac{v\sigma^2}{2(\alpha\sigma+\beta)} \int_0^\infty (\alpha+\beta x)e^{-\sigma x}\left(\frac{2}{v}\right)dx$$

$$= \frac{v\sigma^2}{2(\alpha\sigma+\beta)}\left(\frac{2}{v}\right) \int_0^\infty (\alpha+\beta x)e^{-\sigma x}dx$$

$$= \frac{\sigma^2}{\alpha\sigma+\beta} \int_0^\infty (\alpha+\beta x)e^{-\sigma x}dx$$

$$= \frac{\sigma^2}{\alpha\sigma+\beta}\left(\int_0^\infty \alpha e^{-\sigma x}dx + \int_0^\infty \beta x e^{-\sigma x}dx\right)$$

$$= \frac{\sigma^2}{\alpha\sigma+\beta}\left(\int_0^\infty \alpha e^{-\sigma x}dx + \int_0^\infty \beta x e^{-\sigma x}dx\right)$$

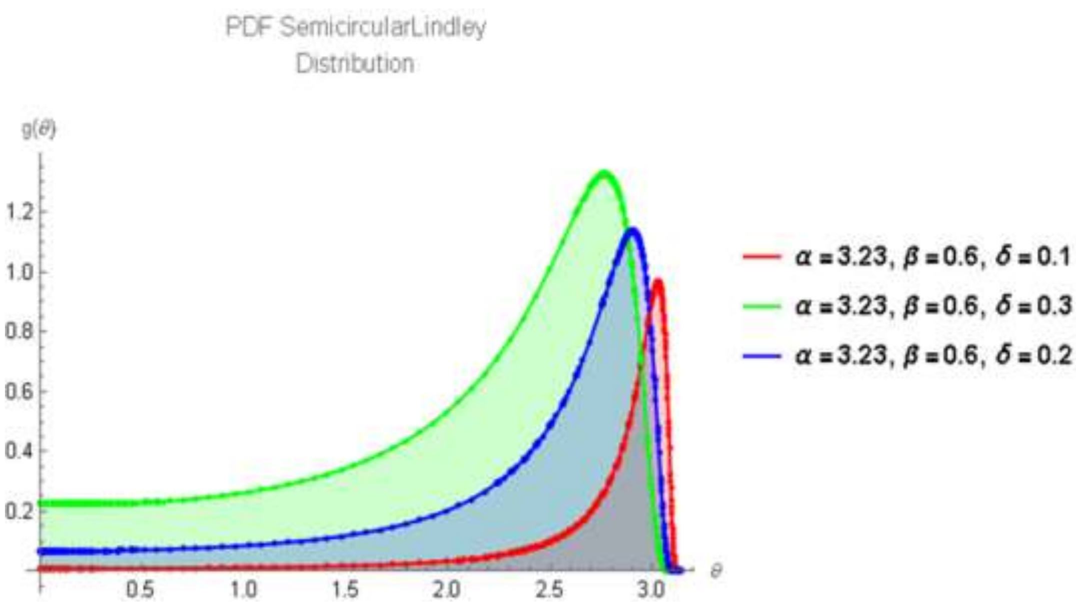
$$= \frac{\sigma^2}{\alpha\sigma+\beta}\left(\frac{\alpha}{\sigma}+\frac{\beta}{\sigma^2}\right)$$

$$= \frac{\sigma^2}{\alpha\sigma+\beta}\left(\frac{\beta+\alpha\sigma}{\sigma^2}\right)$$

$$= \mathbf{1}$$

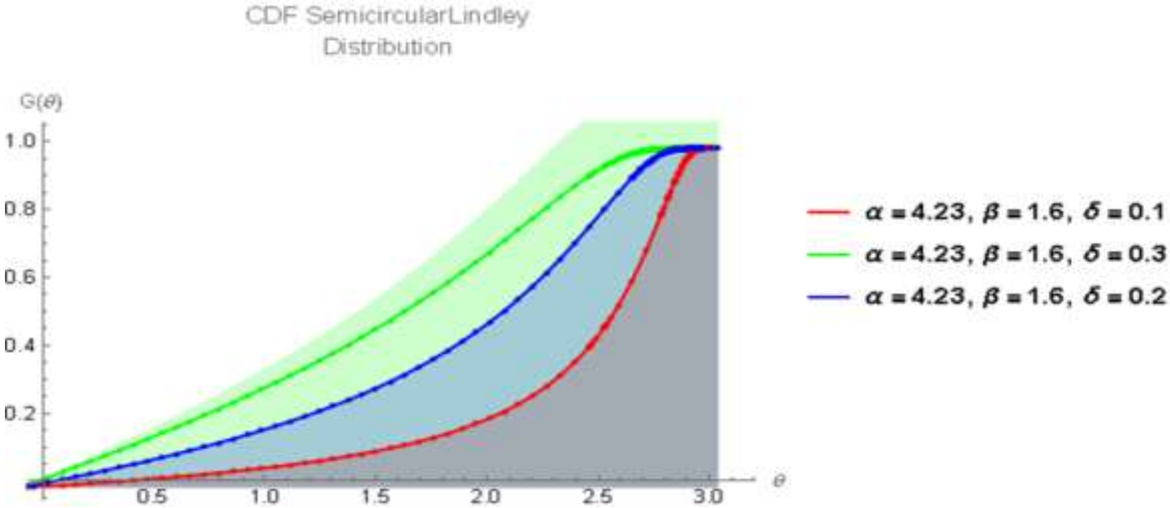
The reasonable shapes of PDF Semi-circular Three Parameter Lindley

The CDF of Three Parameter Lindley distribution given by:



$$F(\theta,\alpha,\beta,\sigma) = 1-\left(1+\frac{\sigma\beta V\tan\left(\frac{\theta}{2}\right)}{\alpha\sigma+\beta}\right)e^{-\sigma V\tan\left(\frac{\theta}{2}\right)} \qquad (6)$$

The reasonable shapes of CDF Semi-circular Three Parameter Lindley

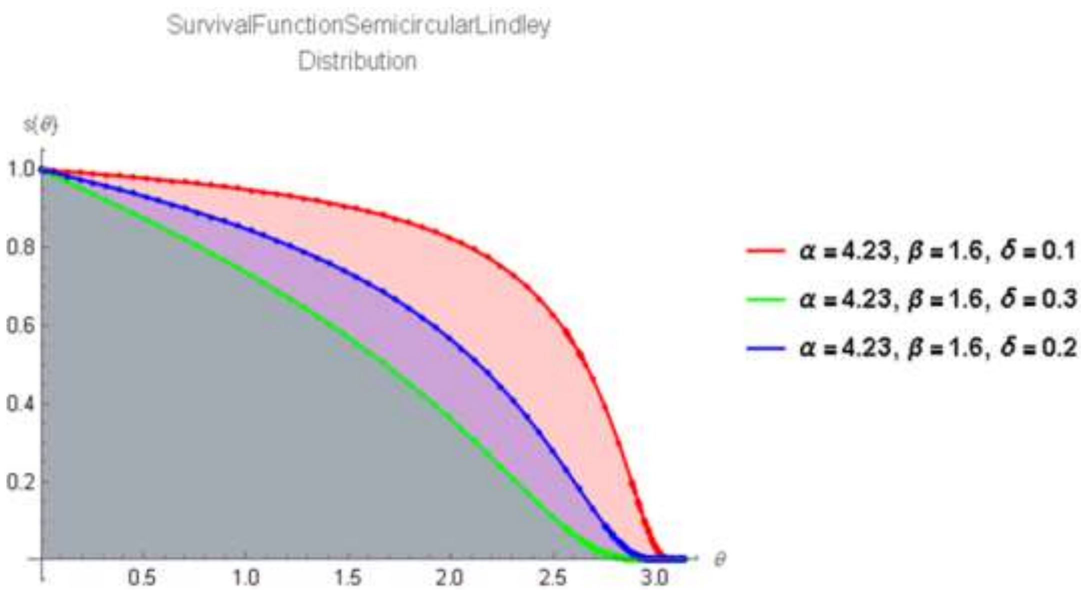


The Survival function of the Three Parameter Lindley distribution given by:

$$R(\theta, \alpha, \beta, \sigma) = 1 - F(\theta, \alpha, \beta, \sigma)$$

$$S(\theta, \alpha, \beta, \sigma) = \left(1 + \frac{\sigma\beta V \tan\left(\frac{\theta}{2}\right)}{\alpha\sigma + \beta}\right) e^{-\sigma V \tan\left(\frac{\theta}{2}\right)} \qquad (7)$$

The reasonable shapes Survival Semi-circular Three Parameter Lindley



The hazard function of the new distribution is

For any random variable X which follows Semi-circular Three Parameter Lindley, its hazard function is given as:

$$h(\theta, \alpha, \beta, \sigma) = \frac{f(\theta, \alpha, \beta, \sigma)}{s(\theta, \alpha, \beta, \sigma)}$$

$$h(\theta, \alpha, \beta, \sigma) = \frac{\frac{v\sigma^2}{2(\alpha\sigma+\beta)}\left(\alpha + \beta V \tan\left(\frac{\theta}{2}\right)\right) e^{-\sigma V \tan\left(\frac{\theta}{2}\right)} \left(\sec^2\left(\frac{\theta}{2}\right)\right)}{\left(1 + \frac{\sigma\beta V \tan\left(\frac{\theta}{2}\right)}{\alpha\sigma+\beta}\right) e^{-\sigma V \tan\left(\frac{\theta}{2}\right)}} \qquad (8)$$

4.Statistical Properties :

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In this section, some of the properties of the Semi-circular Three Parameter Lindley distribution are discussed:

4.1Quantile function

The quintile function or inverse cumulative distribution function. returns the value t such that:

$$\theta = Q(u) = F^{-1}(u), 0 < u < 1$$

$$u = 1 - \left(1 + \frac{\sigma\beta V\tan\left(\frac{\theta}{2}\right)}{\alpha\sigma + \beta}\right) e^{-\sigma V\tan\left(\frac{\theta}{2}\right)}$$

$$\theta = \frac{-v\beta - v\alpha\delta - \beta \mathrm{ProductLog}[\frac{e^{-\frac{v(\beta+\alpha\delta)}{\beta}}(-1+u)v(\beta+\alpha\delta)}{\beta}]}{v^2\beta\delta} \qquad (9)$$

4.2Trigonometric moments of Stereographic Semicircular Three Parameter Lindley

It is customary to derive the trigonometric moments when a new distribution is proposed. Without loss of generality here we assume that μ = 0.The trigonometric moments of the distribution are given by

$\{\varphi_p : p = 0, \pm1, \pm2, \pm3 \ldots..\}$ where$\varphi_p = \varphi_p = \alpha_p + i\beta_p$ with $\alpha_p = E(\cos(p\theta))and\beta_p = E(\sin(p\theta))$

being the p^{th} order cosine and sine moments of the random angle θ , respectively.

Theorem 4.2Under the pdf of Stereographic Semicircular Three Parameter Lindley distribution with μ = 0, the first four α θ $\alpha_p = E(\cos(p\theta))$ and $\beta_p = E(\sin(p\theta))$ are given as follows:

$$\alpha_1 = \int_0^\infty \cos(\theta) \frac{v\sigma^2}{2(\alpha\sigma + \beta)}\left(\alpha + \beta V\tan\left(\frac{\theta}{2}\right)\right) e^{-\sigma V\tan\left(\frac{\theta}{2}\right)}\left(\sec^2\left(\frac{\theta}{2}\right)\right) d\theta$$

Consider the transformation $x = \tan, \cos(\theta) = 1 - \frac{2x^2}{1+x^2}$

$$\alpha_1 = \left(1 - \frac{2\alpha\sigma^2}{\alpha\sigma + \beta}\left[\frac{1}{2\sqrt{\pi}} G_{13}^{31}\left(\frac{\sigma^2}{4}\middle|\begin{matrix}-\frac{1}{2}\\ \frac{-1}{2}, 0, \frac{1}{2}\end{matrix}\right) - \frac{\beta\sigma^2}{(\alpha\sigma + \beta)\sqrt{\pi}} G_{13}^{31}\left(\frac{\sigma^2}{4}\middle|\begin{matrix}-1\\ -1, 0, \frac{1}{2}\end{matrix}\right)\right]\right) \qquad (10)$$

$$\beta_1 = \frac{\alpha^2}{\alpha^2 + 1}\int_0^\infty (\sin(\theta)) \quad (\alpha + x)e^{-\alpha x} dx$$

Consider the transformation $x = \tan, \sin(\theta) = \frac{2x}{1+x^2}$

$$\beta_1 = \frac{\sigma^2 2\alpha}{\alpha\sigma + \beta}\left(\frac{1}{2\sqrt{\pi}} G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}0\\ 0, 0, \frac{1}{2}\end{matrix}\right) + \frac{2}{2\sqrt{\pi}} G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}-\frac{1}{2}\\ \frac{-1}{2}, 0, \frac{1}{2}\end{matrix}\right)\right) (11)$$

Consider the transformation $x = \tan, \sin(\theta) = \left(\frac{2x}{1+x^2}\right)$

$$\alpha_2 = \frac{\sigma^2}{\alpha\sigma + \beta}\left(\int_0^\infty (\alpha + \beta x)e^{-\sigma x} dx + \left(\int_0^\infty \alpha\frac{8x^4 e^{-\sigma x}}{(1 + x^2)^2} dx + \int_0^\infty \frac{8x^4 e^{-\sigma x}}{(1 + x^2)^2}\beta x dx\right) - \left(\int_0^\infty \alpha\frac{8x^2 e^{-\sigma x}}{(1 + x^2)} dx + \int_0^\infty \frac{8x^2 e^{-\sigma x}}{(1 + x^2)}\beta x dx\right)\right)$$

Consider the transformation $x = \tan; \cos 2\theta = 1 + \frac{8x^4}{(1+x^2)^2} - \frac{8x^2}{(1+x^2)}$

$$\alpha_2 = \left(\begin{matrix}1 + \frac{\sigma^2}{\alpha\sigma+\beta}\left[\frac{4\sigma}{\sqrt{\pi}} G_{13}^{31}\left(\frac{\sigma^2}{4}\middle|\begin{matrix}\frac{-3}{2}\\ \frac{-1}{2}, 0, \frac{1}{2}\end{matrix}\right) + \frac{8}{2\sqrt{\pi}} G_{13}^{31}\left(\frac{\sigma^2}{4}\middle|\begin{matrix}-2\\ -1, 0, \frac{1}{2}\end{matrix}\right) - \frac{8\sigma}{2\sqrt{\pi}} G_{13}^{31}\left(\frac{\sigma^2}{4}\middle|\begin{matrix}\frac{-1}{2}\\ \frac{-1}{2}, 0, \frac{1}{2}\end{matrix}\right)\right.\\ \left. - \frac{4}{\sqrt{\pi}} G_{13}^{31} \frac{\sigma^2}{4}\middle|\begin{matrix}1\\ -1, 0, \frac{1}{2}\end{matrix}\right]\end{matrix}\right) \qquad (12)$$

Consider the transformation x = tan, sin2θ $= \frac{4x}{(1+x^2)} - \frac{8x^3}{(1+x^2)^2}$

$$\beta_2 = \left(\begin{array}{c} \frac{\sigma^2}{\alpha\sigma+\beta}\frac{2\sigma}{\sqrt{\pi}} G_{13}^{31}\left(\frac{\sigma^2}{4}\middle|\begin{matrix}0\\0,0,\frac{1}{2}\end{matrix}\right) + \frac{4}{\sqrt{\pi}} G_{13}^{31}\left(\frac{\sigma^2}{4}\middle|\begin{matrix}-\frac{1}{2}\\\frac{-1}{2},0,\frac{1}{2}\end{matrix}\right) - \frac{4\sigma}{\sqrt{\pi}} G_{13}^{31}\left(\frac{\sigma^2}{4}\middle|\begin{matrix}1\\-1,0,\frac{1}{2}\end{matrix}\right) \\ - \frac{4}{\sqrt{\pi}} G_{13}^{31}\left(\frac{\sigma^2}{4}\middle|\begin{matrix}\frac{-3}{2}\\\frac{-1}{2},0,\frac{1}{2}\end{matrix}\right) \end{array} \right) \quad (13)$$

4.3 the Circular mean

Coefficient of mean for Stereographic Semi-circular Three Parameter Lindley distribution is given by:

$$\mu = \tan^{-1}\left(\frac{\beta_1}{\alpha_1}\right)$$

$$\mu = \tan^{-1}\left(\frac{ \frac{\sigma^2 2\alpha}{\alpha\sigma+\beta}\left(\frac{1}{2\sqrt{\pi}} G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}0\\0,0,\frac{1}{2}\end{matrix}\right)\right) + \frac{\sigma^2 2\alpha}{\alpha\sigma+\beta}\left(\frac{2}{2\sqrt{\pi}} G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}-\frac{1}{2}\\\frac{-1}{2},0,\frac{1}{2}\end{matrix}\right)\right) }{ \left(1 - \frac{2\alpha\sigma^2}{\alpha\sigma+\beta}\left[\frac{1}{2\sqrt{\pi}} G_{13}^{31}\left(\frac{\sigma^2}{4}\middle|\begin{matrix}-\frac{1}{2}\\\frac{-1}{2},0,\frac{1}{2}\end{matrix}\right) - \frac{\beta\sigma^2}{(\alpha\sigma+\beta)\sqrt{\pi}} G_{13}^{31}\left(\frac{\sigma^2}{4}\middle|\begin{matrix}-1\\-1,0,\frac{1}{2}\end{matrix}\right)\right]\right) } \right) \quad (14)$$

4.4 the Circular median

Coefficient of median for Stereographic Semi-circular Three Parameter Lindley distribution is given by:

$$\rho = \sqrt{{\alpha_1}^2 + {\beta_1}^2}$$

$$\rho = \sqrt{\left(\begin{array}{c} \left(1 - \frac{2\alpha\sigma^2}{\alpha\sigma+\beta}\left[\frac{1}{2\sqrt{\pi}} G_{13}^{31}\left(\frac{\sigma^2}{4}\middle|\begin{matrix}-\frac{1}{2}\\\frac{-1}{2},0,\frac{1}{2}\end{matrix}\right) - \frac{\beta\sigma^2}{(\alpha\sigma+\beta)\sqrt{\pi}} G_{13}^{31}\left(\frac{\sigma^2}{4}\middle|\begin{matrix}-1\\-1,0,\frac{1}{2}\end{matrix}\right)\right]\right)^2 \\ + \left(\frac{\sigma^2 2\alpha}{\alpha\sigma+\beta}\left(\frac{1}{2\sqrt{\pi}} G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}0\\0,0,\frac{1}{2}\end{matrix}\right)\right) + \frac{\sigma^2 2\alpha}{\alpha\sigma+\beta}\left(\frac{2}{2\sqrt{\pi}} G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}-\frac{1}{2}\\\frac{-1}{2},0,\frac{1}{2}\end{matrix}\right)\right)\right)^2 \end{array} \right)} \quad (15)$$

4.5 the circular variance

Coefficient of variance for Stereographic Semi-circular Three Parameter Lindley distribution is given by:

$$v = 1 - \rho = \sqrt{{\alpha_1}^2 + {\beta_1}^2}$$

$$1 - \sqrt{\left(\begin{array}{c} \left(1 - \frac{2\alpha\sigma^2}{\alpha\sigma+\beta}\left[\frac{1}{2\sqrt{\pi}} G_{13}^{31}\left(\frac{\sigma^2}{4}\middle|\begin{matrix}-\frac{1}{2}\\\frac{-1}{2},0,\frac{1}{2}\end{matrix}\right) - \frac{\beta\sigma^2}{(\alpha\sigma+\beta)\sqrt{\pi}} G_{13}^{31}\left(\frac{\sigma^2}{4}\middle|\begin{matrix}-1\\-1,0,\frac{1}{2}\end{matrix}\right)\right]\right)^2 \\ + \left(\frac{\sigma^2 2\alpha}{\alpha\sigma+\beta}\left(\frac{1}{2\sqrt{\pi}} G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}0\\0,0,\frac{1}{2}\end{matrix}\right)\right) + \frac{\sigma^2 2\alpha}{\alpha\sigma+\beta}\left(\frac{2}{2\sqrt{\pi}} G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}-\frac{1}{2}\\\frac{-1}{2},0,\frac{1}{2}\end{matrix}\right)\right)\right)^2 \end{array} \right)} \quad (16)$$

4.6the circular Standard deviation

Coefficient of Standard deviation for Stereographic Semi-circular Three Parameter Lindley distribution is given by:

$$\sigma = \sqrt{-\log\left({\alpha_1}^2 + {\beta_1}^2\right)}$$

$$\sigma = \sqrt{-\log\left(\begin{array}{c} \left(1 - \frac{2\alpha\sigma^2}{\alpha\sigma+\beta}\left[\frac{1}{2\sqrt{\pi}} G_{13}^{31}\left(\frac{\sigma^2}{4}\middle|\begin{matrix}-\frac{1}{2}\\\frac{-1}{2},0,\frac{1}{2}\end{matrix}\right) - \frac{\beta\sigma^2}{(\alpha\sigma+\beta)\sqrt{\pi}} G_{13}^{31}\left(\frac{\sigma^2}{4}\middle|\begin{matrix}-1\\-1,0,\frac{1}{2}\end{matrix}\right)\right]\right)^2 \\ + \left(\frac{\sigma^2 2\alpha}{\alpha\sigma+\beta}\left(\frac{1}{2\sqrt{\pi}} G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}0\\0,0,\frac{1}{2}\end{matrix}\right)\right) + \frac{\sigma^2 2\alpha}{\alpha\sigma+\beta}\left(\frac{2}{2\sqrt{\pi}} G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}-\frac{1}{2}\\\frac{-1}{2},0,\frac{1}{2}\end{matrix}\right)\right)\right)^2 \end{array} \right)} \quad (17)$$

4.7 the Circular skewness

Coefficient of Sleekness for Stereographic Semi-circular Three Parameter Lindley distribution is given by:

$$\text{Circular skewness} = \frac{\beta_2}{(1-\rho)^{\frac{2}{3}}}$$

$$S_k = \frac{\left(\frac{\sigma^2}{\alpha\sigma+\beta}\frac{2\sigma}{\sqrt{\pi}}G_{13}^{31}\left(\frac{\sigma^2}{4}\middle|\begin{matrix}0\\0,0,\frac{1}{2}\end{matrix}\right)+\frac{4}{\sqrt{\pi}}G_{13}^{31}\left(\frac{\sigma^2}{4}\middle|\begin{matrix}-\frac{1}{2}\\\frac{-1}{2},0,\frac{1}{2}\end{matrix}\right)-\frac{4\sigma}{\sqrt{\pi}}G_{13}^{31}\left(\frac{\sigma^2}{4}\middle|\begin{matrix}1\\-1,0,\frac{1}{2}\end{matrix}\right)-\frac{4}{\sqrt{\pi}}G_{13}^{31}\left(\frac{\sigma^2}{4}\middle|\begin{matrix}\frac{-3}{2}\\\frac{-1}{2},0,\frac{1}{2}\end{matrix}\right)\right)}{\left(\sqrt{\left(\left(1-\frac{2\alpha\sigma^2}{\alpha\sigma+\beta}\left[\frac{1}{2\sqrt{\pi}}G_{13}^{31}\left(\frac{\sigma^2}{4}\middle|\begin{matrix}-\frac{1}{2}\\\frac{-1}{2},0,\frac{1}{2}\end{matrix}\right)-\frac{\beta\sigma^2}{(\alpha\sigma+\beta)\sqrt{\pi}}G_{13}^{31}\left(\frac{\sigma^2}{4}\middle|\begin{matrix}-1\\-1,0,\frac{1}{2}\end{matrix}\right)\right]\right)^2+\left(\frac{\sigma^2 2\alpha}{\alpha\sigma+\beta}\left(\frac{1}{2\sqrt{\pi}}G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}0\\0,0,\frac{1}{2}\end{matrix}\right)\right)+\frac{\sigma^2 2\alpha}{\alpha\sigma+\beta}\left(\frac{2}{2\sqrt{\pi}}G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}-\frac{1}{2}\\\frac{-1}{2},0,\frac{1}{2}\end{matrix}\right)\right)\right)^2\right)}\right)^{\frac{2}{3}}} \quad (18)$$

4.8 Coefficient of Kurtosis

The Coefficient of Kurtosis of for Stereographic Semi-circular Three Parameter Lindley distribution is given by:

$$\text{Circular kurtosis} = \frac{\alpha_2-\rho^4}{(1-\rho)^4}$$

$$= \frac{\left(1+\frac{\sigma^2}{\alpha\sigma+\beta}\left[\frac{4\sigma}{\sqrt{\pi}}G_{13}^{31}\left(\frac{\sigma^2}{4}\middle|\begin{matrix}\frac{-3}{2}\\\frac{-1}{2},0,\frac{1}{2}\end{matrix}\right)+\frac{8}{2\sqrt{\pi}}G_{13}^{31}\left(\frac{\sigma^2}{4}\middle|\begin{matrix}-2\\-1,0,\frac{1}{2}\end{matrix}\right)-\frac{8\sigma}{2\sqrt{\pi}}G_{13}^{31}\left(\frac{\sigma^2}{4}\middle|\begin{matrix}\frac{-1}{2}\\\frac{-1}{2},0,\frac{1}{2}\end{matrix}\right)-\frac{4}{\sqrt{\pi}}G_{13}^{31}\frac{\sigma^2}{4}\middle|\begin{matrix}1\\-1,0,\frac{1}{2}\end{matrix}\right]\right)-\left(\left(1-\frac{2\alpha\sigma^2}{\alpha\sigma+\beta}\left[\frac{1}{2\sqrt{\pi}}G_{13}^{31}\left(\frac{\sigma^2}{4}\middle|\begin{matrix}-\frac{1}{2}\\\frac{-1}{2},0,\frac{1}{2}\end{matrix}\right)-\frac{\beta\sigma^2}{(\alpha\sigma+\beta)\sqrt{\pi}}G_{13}^{31}\left(\frac{\sigma^2}{4}\middle|\begin{matrix}-1\\-1,0,\frac{1}{2}\end{matrix}\right)\right]\right)^2+\left(\frac{\sigma^2 2\alpha}{\alpha\sigma+\beta}\left(\frac{1}{2\sqrt{\pi}}G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}0\\0,0,\frac{1}{2}\end{matrix}\right)\right)+\frac{\sigma^2 2\alpha}{\alpha\sigma+\beta}\left(\frac{2}{2\sqrt{\pi}}G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}-\frac{1}{2}\\\frac{-1}{2},0,\frac{1}{2}\end{matrix}\right)\right)\right)^2\right)^2}{\left(1-\sqrt{\left(\left(1-\frac{2\alpha\sigma^2}{\alpha\sigma+\beta}\left[\frac{1}{2\sqrt{\pi}}G_{13}^{31}\left(\frac{\sigma^2}{4}\middle|\begin{matrix}-\frac{1}{2}\\\frac{-1}{2},0,\frac{1}{2}\end{matrix}\right)-\frac{\beta\sigma^2}{(\alpha\sigma+\beta)\sqrt{\pi}}G_{13}^{31}\left(\frac{\sigma^2}{4}\middle|\begin{matrix}-1\\-1,0,\frac{1}{2}\end{matrix}\right)\right]\right)^2+\left(\frac{\sigma^2 2\alpha}{\alpha\sigma+\beta}\left(\frac{1}{2\sqrt{\pi}}G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}0\\0,0,\frac{1}{2}\end{matrix}\right)\right)+\frac{\sigma^2 2\alpha}{\alpha\sigma+\beta}\left(\frac{2}{2\sqrt{\pi}}G_{13}^{31}\left(\frac{\alpha^2}{4}\middle|\begin{matrix}-\frac{1}{2}\\\frac{-1}{2},0,\frac{1}{2}\end{matrix}\right)\right)\right)^2\right)}\right)^4} \quad (19)$$

5.Parameter estimation:

the Method of Maximum Likelihood Estimate is Used for Estimating The Parameters of The Newly Proposed Distribution Known as of Semi-circular Three Parameter Lindley. Let x1, x2,…,xn be a Random Sample of Ssize n From of The Semi-circular Three Parameter Lindley, Fhen the Corresponding likelihood Function is Given By:

Let x1 ,x2,x3, x4,........xn be a random sample of size n from Semi-circular Three Parameter Lindley distribution The likelihood function, L Semi-circular Three Parameter Lindley distribution is given by:

$$L(\theta_1,\theta_2.\theta_n,\alpha) = f(\theta_1,\sigma,\alpha,\beta).\ f(\theta_2,\sigma,\alpha,\beta).f(\theta_3,\sigma,\alpha,\beta)\ ...\ f(\theta_n,\sigma,\alpha,\beta)$$

$$L(\theta,\sigma,\alpha,\beta) = \prod_{i=1}^{n} f(\theta_i,\sigma,\alpha,\beta)$$

$$L(\theta_i,\sigma,\alpha,\beta) = n\log(V\sigma^2) - n\log\big(2(\alpha\sigma+\beta)\big) - \sigma V\sum_{i=1}^{n}\tan\left(\frac{\theta_i}{2}\right) + \log\sum_{i=1}^{n}\left(\alpha Sec^2[\frac{\theta_i}{2}] + v\beta Sec^2[\frac{\theta_i}{2}]Tan[\frac{\theta_i}{2}]\right)$$

$$\log L = n\log(V\sigma^2) - n\log\big(2(\alpha\sigma+\beta)\big) - \sigma V\sum_{i=1}^{n}\tan\left(\frac{\theta_i}{2}\right) + \log\sum_{i=1}^{n}\left(\alpha Sec^2[\frac{\theta_i}{2}] + v\beta Sec^2[\frac{\theta_i}{2}]Tan[\frac{\theta_i}{2}]\right)$$

$$\frac{\partial \log L}{\partial\alpha} = -\frac{n\sigma}{\beta+\alpha\sigma} + \frac{nSec^2[\frac{\theta i}{2}]}{n\alpha Sec^2[\frac{\theta i}{2}] + v\beta\sigma^2 Sec^2[\frac{\theta i}{2}]Tan[\frac{\theta i}{2}]} = 0 \quad (20)$$

$$\frac{\partial \log L}{\partial \beta} = -\frac{n}{\beta + \alpha\sigma} + \frac{v\sigma^2 Sec^2\left[\frac{\theta i}{2}\right] Tan\left[\frac{\theta i}{2}\right]}{n\alpha Sec^2\left[\frac{\theta i}{2}\right] + v\beta\sigma^2 Sec^2\left[\frac{\theta i}{2}\right] Tan\left[\frac{\theta i}{2}\right]} = 0 \quad (21)$$

$$\frac{\partial \log L}{\partial \sigma} = \frac{2n}{\sigma} - \frac{n\alpha}{\beta + \alpha\sigma} - nvTan\left[\frac{\theta i}{2}\right] + \frac{2v\beta\sigma Sec^2\left[\frac{\theta i}{2}\right] Tan\left[\frac{\theta i}{2}\right]}{n\alpha Sec^2\left[\frac{\theta i}{2}\right] + v\beta\sigma^2 Sec^2\left[\frac{\theta i}{2}\right] Tan\left[\frac{\theta i}{2}\right]} = 0 \quad (22)$$

The maximum likelihood estimates $(\widehat{\alpha}, \widehat{\sigma}, \hat{\beta})$ equations $\frac{dLogL}{d\alpha} = 0$, ,The Equation (20,21,22) cannot be solved as they both are in closed forms. So we compute the parameters of the Semi-circular Three Parameter Lindley.

6.Conclusions

In this paper, we discussed circular distribution resulting from extending Stereographic Semi-circular Three Parameter Lindley distribution on unit semicircle which is obtained by inducing Inverse Stereographic Projection on the real line. The density and distribution functions of Journal of New Theory Stereographic Circular Three Parameter Lindley distribution admit explicit forms, as do trigonometric moments and observed that in similar to linear case Stereographic Circular are Special cases to proposed Semi-circular Three Parameter Lindley. As this distribution is asymmetric, promising for modelling asymmetrical directional data. Semi-circular Three Parameter Lindley.

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