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Estimate The Stress Peaks By Using Double Complex ZZ Transform

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Abstract: *the stress peaks of the human knee joint in standing and swing phase is presented in this article , as well as relationship between the stress peaks in the knee joint and determines the state of health or disease of the adult knee for resultant joint force using mathematical model is developed to calculating where using a relatively simple single algebraic equation and used the double complex ZZ transformation as a powerful tool to obtain a solution to the stress peaks and the model .It can be readily applied to estimate the stress peaks in knee joint in the most everyday activities . This is demonstrated by analyzing the data on the resultant knee force obtained from using a 12 mathematical program where a stance period of gait of cycle is considered .*

Keywords: Integral Transform , Complex double ZZ , Stress peaks

1.Introduction

Differential equations are still in use since Newton's time in understanding the physical, engineering and biological sciences. In addition to their contribution to the study of mathematical analysis. It can be said without overriding or exaggeration that differential equations extend their influence to many medical and social sciences such as psychology, economics and sociology, where most of the relationships and laws between variables in any engineering or physical issue appear in the form of differential equations. In order to be able to solve differential equations, integral transformations have been widely used. The importance of integral transformations is that give us both power and ease to solve initial value problems. The fact that this initial value is one of the main problems of linear and integral equations.

Many integral transforms, such as Laplace, Elzaki, Fourier, Aboodh, and Mahgoub, have been devised over the course of mathematical history. They were used to solve ordinary differential equations and their applications [1, 2, 3, 4, 5], integral equations [6, 7, 8], partial integral equations [9], as well as many researchers used some of these integral transforms to solve ordinary differential equation systems [10, 11, 12].

In 2016, The new integral transform “ZZ Transform Method’’ was introduced by Zain Ul Abadin Zafar [13] and it applied this new integral transform technique for solving first order linear differential equation, for instance Law of Natural Growth or Law of Natural Decay [14]

The double ZZ transform was developed in 2022 by researchers as a two-dimensional generalization of the one-dimensional ZZ transform .We have also established a few key theorems and properties. The suggested transform was used to find a solution to the typical squeeze action for unstable and incompressible fluid flow through the elastic porosity of layers in order to demonstrate the effectiveness and high accuracy of the transform. Squeeze lubrication involves compressing the synovial fluid, which produces hydrodynamic pressure. Both human mobility and the synovial joints depend on one other [15-16].

The double complex ZZ integral transform can be used to solve partial differential equations with initial conditions, the goal of this research. We first prove some of the double complex ZZ integral transform's key properties and theorems, which we apply to estimate the stress peaks in the most in knee joint of everyday activities.

That knowledge intrinsic path mechanical changes in articulator cartilage depends upon local stress levels and joint loading, and the abnormal mechanical stress upon joint cartilage is one of the main causes of osteoarthritis. In other words, it is essential to estimate the stress levels and peak stress on the joint surface in order to understand the path mechanical changes in the joint cartilage. In another aspect, the treatments of osteoarthritis of joint vary from conservative rehabilitative exercises to surgical treatments such as osteotomies or total arthroplasties. Therefore, the estimation of joint stress level and peak stress is very useful for both preoperative planning and postoperative rehabilitation. Furthermore, knowing the joint stress level is also very helpful in understanding the mechanics of the normal joint and understanding the pathology of the articular cartilage of the joint [17] .

2.<u>Basic Definition of Fractional Calculus</u>

In this section we provide some basic definitions and property of ZZ transform

Definition (2.1) [39]:let f be defined function at an open interval (a, b) then the integral transform of f whose its symbol $F(p)$is defined as follows:

$$I\{f(x)\} = F(p) = \int_a^b k(p\,, x) f(x) dx$$

Where k is a function of two variables, called the kernel of the transform, and a, b are real numbers or $\pm\infty$ such that the above integral converges.

Definition (2.2)[24] :- Let $\varphi\ (t)$be a function defined for all $t \geq 0$, then ZZ transform of $\varphi\ (t)$ is the function $Z(\gamma\,, \rho)$ is defined by

$$Z(\gamma\,, \rho) = \beta\ \{\varphi\ (t)\} = \frac{\rho}{\gamma} \int_0^\infty \varphi\ (t)\ e^{-\frac{\rho}{\gamma}t}\ dt$$

the inverse ZZ−transform of $Z(\gamma\,, \rho)$and it is defined as$\beta\ \{\varphi\ (t)\} = Z^{-1}\left(Z(\gamma\,, \rho)\right)$, where Z^{-1} is the inverse ZZ−transform operator.

Equivalently

$$\beta\ \{\varphi\ (t)\} = Z^{-1}\left(Z(\gamma\,, \rho)\right) = \frac{1}{2\pi\, i} \int_{\delta - i\infty}^{\delta + i\infty} \frac{\rho}{\gamma}\ Z(\gamma\,, \rho)\ e^{-\frac{\rho}{\gamma}t}\ d\gamma$$

Where $(\gamma\,, \rho)$ are complex number .

Linear Property ZZ Transform (2.3) [24] : If a, b are constants and $f(t)$, $g(t)$ are functions, then $\beta\ \{af(t)$+ $bg(t)\} =\ a\ \beta f(t)\}$ + $b\ \beta\ \{g(t)\}$.

_**Definition(2.4) [40]:-** The set A of functions for exponential order is defined as: $A = \{\varphi(t) \colon \exists\ N\,, \lambda_1\,, \lambda_2\ > 0\,, |\varphi(t)| > \mu\ e^{i\lambda_j |t|}, t \in (-1)^{j-1} \times (0\,, \infty), i \in \complement\ , j = 1{,}2\}$ (2.1)

For set A in equation (2.1), M is a finite random constant, $\lambda_1\,, \lambda_2$could be finite or infinite and $\varphi(t)$is a function that is defined for all $t \geq 0$. The new **SEA** integral transform is defined as:

$$\beta^c\ \{\varphi\ (t)\} = \frac{\rho}{\gamma} \int_0^\infty \varphi\ (t)\ e^{-i\left(\frac{\rho}{\gamma}\right)t}\ dt$$

Linearity Property SEA Transform (2.5) [40]: Considering the functions $f(t)$ and $g(t)$ and the constants a and b, then: β^c $\{af(t)$+ $bg(t)\} =\ a\beta^c\{f(t)\}$ + $b\ \beta^c\ \{g(t)\}$.

Definition (2.6): The double ZZ transform of the function $\varphi\ (x, t)$ is defined by the double integral as :

$$\beta^{2c}\ \{\varphi\ (x, t)\} = Z\big((\theta\,, \rho), (\omega, \gamma)\big)$$

$$= \frac{\theta}{\omega}\ \frac{\rho}{\gamma} \int_0^\infty \int_0^\infty \varphi\ (x\,, t)\ e^{-i\left(\frac{\theta}{\omega}\ x + \frac{\rho}{\gamma}\, t\right)}\ dx\, dt = Z\big((i\theta\,, i\rho), (\omega, \gamma)\big)$$

the inverse double ZZ−transform of $Z\big((\theta\,, \rho), (\omega, \gamma)\big)$and it is defined as $\beta^{2c}\ \{\varphi\ (x, t)\} = Z^{-1}\ \Big(Z\big((\theta\,, \rho), (\omega, \gamma)\big)\Big)$, where Z^{-1} is the inverse ZZ−transform operator.

Equivalently

$\beta^{2c}\{\varphi(x,t)\} = Z^{-1}\Big(Z\big((\theta i,\rho i),(\omega,\gamma)\big)\Big)$

$$= \frac{1}{2\pi i}\int_{\delta-i\infty}^{\delta+i\infty}\frac{\rho}{\gamma}\; Z\big((\theta i,\rho i),(\omega,\gamma)\big)\; e^{-\frac{i\rho}{\gamma}t}\; d\gamma \,.\, \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{\theta}{\omega}\; Z\big((\theta i,\rho i),(\omega,\gamma)\big)\; e^{-\frac{i\theta}{\omega}t}\; d\omega$$

Where $\theta, \rho, \omega, \gamma$ are complex value

3. Double Complex of ZZ integral transform for some Basic functions :

1- If $\varphi(x,t) = k$, then $\beta^{2c}\{1\} = -ik$

Proof: by definition (2.6)

$$\beta^{2c}\{1\} = \frac{\theta}{\omega}\frac{\rho}{\gamma}\int_0^\infty\int_0^\infty k\; e^{-i(\frac{\theta}{\omega}x+\frac{\rho}{\gamma}t)}\;dx\,dt = \frac{\theta}{\omega}\frac{\rho}{\gamma}\int_0^\infty\left[\frac{-\omega}{\theta}\; e^{-i\frac{\rho}{\gamma}t}\left(e^{-\frac{\theta}{i\omega}x}\Big|_0^\infty\right)\right]dt =$$

$$= \frac{\rho}{i\gamma}\int_0^\infty e^{-\frac{\rho}{i\gamma}t}\,\mathrm{dt} = \frac{\rho}{i\gamma}\,\frac{\gamma}{i\rho} = -1$$

In general

$\beta^{2c}\{k\} = -ik$, k is constant

2- If $\varphi(x,t) = e^{(\alpha x+\epsilon t)}$, then $\beta^{2c}\left\{e^{(\alpha x+\epsilon t)}\right\} = \frac{\theta\;\rho(\alpha\omega\epsilon\gamma-\theta\rho)}{(\alpha\epsilon\gamma\omega-\theta\rho)^2+(\theta\epsilon\gamma-\alpha\omega\rho)^2} + i\,\frac{\theta\;\rho(\theta\epsilon\gamma-\alpha\omega\rho)}{(\alpha\epsilon\gamma\omega-\theta\rho)^2+(\theta\epsilon\gamma-\alpha\omega\rho)^2}$

Proof: by definition (2.6)

$$\beta^{2c}\left\{e^{(\alpha x+\epsilon t)}\right\} = \frac{\theta}{\omega}\frac{\rho}{\gamma}\int_0^\infty\int_0^\infty e^{(\alpha x+\epsilon t)}\quad e^{-i\left(\frac{\theta}{\omega}x+\frac{\rho}{\gamma}t\right)}\;dx\,dt$$

$$= \frac{\theta}{\omega}\frac{\rho}{\gamma}\int_0^\infty\int_0^\infty e^{-(\frac{\theta}{\omega}i-\alpha)x}\quad e^{-\left(\frac{\rho}{\gamma}i-\epsilon\right)t}\;dx\,dt = \frac{\rho}{\gamma}\int_0^\infty\left[\int_0^\infty\frac{\theta}{\omega}\;e^{-(\frac{\theta}{\omega}i-\alpha)x}\,\mathrm{dx}\right]e^{-\left(\frac{\rho}{\gamma}i-\epsilon\right)t}\;dt$$

$$= \frac{\theta}{(\theta i-\alpha\omega)}\left[\int_0^\infty\frac{\rho}{\gamma}\;e^{-\left(\frac{\rho}{\gamma}-\epsilon\right)t}\;dt\right] = \frac{\theta}{(\theta i-\alpha\omega)}\;\frac{\rho}{(\rho i-\epsilon\gamma)}$$

$$= \frac{\theta\,\rho(\alpha\omega\epsilon\gamma-\theta\rho)}{(\alpha\epsilon\gamma\omega-\theta\rho)^2+(\theta\epsilon\gamma-\alpha\omega\rho)^2} + i\,\frac{\theta\,\rho(\theta\epsilon\gamma-\alpha\omega\rho)}{(\alpha\epsilon\gamma\omega-\theta\rho)^2+(\theta\epsilon\gamma-\alpha\omega\rho)^2}$$

3- If $\varphi(x,t) = x^\delta\; y^\varepsilon \quad \varepsilon,\delta = 0,1,2,\dots$, then $\beta^{2c}\left\{x^\delta\; y^\varepsilon\right\} = \delta!\;\varepsilon!\left(\frac{\omega}{\theta}i\right)^\delta\;\left(\frac{\gamma}{\rho}\,i\right)^\varepsilon$

Proof: by definition (2.6)

$$\beta^{2c}\left\{x^\delta\; y^\varepsilon\right\} = \frac{\theta}{\omega}\frac{\rho}{\gamma}\int_0^\infty\int_0^\infty x^\delta\; y^\varepsilon\quad e^{-i\left(\frac{\theta}{\omega}x+\frac{\rho}{\gamma}t\right)}\;dx\,dt$$

$$\beta^{2c}\left\{x^\delta\; y^\varepsilon\right\} = \frac{\theta}{\omega}\int_0^\infty e^{-i\frac{\theta}{\omega}x}\;x^\delta dx\;\;\frac{\rho}{\gamma}\int_0^\infty e^{-\frac{i\rho}{\gamma}t}\;y^\varepsilon\;dt = \delta!\;\varepsilon!\left(\frac{\omega}{\theta}\,i\right)^\delta\;\left(\frac{\gamma}{\rho}\,i\right)^\varepsilon$$

4- if $\varphi(x,t) = \cos(\alpha x+\epsilon t)$, then $\beta^{2c}\{\cos(\alpha x+\epsilon t)\} = \frac{-\theta\rho(\theta\rho+\alpha\omega\epsilon\gamma)}{(\theta^2-\alpha^2\omega^2)(\rho^2-\varepsilon^2\gamma^2)}$

Proof: by definition (2.6)

$$\beta^{2c}\{\cos(\alpha x+\epsilon t)\} = \frac{\theta}{\omega}\frac{\rho}{\gamma}\int_0^\infty\int_0^\infty\cos(\alpha x+\epsilon t)\quad e^{-i\left(\frac{\theta}{\omega}x+\frac{\rho}{\gamma}t\right)}\;dx\,dt$$

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$$\beta^{2c}\{\cos(\alpha x+\epsilon t)\}=\frac{\theta}{\omega}\ \frac{\rho}{\gamma}\int_0^\infty\int_0^\infty\left(\frac{e^{i(\alpha x+\epsilon t)}+e^{-i(\alpha x+\epsilon t)}}{2}\right)\ e^{-i\left(\frac{\theta}{\omega}\ x+\frac{\rho}{\gamma}t\right)}\ dx\,dt$$

$$\frac{1}{2}\left(\frac{\theta}{\omega}\ \frac{\rho}{\gamma}\int_0^\infty\int_0^\infty e^{i(\alpha x+\epsilon t)}\ e^{-i\left(\frac{\theta}{\omega}\ x+\frac{\rho}{\gamma}t\right)}\ dx\,dt+\frac{\theta}{\omega}\ \frac{\rho}{\gamma}\int_0^\infty\int_0^\infty e^{-i(\alpha x+\epsilon t)}\ e^{-i\left(\frac{\theta}{\omega}\ x+\frac{\rho}{\gamma}t\right)}dx\,dt\right)$$

$$=\frac{1}{2}\left(\frac{\rho}{\gamma}\int_0^\infty e^{-i\left(\frac{\rho}{\gamma}-\epsilon\right)t}\left(\frac{\theta}{\omega}\int_0^\infty e^{-\left(\frac{\theta}{\omega}-\alpha\right)x}\,dx\right)dt+\frac{\rho}{\gamma}\int_0^\infty e^{-i\left(\frac{\rho}{\gamma}+\epsilon\right)t}\left(\frac{\theta}{\omega}\int_0^\infty e^{-i\left(\frac{\theta}{\omega}+\alpha\right)x}\,dx\right)dt\right)$$

$$=\frac{1}{2}\ \left(\frac{\theta}{(\theta-\alpha\omega)\,i}\ \ \frac{\rho}{(\rho-\epsilon\gamma)\,i}+\frac{\theta}{(\theta+\alpha\omega)\,i}\ \ \frac{\rho}{(\rho+\epsilon\gamma)\,i}\right)=\frac{1}{2}\ \left(\frac{-\theta\rho(2\theta\rho+2\alpha\omega\epsilon\gamma)}{(\theta^2-\alpha^2\omega^2)(\rho^2-\epsilon^2\gamma^2)}\right)$$

$$=\frac{-\theta\rho(\theta\rho+a\omega\epsilon\gamma)}{(\theta^2-\alpha^2\omega^2)(\rho^2-\varepsilon^2\gamma^2)}$$

Similarly , we can proof

$$\beta^{2c}\{\sin(\alpha x+\epsilon t)\}=\frac{\theta}{\omega}\ \frac{\rho}{\gamma}\int_0^\infty\int_0^\infty\sin(\alpha x+\epsilon t)\quad e^{-i\left(\frac{\theta}{\omega}\ x+\frac{\rho}{\gamma}t\right)}\ dx\,dt$$

$$\beta^{2c}\{\sin(\alpha x+\epsilon t)\}=\frac{\theta}{\omega}\ \frac{\rho}{\gamma}\int_0^\infty\int_0^\infty\left(\frac{e^{i(\alpha x+\epsilon t)}-e^{-i(\alpha x+\epsilon t)}}{2i}\right)\ e^{-i\left(\frac{\theta}{\omega}\ x+\frac{\rho}{\gamma}t\right)}\ dx\,dt$$

$$\frac{1}{2}\left(\frac{\theta}{\omega}\ \frac{\rho}{\gamma}\int_0^\infty\int_0^\infty e^{i(\alpha x+\epsilon t)}\ e^{-i\left(\frac{\theta}{\omega}\ x+\frac{\rho}{\gamma}t\right)}\ dx\,dt-\frac{\theta}{\omega}\ \frac{\rho}{\gamma}\int_0^\infty\int_0^\infty e^{-i(\alpha x+\epsilon t)}\ e^{-i\left(\frac{\theta}{\omega}\ x+\frac{\rho}{\gamma}t\right)}dx\,dt\right)$$

$$=\frac{1}{2i}\left(\frac{\rho}{\gamma}\int_0^\infty e^{-i\left(\frac{\rho}{\gamma}-\epsilon\right))t}\left(\frac{\theta}{\omega}\int_0^\infty e^{-i\left(\frac{\theta}{\omega}-\alpha\right)x}\,dx\right)dt-\frac{\rho}{\gamma}\int_0^\infty e^{-i\left(\frac{\rho}{\gamma}+\epsilon\right)t}\left(\frac{\theta}{\omega}\int_0^\infty e^{-i\left(\frac{\theta}{\omega}+\alpha\right)x}\,dx\right)dt\right)$$

$$=\frac{1}{2i}\ \left(\frac{\theta}{(\theta-\alpha\omega)\,i}\ \ \frac{\rho}{(\rho-\epsilon\gamma)\,i}+\frac{\theta}{(\theta+\alpha\omega)\,i}\ \ \frac{\rho}{(\rho+\epsilon\gamma)\,i}\right)=\frac{1}{2i}\ \frac{-\theta\rho(2\epsilon\theta\gamma+2\alpha\omega\rho)}{(\theta^2-\alpha^2\omega^2)(\rho^2-\varepsilon^2\gamma^2)}$$

$$\frac{i\theta\rho(2\epsilon\theta\gamma+2\alpha\omega\rho)}{(\theta^2-\alpha^2\omega^2)(\rho^2-\varepsilon^2\gamma^2)}$$

5- if $\varphi\,(x,t)=\cos h(\alpha x+\epsilon t)$, then $\beta^{2c}\{\cosh(\alpha x+\epsilon t)\}=\frac{\theta\rho(\alpha\epsilon\omega\gamma-\theta\rho)}{(\theta^2+\alpha^2\omega^2)(\rho^2+\epsilon^2\gamma^2)}$

Proof: by definition (2.6)

$$\beta^{2c}\{\cosh(\alpha x+\epsilon t)\}=\frac{\theta}{\omega}\ \frac{\rho}{\gamma}\int_0^\infty\int_0^\infty\cosh(\alpha x+\epsilon t)\quad e^{-i\left(\frac{\theta}{\omega}\ x+\frac{\rho}{\gamma}t\right)}\ dx\,dt$$

$$\beta^{2c}\{\cos h(\alpha x+\epsilon t)\}=\frac{\theta}{\omega}\ \frac{\rho}{\gamma}\int_0^\infty\int_0^\infty\left(\frac{e^{(\alpha x+\epsilon t)}+e^{-(\alpha x+\epsilon t)}}{2}\right)\ e^{-i\left(\frac{\theta}{\omega}\ x+\frac{\rho}{\gamma}t\right)}\ dx\,dt$$

$$\frac{1}{2}\left(\frac{\theta}{\omega}\ \frac{\rho}{\gamma}\int_0^\infty\int_0^\infty e^{(\alpha x+\epsilon t)}\ e^{-i\left(\frac{\theta}{\omega}\ x+\frac{\rho}{\gamma}t\right)}\ dx\,dt+\frac{\theta}{\omega}\ \frac{\rho}{\gamma}\int_0^\infty\int_0^\infty e^{-(\alpha x+\epsilon t)}\ e^{-i\left(\frac{\theta}{\omega}\ x+\frac{\rho}{\gamma}t\right)}dx\,dt\right)$$

$$= \frac{1}{2}\left(\frac{\rho}{\gamma}\int_0^\infty e^{-\left(\frac{\rho}{\gamma}i-\epsilon\right)t}\left(\frac{\theta}{\omega}\int_0^\infty e^{-\left(\frac{\theta}{\omega}i-\alpha\right)x}\,dx\right)dt + \frac{\rho}{\gamma}\int_0^\infty e^{-\left(\frac{\rho}{\gamma}i+\epsilon\right)t}\left(\frac{\theta}{\omega}\int_0^\infty e^{-\left(\frac{\theta}{\omega}i+\alpha\right)x}\,dx\right)dt\right)$$

$$= \frac{1}{2}\left(\frac{\theta}{\theta i-\alpha\omega}\ \frac{\rho}{\rho i-\epsilon\gamma} + \frac{\theta}{\theta i+\alpha\omega}\ \frac{\rho}{\rho i+\epsilon\gamma}\right) = \frac{1}{2}\left(\frac{\theta\rho(-2\theta\rho+2\alpha\epsilon\omega\gamma)}{(\theta^2+\alpha^2\omega^2)(\rho^2+\epsilon^2\gamma^2)}\right)$$

$$= \frac{\theta\rho(\alpha\epsilon\omega\gamma-\theta\rho)}{(\theta^2+\alpha^2\omega^2)(\rho^2+\epsilon^2\gamma^2)}$$

Similarly , we can proof

$$\beta^{2c}\{\sinh(\alpha x+\epsilon t)\} = \frac{\theta}{\omega}\ \frac{\rho}{\gamma}\int_0^\infty\int_0^\infty \sinh(\alpha x+\epsilon t)\quad e^{-i\left(\frac{\theta}{\omega}x+\frac{\rho}{\gamma}t\right)}\,dx\,dt$$

$$\beta^{2c}\{\sinh(\alpha x+\epsilon t)\} = \frac{\theta}{\omega}\ \frac{\rho}{\gamma}\int_0^\infty\int_0^\infty \left(\frac{e^{(\alpha x+\epsilon t)}-e^{-(\alpha x+\epsilon t)}}{2}\right)\ e^{-i\left(\frac{\theta}{\omega}x+\frac{\rho}{\gamma}t\right)}\,dx\,dt$$

$$\frac{1}{2}\left(\frac{\theta}{\omega}\ \frac{\rho}{\gamma}\int_0^\infty\int_0^\infty e^{(\alpha x+\epsilon t)}\ e^{-i\left(\frac{\theta}{\omega}x+\frac{\rho}{\gamma}t\right)}\,dx\,dt + \frac{\theta}{\omega}\ \frac{\rho}{\gamma}\int_0^\infty\int_0^\infty e^{-(\alpha x+\epsilon t)}\ e^{-i\left(\frac{\theta}{\omega}x+\frac{\rho}{\gamma}t\right)}\,dx\,dt\right)$$

$$= \frac{1}{2}\left(\frac{\rho}{\gamma}\int_0^\infty e^{-\left(\frac{\rho i}{\gamma}-\epsilon\right)t}\left(\frac{\theta}{\omega}\int_0^\infty e^{-\left(\frac{\theta i}{\omega}-\alpha\right)x}\,dx\right)dt - \frac{\rho}{\gamma}\int_0^\infty e^{-\left(\frac{\rho i}{\gamma}+\epsilon\right)t}\left(\frac{\theta}{\omega}\int_0^\infty e^{-\left(\frac{\theta i}{\omega}+\alpha\right)x}\,dx\right)dt\right)$$

$$= \frac{1}{2}\left(\frac{\theta}{\theta i-\alpha\omega}\ \frac{\rho}{\rho i-\epsilon\gamma} - \frac{\theta}{\theta i+\alpha\omega}\ \frac{\rho}{\rho i+\epsilon\gamma}\right) = \frac{1}{2}\left(\frac{i\theta\rho(2\epsilon\gamma\theta+2\alpha\epsilon\omega)}{(\theta^2+\alpha^2\omega^2)\,(\rho^2+\epsilon^2\gamma^2)}\right)$$

$$= \frac{i\theta\rho(\epsilon\gamma\theta+\alpha\rho\omega)}{(\theta^2+\alpha^2\omega^2)\,(\rho^2+\epsilon^2\gamma^2)}$$

4. Some property and theorm for the double complex ZZ transform

Properties 4.1 :- The linear properties of ZZ double complex transform .

Let $\varphi\,(x,t)$ and $\mu\,(x,t)$ are a functions for x , t and α , ε are constant , then

$$\beta^{2c}\{\alpha\varphi\,(x,t)+\varepsilon\,\mu(x,t)\} = \alpha\ \beta^{2c}\big(\varphi\,(x,t)\big)+\varepsilon\beta^{2c}\big(\mu\,(x,t)\big)$$

Proof :-

$$\beta^{2c}\{\alpha\varphi\,(x,t)+\varepsilon\,\mu(x,t\} = \frac{\theta}{\omega}\ \frac{\rho}{\gamma}\int_0^\infty\int_0^\infty \{\alpha\varphi\,(x,t)+\varepsilon\,\mu(x,t\}\quad e^{-i\left(\frac{\theta}{\omega}x+\frac{\rho}{\gamma}t\right)}\,dx\,dt$$

$$= \frac{\theta}{\omega}\ \frac{\rho}{\gamma}\int_0^\infty\int_0^\infty \{\alpha\varphi\,(x,t)\}\quad e^{-i\left(\frac{\theta}{\omega}x+\frac{\rho}{\gamma}t\right)}\,dx\,dt + \frac{\theta}{\omega}\ \frac{\rho}{\gamma}\int_0^\infty\int_0^\infty \{\varepsilon\,\mu(x,t)\}\quad e^{-i\left(\frac{\theta}{\omega}x+\frac{\rho}{\gamma}t\right)}\,dx\,dt +$$

$$= \alpha \frac{\theta}{\omega}\ \frac{\rho}{\gamma} \int_0^\infty \int_0^\infty \{\varphi\ (x,t)\}\quad e^{-i\left(\frac{\theta}{\omega}\ x+\frac{\rho}{\gamma} t\right)}\ dx\ dt + \ \varepsilon \frac{\theta}{\omega}\ \frac{\rho}{\gamma} \int_0^\infty \int_0^\infty \{\ \mu(x,t)\}\quad e^{-i\left(\frac{\theta}{\omega}\ x+\frac{\rho}{\gamma} t\right)}\ dx\ dt$$

$$= \ \alpha\ \beta^{2c}\big(\ \varphi\ (x,t)\big) + \varepsilon\beta^{2c}\big(\ \mu\ (x,t)\big)$$

Theorem 4.2 :- Double complex ZZ transform of first and second order partial derivatives are in the form

1) $\beta^2 \left\{\ \frac{\partial\varphi\ (x,t)}{\partial x}\right\} \ = \frac{\theta}{\omega}\ \left(iZ((\theta\ ,\rho)\ ,(\omega\ ,\gamma)) - \beta^c(\varphi(0\ ,t)\right)$

2) $\beta^2 \left\{\ \frac{\partial^2\varphi\ (x,t)}{\partial x^2}\right\}$

$= \frac{\theta^2}{\omega^2}\ \left(-Z((\theta\ ,\rho)\ ,(\omega\ ,\gamma)) - i\beta^c\big(\varphi(0\ ,t)\big)\right) - \ \frac{\theta}{\omega}\ \beta^c\left(\frac{\partial}{\partial x}\varphi(0\ ,t)\right)$

3)$\beta^2 \left\{\ \frac{\partial\varphi\ (x,t)}{\partial t}\right\} = \frac{\rho}{\gamma}\ \left(iZ((\theta\ ,\rho)\ ,(\omega\ ,\gamma)) - \beta^c(\varphi(x\ ,0)\right)$

4) $\beta^2 \left\{\ \frac{\partial^2\varphi\ (x,t)}{\partial t^2}\right\} = \frac{\rho^2}{\gamma^2}\ \ (-Z((\theta\ ,\rho)\ ,(\omega\ ,\gamma)) - iZ(x\ ,0)) - \ \frac{\rho}{\gamma}\ \left(\frac{\partial}{\partial t}\ Z(\ x\ ,0)\right)$

5) $\beta^2 \left\{\ \frac{\partial^2\varphi\ (x,t)}{\partial x\partial t}\right\} = \frac{\theta}{\omega}\ Z(0,0) - \frac{\theta^2}{\omega^2}\ \ Z\ (0,t) + \frac{\theta}{\omega}\ \left(\frac{\rho}{\gamma}\ \big(Z((\theta\ ,\rho)\ ,(\omega\ ,\gamma)) - Z(x\ ,0)\big)\right)$

Proof :- by using definition (2.6) of double ZZ transform , we get

1) $\beta^2 \left\{\ \frac{\partial\varphi\ (x,t)}{\partial x}\right\} = \frac{\theta}{\omega}\ \ \frac{\rho}{\gamma}\ \int_0^\infty\int_0^\infty \frac{\partial\varphi\ (x,t)}{\partial x}\quad e^{-i\left(\frac{\theta}{\omega}\ x+\frac{\rho}{\gamma} t\right)}\ \ dx\ dt = \frac{\theta}{\omega}\ \ \left(iZ((\theta\ ,\rho)\ ,(\omega\ ,\gamma)) - \beta^c(\varphi(0\ ,t)\right)$

2)$\beta^2 \left\{\ \frac{\partial^2\varphi\ (x,t)}{\partial x^2}\right\} = \frac{\theta}{\omega}\ \ \frac{\rho}{\gamma}\ \int_0^\infty \int_0^\infty \ \frac{\partial^2\varphi\ (x,t)}{\partial x^2}\quad e^{-i\left(\frac{\theta}{\omega}\ x+\frac{\rho}{\gamma} t\right)}\ \ dx\ dt$

$\frac{\theta^2}{\omega^2}\ \left(\ i^2Z((\theta\ ,\rho)\ ,(\omega\ ,\gamma)) - \beta^c\big(\varphi(0\ ,t)\big)\right) - \ \frac{\theta}{\omega}\ \beta^c\left(\frac{\partial}{\partial x}\varphi(0\ ,t)\right)$

3) $\beta^2 \left\{\ \frac{\partial\varphi\ (x,t)}{\partial t}\right\} = \frac{\theta}{\omega}\ \ \frac{\rho}{\gamma}\ \int_0^\infty \int_0^\infty \frac{\partial\varphi\ (x,t)}{\partial t}\quad e^{-i\left(\frac{\theta}{\omega}\ x+\frac{\rho}{\gamma} t\right)}\ \ dx\ dt = \frac{\rho}{\gamma}\ \left(iZ((\theta\ ,\rho)\ ,(\omega\ ,\gamma)) - \beta^c(\varphi(x\ ,0)\right)$

4) $\beta^2 \left\{\ \frac{\partial^2\varphi\ (x,t)}{\partial t^2}\right\} = \frac{\theta}{\omega}\ \ \frac{\rho}{\gamma}\ \int_0^\infty \int_0^\infty \ \frac{\partial^2\varphi\ (x,t)}{\partial t^2}\quad e^{-i\left(\frac{\theta}{\omega}\ x+\frac{\rho}{\gamma} t\right)}\ \ dx\ dt$

$$\frac{\rho^2}{\gamma^2}\ \left(i^2 Z((\theta,\rho),(\omega,\gamma)) - \beta^c\big(\varphi(x,0)\big)\right) - \frac{\rho}{\gamma}\ \beta^c\left(\frac{\partial}{\partial t}\varphi(x,0)\right)$$

5) $$\beta^2\left\{\frac{\partial^2\varphi(x,t)}{\partial x\partial t}\right\} = \frac{\theta}{\omega}\ \frac{\rho}{\gamma}\int_0^\infty\int_0^\infty \frac{\partial^2\varphi(x,t)}{\partial x\partial t}\quad e^{-i\left(\frac{\theta}{\omega}\ x+\frac{\rho}{\gamma}t\right)}\ dx\,dt$$

$$= \frac{\theta}{\omega}\ Z(0,0) - \frac{\theta}{\omega} i\ \beta(\varphi(0,t)) + \frac{\theta}{\omega}\ i\ \left(\frac{\rho}{\gamma}\ \left(iZ((\theta,\rho),(\omega,\gamma)) - \beta^c(\varphi(x,0)\right)\right)$$

Table 1 : ZZ transform for some special functions [40,24]

$\varphi(t)$	$\beta\{\varphi(t)\}$	$\varphi(t)$	$\beta^c\{\varphi(t)\}$
k	k	k	$-ik$
e^{at}	$\frac{\rho}{\rho - a\gamma}$	$\sin at$	$\frac{-a\rho\gamma}{\rho^2 - a^2\gamma^2}$
t^n	$n!\ (\frac{\gamma}{\rho})^n$	t^n	$(-1)^n\ (i)^{n-1}\ n!\ (\frac{\gamma}{\rho})^n$
$\sin at$	$\frac{a\rho\gamma}{\rho^2 + a^2\gamma^2}$	e^{at}	$-\left[\frac{a\rho\gamma}{a^2\gamma^2+\rho^2} + i\ \frac{\rho^2}{a^2\gamma^2+\rho^2}\right]$
$\cos at$	$\frac{\rho^2}{\rho^2 + a^2\gamma^2}$	$\cos at$	$\frac{-i\rho^2}{\rho^2 - a^2\gamma^2}$

5. Application

In this section , applied complex double ZZ transform to estimation of knee joint stress level and peak stress is very useful Furthermore, knowing the knee stress level is also very helpful in understanding the mechanics of the normal knee and understanding the pathology of the articular cartilage of the knee .To calculate the stress level and analyze the peak stress at knee joint for the human routine activities through the equation are given in the following:

$$\frac{\partial P_e}{\partial F} = \frac{l}{\bar{u}\,.\,t}\left(\frac{D_u\,\beta_0}{\tau}\ \ \frac{\partial W^*}{dr^*} + \frac{\mu\ \bar{u}}{l\ H_0}\ \ F\right) + \ (F^2 - FR)\ \ \frac{\partial p^*}{dr^*} \qquad (1)$$

Now, derivative the dimensionless squeeze pressure (p^*) and load carry capacity respect to r^* substitute in equation (5.1) , we get

$$\frac{\partial W^*}{dr^*} = \frac{6\pi\ r^*\big(1 + 0.25\ (3+\bar{U})\big)\ \ (F * h^{*2} * Sl)}{H_0^* * R^3\ \left(1 - \frac{L\ w\ H}{N} - \frac{\bar{w}\ N\ R}{SL} + C_0\right) + \ \beta\ R\ h_c(0.1\,\beta\ R\ h_0\ S_r + 0.125\ C_1)} \qquad (2)$$

$$\frac{\partial p}{\partial r^*} = \frac{(\,-3r^*)(1 + 0.25\ (3+\bar{U})}{R^3\ \left(1 - \frac{L\ w\ H}{N} - \frac{\bar{w}\ N\ R}{SL} + C_0\right) + \ \beta\ R\ h_c(0.1\,\beta\ R\ h_0\ S_r + 0.125\ C_1)} \qquad (3)$$

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Now, substitute (5.2) and (5.3) in equation (5.1) , we get

$$\frac{\partial P_e}{\partial F} = \frac{l}{\bar{u}\,.\,t}$$

$$\frac{\partial P_e}{\partial F} = \left(\frac{D_u\,\beta_0}{\tau} \quad \frac{6\pi\, r^*\left(1 + 0.25\,(3 + \bar{U})\right)\;(F * h^{*2} * Sl)}{H_0^* * R^3 \begin{pmatrix} 1 - \frac{L\;w\,H}{N} - \frac{\bar{w}\,N\,R}{SL} + C_0 + \\ + \beta\,R\,h_c(0.1\,\beta\,R\,h_0\,S_r + 0.125\,C_1) \end{pmatrix}} + \frac{\mu\,l\;\bar{u}}{W}\;\; F \right)$$

$$+ \left((F^2 - FR\,)\frac{(\,-3r^*)(1 + 0.25\,(3 + \bar{U})}{R^3 \begin{pmatrix} 1 - \frac{L\;w\,H}{N} - \frac{\bar{w}\,N\,R}{SL} + C_0 + \\ + \beta\,R\,h_c(0.1\,\beta\,R\,h_0\,S_r + 0.125\,C_1) \end{pmatrix}} \right) \qquad (4)$$

Now , we have been presented dimensionless stress peaks

$$\bar{l} = \frac{l}{\bar{u}\,.\,t}, \qquad \bar{\mu} = \frac{\mu\,l\;\bar{u}}{W} \qquad (5)$$

Substitute the equation (5.5) in equation (5.4) , we obtain

$$\frac{\partial P_e}{\partial F} = \bar{l}\left(\frac{D_u\,\beta_0}{\tau} \quad \frac{6\pi\, r^*\left(1 + 0.25\,(3 + \bar{U})\right)\;(F * h^{*2} * Sl)}{H_0^* * R^3 \begin{pmatrix} 1 - \frac{L\;w\,H}{N} - \frac{\bar{w}\,N\,R}{SL} + C_0 + \\ + \beta\,R\,h_c(0.1\,\beta\,R\,h_0\,S_r + 0.125\,C_1) \end{pmatrix}} + \bar{\mu}\;\; F \right)$$

$$+ \left((F^2 - FR\,)\frac{(\,-3r^*)(1 + 0.25\,(3 + \bar{U})}{R^3 \begin{pmatrix} 1 - \frac{L\;w\,H}{N} - \frac{\bar{w}\,N\,R}{SL} + C_0 + \\ + \beta\,R\,h_c(0.1\,\beta\,R\,h_0\,S_r + 0.125\,C_1) \end{pmatrix}} \right) \qquad (6)$$

taking double complex ZZ transform to both equation (5.6) with condition $P_e(r\,,F) = 0$, we get

$$\frac{\rho}{\gamma}\;\left(iP_e((\theta\,,\rho)\,,(\omega\,,\gamma)) - \beta^c(\varphi(r\,,0)\right)$$

$$= \bar{l}\left(- \frac{D_u\,\beta_0}{\tau} \quad \frac{6\pi\, r^*\left(1 + 0.25\,(3 + \bar{U})\right)\;(F * h^{*2} * Sl)}{H_0^* * R^3 \begin{pmatrix} 1 - \frac{L\;w\,H}{N} - \frac{\bar{w}\,N\,R}{SL} + C_0 + \\ + \beta\,R\,h_c(0.1\,\beta\,R\,h_0\,S_r + 0.125\,C_1) \end{pmatrix}} - \bar{\mu}\;\; \frac{\gamma}{\rho i} \right)$$

$$+\left(\left(2\left(\frac{\gamma}{\rho}\right)^{2}+\frac{\gamma}{\rho i}\right)\frac{(-3r^{*})(1+0.25\,(3+\bar{U})}{R^{2}\begin{pmatrix}1-\frac{L\ w\ H}{N}-\frac{\bar{w}\ N\ R}{SL}+C_{0}+\\+\beta\ R\ h_{c}(0.1\,\beta\ R\ h_{0}\ S_{r}+0.125\ C_{1})\end{pmatrix}}\right) \quad (7)$$

After performing simple mathematical operations we get:

$$P_{e}(r\ ,F)=\bar{l}\left(\frac{D_{u}\ \beta_{0}}{\tau}\ \frac{6\pi\ r^{*}\big(1+0.25\ (3+\bar{U})\big)\ (F*h^{*2}*Sl)\ F}{H_{0}^{*}*R^{3}\begin{pmatrix}1-\frac{L\ w\ H}{N}-\frac{\bar{w}\ N\ R}{SL}+C_{0}+\\+\beta\ R\ h_{c}(0.1\,\beta\ R\ h_{0}\ S_{r}+0.125\ C_{1})\end{pmatrix}}+\frac{1}{2}\bar{\mu}\ F^{2}\right)$$

$$+\left(\left(\frac{1}{3}\ F^{3}-\frac{1}{2}\ F^{2}\right)\frac{(-3r^{*})(1+0.25\,(3+\bar{U})}{R^{3}\begin{pmatrix}1-\frac{L\ w\ H}{N}-\frac{\bar{w}\ N\ R}{SL}+C_{0}+\\+\beta\ R\ h_{c}(0.1\,\beta\ R\ h_{0}\ S_{r}+0.125\ C_{1})\end{pmatrix}}\right) \quad (8)$$

5.1 <u>Factors affecting stress of peaks</u>

Stress peaks is associated with skeletal muscle weakness and decreased muscle function. People of a certain age usually suffer from joint and knee pain as a result of excess weight, as obesity increases the pressure on the knee and increases the exposure of the knee cartilage to deterioration and joint osteoarthritis, and this affects the practice of daily activities such as walking or climbing stairs. As well as a lack of flexibility muscle , this leads to You may be exposed to knee injuries, as flexible muscles give full range of motion and strong muscles have the ability to protect the joints. . This study aims to clarify the effect of stress on muscle function and its effect on the knee joint ,we have dealt with a set of factors that affect the stress of peaks and after applying equation (5.8)

Figure (1) shows the variation of stress peaks (P) as a function of step length (Sl) for different values of particle velocity ($\bar{u}$). After applying equation (5.8), the effective stress peaks increase with increasing particle speed, as shown in table (2)in the case of a healthy and injured person. This is because increasing the velocity of molecules leads to increasing the velocity synovial fluid, that is, there is an increase in the kinetic energy of a person, which consequently leads to an increase in the length of the step and an increase in its frequency. The flexibility of the muscles on the joint improves the depth of neuromuscular coordination and the ability to relax the muscles working on it, which helps in rapid muscle contraction and shows the velocity element's connection with the flexibility elements. The exist another factor play a role in influential on peaks stress been Cohesive force of particle as seen in figure (2) .it observed peaks stress decreasing with increasing cohesive force of particle ,since during gait cycle (swing phase - stance phase) the particle synovial fluid surface of approach each other , which in turn increasing the viscosity in normal joint , in case diseases joint the viscosity synovial fluid increasing then shear rate lower this lead the lower in flow rate synovial fluid we see that the maximum peaks stress during active in case healthy joint is 11.766 at Cohesive force of particle of the synovial fluid (N=6) , while maximum peaks steress during active is 1.77565 at Cohesive force of particle of the synovial fluid (N=6).

In figure (3) represents the effect of the body weight on the stress of peaks, where it was found that increasing the stress of peaks depends on increasing body weight and when we compared stress of peaks in healthy joint and pathological joint with variant body weight , it was found the percentage rate of increase in stress of peaks in healthy joint is 0.10% , while the percentage rate of increase in stress of peaks in pathological joint is 0.05 % it was found at patients injured with osteoarthritis in the knee joint.

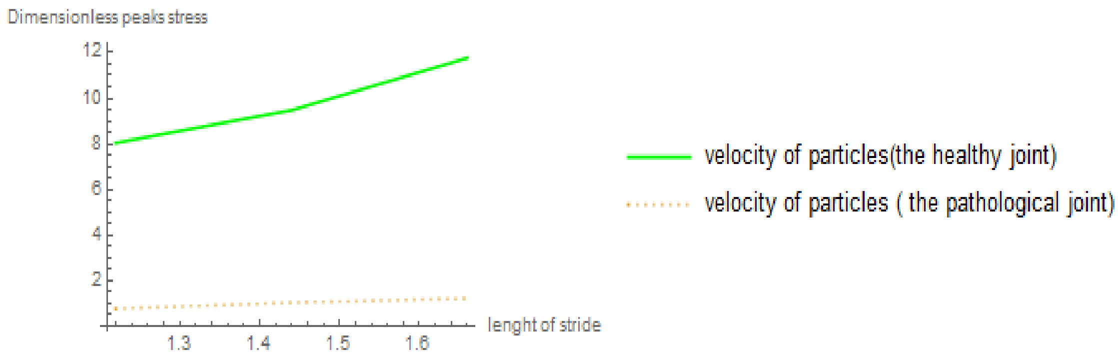

Figure 1 : shows the variation of dimensionless stress of peaks (P_e) for different values of particle velocity $(\bar{u})$

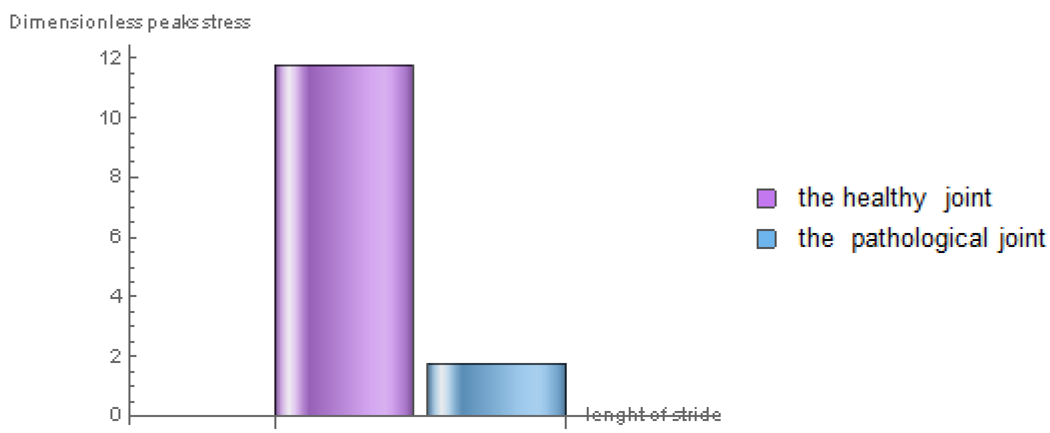

Figure 2 : shows the variation of dimensionless stress of peaks (P_e) for different values of Cohesive force of particle(N)

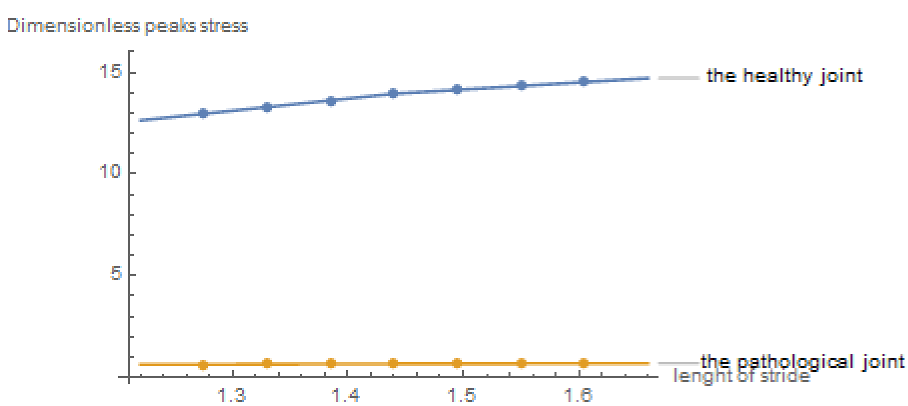

Figure 3 : shows the variation of dimensionless stress of peaks (P_e) for different values of weight of body (w)

Table 2 : Relationship between stress of peaks (P_e) for different parameters $(\vec{u}\ , C_0, w)$

The healthy joint Muscles Force (F=2)					
Velocity of particle	Stress Peaks	Cohesive force of particle (N)	Stress Peaks	Weight of body	Stress Peaks
2	8.04045	2	16.328	600	12.6464
4	9.82278	4	13.6829	800	13.9554
6	11.7904	6	11.766	900	14.7173
The disease joint Muscles Force (F=1.5)					
Velocity of particle	Stress Peaks	Cohesive force of particle (N)	Stress Peaks	Weight of body	Stress Peaks
2	0.8131	2	1.6277	600	0.6414
4	0.9316	4	1.2809	800	0.6795
6	1.0990	6	1.2002	900	0.7003

Reference

1. H. A. Agwa, F. M. Ali and A. Kilicman; A new integral transform on time scales and its applications, Advances in Difference Equations, 2012, pp. 1-14.

2. M. E. Benattia, K. Belghaba; Application of Aboodh Transform For Solving First order Constant Coefficients Complex Equation, General Letters in Mathematics Vol. 6, No. 1, Mar 2019, pp.28-34.

3. P. S. Kumar and A. Viswanathan; Application of Mahgoub Transform to Mechanics , electrical Circuit Problems, International Journal of Science and Research,Vol. 7, Issue 7 (2018) pp 195-197.

4. P. V. Pawani, U. L. Priya and B. A. Reddy: Solving Differential Equations by using Laplace Transform, International Journal of Research and Analytical Reviews, Vol. 5, Issue 3 , pp 1796-1799.

5. Khalid Suliman Aboodh, Application of new transform `Aboodh Transform ', to partial differential equations, Global Journal of Pure and Applied Mathematics, 10(2), 249-254, (2014).

6. S. Aggarwal, R. Chauhan and N Sharma: A New Application of Mahgoub Transform for Solving Linear Volterra Integral Equations, Aasian Resonance ,Vol 7 Issue 2 (2018) pp 46-48.

7. S. Aggarwal, R. Chauhan and N Sharma, :Application of Aboodh Transform for Solving Linear Volterra Integro-Differential Equations of Second Kind, International Journal of Research in Advent Technology, Vol.6, No.6, June (2018),pp 1186-1190.

8. S.A. Pourreza Ahmadi, H. Hosseinzadeh and A. Yazdani Cherati; A New IntegralTransform for Solving Higher OrderOrdinary Differential Equations, Nonlinear Dynamics and Systems Theory, Vol 19(2) (2019)pp. 243252.

9. K.S. Aboodh, R. A. Farah, I. A. Almardy and F.A,. ALmostafa, Solution of Partial Integro-differential Equations by using Aboodh and Double Aboodh Transform Methods, Global Journal of Pure and Applied Mathematics, Volume 13, Number 8 (2017), pp. 4347-4360.

10. T. M. Elzaki and S. M. Elzaki; On the ELzaki Transform and Systems of Ordinary Differential Equations

11. Global Journal of Pure and Applied Mathematics, Volume 7, Number 1 (2011), pp. 113119.

12. Sadiq A. Mehdi, Emad A. Kuffi, Jinan A. Jasim, " Solving Ordinary Differential Equations Using a New General Complex Integral Transform", Journal of interdisciplinary mathematics, (2022).

13. Zain U. Zafar, (2016), "ZZ Transform Method", International Journal of Advanced Engineering and Global Technology, Vol.4, No.1, pp.1605-1611.

14. Rao, R. U. (2017). ZZ transform method for natural growth and decay problems. International Journal of Progressive Sciences and Technologies (IJPSAT), 5(2), 147-150.

15. Shaban, R. A., Abdullah, E. Y., & Albukhuttar, A. N. (2022). Study unsteady flow of synovial fluid in the elastic porosity of layers using double ZZ transformation. Journal of Interdisciplinary Mathematics, 25(6), 1933-1950.https://doi.org/10.1080/09720502.2022.2093438

16. Shaban, R. A., Abdullah, E. Y., & Albukhuttar, A. N. "Theoretical Analysis of Squeeze Lubrication Using Double ZZ Transform: Application of non-Newtonian fluids", Mathematical Modelling of Engineering Problems (accepted for publication) , 2023**.**

17. Wang X. , Tian Y. & Yixiang .D ,(2005) , " The Hip Stress Level Analysis For Human Routine Activities" , Biomedical EngineeringApplications, Basis & Communications, Vol.(17), pp. 153-158.