

Heaviside Step Function with Differential Equations Laplace Transforms In Matlab

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Abstract: Many researchers have presented studies on the definition of the Heaviside function and its uses. In this paper, we mention one of the uses of the Heaviside function in solving differential equations using Laplace transforms, and then we represent it in MATLAB.

Keywords: Heaviside Step Function, differential equations, Matlab.

1. INTRODUCTION

The Heaviside gradient function is a mathematical concept that has been developed and used by many scientists. In 1890,[1] Oliver Heaviside presented a study of the Heaviside function in circuit analysis and signal theory. In 1913, [2] Andrei Markov used the Heaviside function to develop theories about stochastic processes. In 1995,[3] Edward Wigner presented a study of the Heaviside function in the context of quantum theory. In 2001,[4] B. Rousseff and D. Mischev used the modified Heaviside function to examine the transmission properties of an optical detector operating in an electromotive tension system. In 2002,[5] Eric W. Weisstein presented a more in-depth study of the Heaviside function. In 2010,[6] Jun Wang presented a study in which he used the Heaviside function as an activation function to analyze and design a winner model. In 2019,[7] J. Venetis obtained a precise analytical form of the Heaviside gradient function. In 2021,[8] John Venetis presented a study in which he derived an explicit form for the Heaviside gradient function, which is clearly a fundamental concept in calculus and also participates in many other areas of applied and engineering mathematics.

2. Heaviside step function in differential equations Laplace transforms

Heaviside step function can be used when dealing with differential equations because it describes systems that experience sudden signals or changes in time. For example, if we have a differential equation that includes an input function that starts at a certain time, we can use the Heaviside function to facilitate the solution.

2.1 Definition:

The Heaviside Step function is a mathematical function used in many fields such as mathematics, engineering, and physics. It is symbolized by the symbol $H(t)$, and is known by the following formula:

$$H(t) = \begin{cases} 0, & \text{if } t < 0 \\ 1, & \text{if } t \geq 0 \end{cases} \quad (1)$$

- The Heaviside function (1) is a powerful and flexible tool due to its many uses in different fields. Here we will show its use in solving partial differential equations and how to represent it in MATLAB.

- To plot a Heaviside Step Function in MATLAB, you can use the following code. This code plots the function over a specific time period:

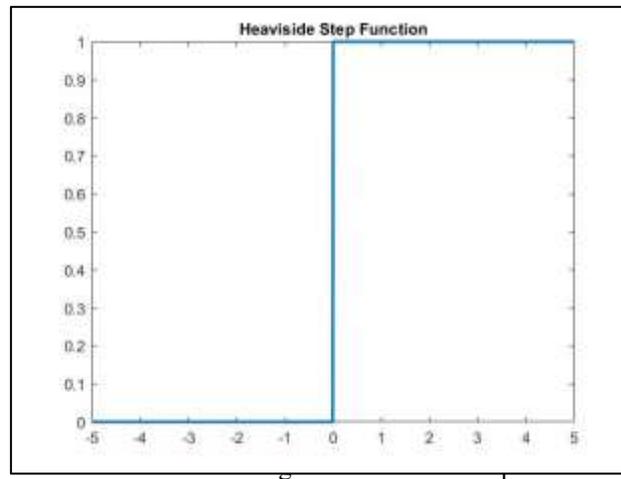
```
t = -5:0.01:5;  
H = t >= 0;  
figure;  
plot(t, H, 'LineWidth', 2);  
title('Heaviside Step Function');  
y label('Time (t)');  
x label('H(t)');  
grid on;
```

x lim([-0.5 1.5]);

y line(0, '-r');

2.2 Code Explanation

- Define the time domain: A vector t is created from -5 to 5 with a step of 0.01.
- Calculate the Heaviside function: The expression $H = t \geq 0$ is used to create a vector H whose value is 1 when t is greater than or equal to 0, and 0 when it is less than 0.
- Plot the function: The plot function is used to plot the function with assignments to the labels and axes.
- Add a vertical line: A vertical line is added at $t=0$ to indicate the change point



3. Heaviside step function in differential equations Laplace transforms

We use the Heaviside function to carry differential equations because it represents signals that start at a certain time. This makes it useful in analyzing systems that undergo sudden changes, and also facilitates solutions. When dealing with differential equations, the initial conditions or boundaries may be discontinuous. Using the Heaviside function, complex equations can be transformed into simpler forms, which facilitates the solution process. In addition, it helps in analyzing the response of systems in time, as it can be used to describe how the system responds to sudden changes in inputs. In short, the Heaviside function is a powerful and flexible tool that facilitates the processing of differential equations and the analysis of dynamic systems more accurately and efficiently.

To graph the Heaviside Step Function and use it to solve differential equations using Laplace transforms in MATLAB, you can follow the steps below.

I will give a simple example of solving a differential equation using the Laplace transform, and then graph the solution with the Heaviside function.

3.1 Example

Solve the following differential equation $[x'' + 2x' + x = H(t)]$ with Laplace transform and its equivalent in MATLAB using the Heaviside Step function

Sol :

Suppose we have the following differential equation: $[x'' + 2x' + x = H(t)]$

t = -1:0.01:5;

```
H = zeros(size(t));  
  
H(t >= 1) = 1;  
  
figure;  
  
plot(t, H, 'Linewidth', 2);  
  
title('Heaviside Step Function H(t - 1)');  
  
y label('t');  
  
x label('H(t - 1)');  
  
grid on;  
  
axis([-1 5 -0.5 1.5]);  
  
sym x(t) X(s)  
  
Dx = diff(x, t);  
  
D2x = diff(x, t, 2);  
  
eq = D2x + 2*Dx + x == heaviside(t - 1);  
  
L_eq = laplace(eq);  
  
X_s = solve(L_eq, X);  
  
x_t = ilaplace(x_s);  
  
disp('The solution X(t) is:');  
  
disp(x_t);
```

3.2 Code Explanation:

- Draw the Heaviside function:
- We created an array of zeros and then set the values to 1 when (t) is greater than or equal to 1.
- Solve the differential equation
- We used syms to define the symbolic variables.
- We defined the differential equation using heaviside.
- Next, we used laplace to transform the equation into the domain(s).
- We solved the equation for $(Y(s))$.
- Finally, we used ilaplace to get the solution in the time domain.

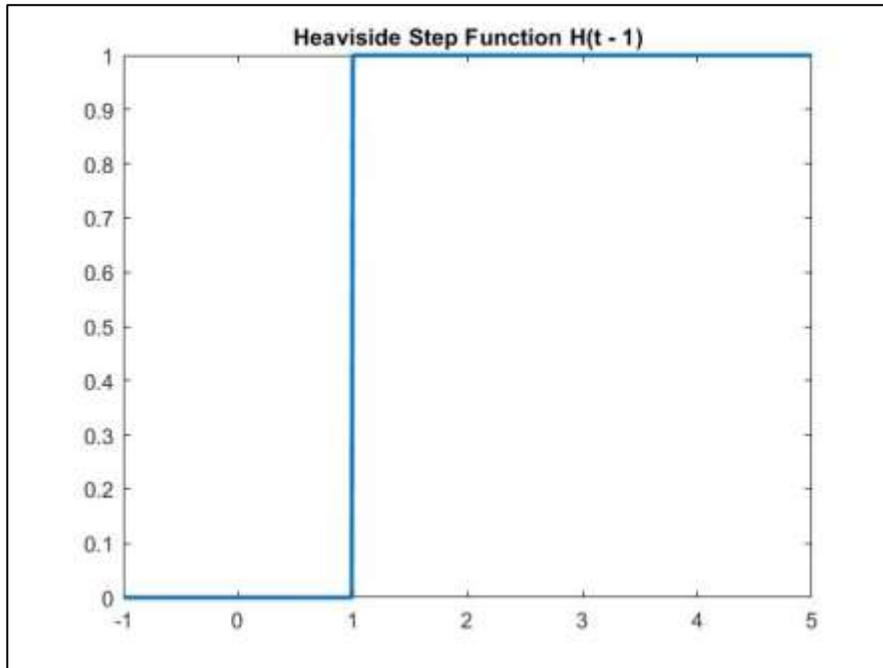


Figure2: Heaviside step function in differential equations Laplace transforms.

4. Conclusion

When using the Heaviside function to solve differential equations, we can transform complex equations into simpler forms, making the solution process easier. Representing this function in programs such as MATLAB allows engineers and researchers to perform calculations and analyses with high efficiency. The tools available in MATLAB, such as the Heaviside function, contribute to speeding up the modeling and simulation process, which enhances the understanding of the behavior of systems under the influence of sudden changes.

5. References

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