International Journal of Engineering and Information Systems (IJEAIS)
ISSN: 2643-640X
Vol. 8 Issue 10 October - 2024, Pages: 124-130

Confused Crisp Set Stable Neutrosophic Topological Spaces

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Abstract—*The space was portioned into three separate parts, hence it is concluded that we can divide it by of interior and exterior crisp sets. The scientific problems that need solving determine the part whether it is located in the inner or outer area or in between. Therefore, we decided to find a new type of sets and named it confused crisp set, which divided the space into three separate partitions. Moreover, their properties and their connection with the two concepts of interior and exterior were highlighted.*

Keywords—Stable Neutrosophic Topological Spaces, Stable Neutrosophic Crisp Interior Set, Stable Neutrosophic Crisp Exterior Set, Confused Crisp Set.

1. Introduction

It is no secret to researchers in the field of the neutrosophic concept that it is one of the important and modern topics that is intertwined with the rest of the natural and engineering sciences alike [1-5], and this importance was centered on the conditions and characteristics on which it was built, as the researchers Salama and Florentin explained them in their books [6,7] to make it easier for the recipient and the researcher in this matter. The concept of neutrosophic has been developing rapidly, especially in the subject of topological spaces [9].

The topic is to explain the various characteristics of these groups that work in the field of $(p(x) \times p(x) \times p(x))$. In the current research [10-15], a new type of topological space was identified, which we called (Stable Neutrosophic Crisp Topological Space (SNCT-space) in three conditions. Its topological concepts were built as a stable interior concept, symbolized $(Si_{ij}(A^N) = \cup_i \{S^N \in \wr, S^N \subseteq_j A^N\}\, i,j = 1,2)$. The symbol i represents an index of union types, and j represents an index of subset types. In this research, the definition of stable exterior is discussed through stable interior, and the definition of Confused crisp set is discussed through stable exterior and stable interior. For more properties, the research above has more details.

2. Basic concepts

In this part, we will review the most important concepts in SNCT-space and the results related to them that we need.

Definition 1 [7]: There are Three type of neutrosophic crisp Set ($NC - sets$):

Type 1: The NC-set of the first class, where their members are satisfied: $C_1 \cap C_2 = \emptyset, C_1 \cap C_3 = \emptyset\ , C_2 \cap C_3 = \emptyset$

Type 2: The NC- set $C^N$of the second class, where their members are satisfied:

$$C_1 \cap C_2 = \emptyset\, , C_1 \cap C_3 = \emptyset\, , C_2 \cap C_3 = \emptyset\ and\ \ C_1 \cup C_2 \cup C_3 = X$$

Type 3: The NC-set $C^N$of the third class,where their members are satisfied $C_1 \cap C_2 \cap C_3 = \emptyset\ and\ \ C_1 \cup C_2 \cup C_3 = X$

Definition 2 [7]: take X be a non-empty fixed set, there are four types of NC- empty sets, they are:

$$\emptyset_1^N =< \emptyset, \emptyset, X >, \emptyset_2^N =< \emptyset, X, \emptyset >, \emptyset_3^N =< \emptyset, X, X >, \emptyset_4^N =< \emptyset, \emptyset, \emptyset >.$$

And four type of NC-universal sets: $X_1^N =< X, \emptyset, \emptyset >$, $X_2^N =< X, X, \emptyset >, X_3^N =< X, \emptyset, X >, X_4^N =< X, X, X >$

Definition 3 [7]: Let C^N be a NC-set .Then the complement $C^N$are three type$s$:

$$(C^N)^{C1} =< C_1^C, C_2^C, C_3^C >,\ (C^N)^{C2} =< C_3, C_2, C_1 >, (C^N)^{C3} =< C_3, C_2^C, C_1 >$$

Definition 4 [7]: Let $L^N and\ K^N$ be two NC- sets .Then there are forms of the relation of subsets between two NC-sets

1. $L^N \subseteq_1 K^N \leftrightarrow L_1 \subseteq K_1, L_2 \subseteq K_2, L_3 \supseteq K_3$
2. $L^N \subseteq_2 K^N \leftrightarrow L_1 \subseteq K_1, L_2 \supseteq K_2, L_3 \supseteq K_3$

Definition 5 [7]: Let $L^N andK^N$ be two NC- sets,

1. The union between two NC-sets as $L^N \cup_1 K^N =< L_1 \cup K_1, L_2 \cup K_2, L_3 \cap K_3 >$ and
$$L^N \cup_2 K^N =< L_1 \cup K_1, L_2 \cap K_2, L_3 \cap K_3 >$$
2. The intersection between two NC- sets as $L^N \cap_1 K^N =< L_1 \cap K_1, L_2 \cap K_2, L_3 \cup K_3 >$ and

$$L^N \cap_2 K^N =< L_1 \cap K_1, L_2 \cup K_2, L_3 \cup K_3 >$$

Definition 6[8]: Let X be a fixed set that is not empty, a (SNCT) is a family $\wr$ satisfies the following condition:

1. $\emptyset_1^N, X_1^N \in \wr$
2. $\forall \mathbb{A}^N, \mathbb{B}^N \in \wr, \exists\ \mathbb{K}^N \in \wr, \ni\ \mathbb{K}^N \subseteq_1 \mathbb{A}^N \cap_1 \mathbb{B}^N$
3. $\forall\ \mathbb{A}_i^N \in \wr, \exists\ \mathbb{F}^N \in \wr\ \ni\ \mathbb{F}^N \subseteq_1 {}_{i=1}^{n}\cup_2 \mathbb{A}_i^N$

Then $(X, \wr)$ is (SNCT-space). For any $\mathbb{A}^N \in \wr$ is stable neutrosophic crisp open set and its denoted by(SNC$O - set$), the complement of type 2 for (SNC$O - set$) is stable neutrosophic crisp closed set and denoted by (SNC$C - set$).

Definition 7[8]: Let$(X, \wr)$ be a (SNCT-space), $\mathbb{A}^N$is a NC- set, then the stable interior of $\mathbb{A}^N$ denoted by $Si_{ij}(\mathbb{A}^N)$ and define as: $Si_{ij}(\mathbb{A}^N) = \cup_i \{\mathbb{S}^N \in \wr, \mathbb{S}^N \subseteq_j \mathbb{A}^N\}$ $\quad i, j = 1,2$. It can be noted that the index i is an indication of the type of union and the index j is an indication of the type of the subsets.

3. STABLE EXTERIOR AND CONFUSED CRISP SET

The tool adopted in this research is NC-sets, where a topological space was defined out of the general perspective of the conventional topological spaces. The procedures of research are about a new definition of internal spaces, which we called stable exterior. Its algebraic and topological structures were shown. Moreover, the necessary and sufficient conditions were described for the union of stable interior to be closed. Also, many properties were discussed for this concept.

Definition 8: Let (X, $\wr$) be a SNCT-space and L^N $be\ a$ NC-set Then, the exterior of L^Ndenoted by $Se_{ij}(L^N)$ and define as: $Se_{ij}(L^N) = Si_{ij}((L^N)^{C_2})$ $\quad i, j = 1,2$

Example 9: Let (X,$\wr$) be a SNCT − space, $\wr = \{A^N, B^N, C^N, D^N, E^N, F^N, {\emptyset_1}^N, {X_1}^N\}$, such that:

$A^N =< \{a\}, \{b, c, d\}, \{e\} >, B^N =< \{a\}, \{e, f\}, \{c\} >, C^N =< \{a\}, \emptyset, \{e, c, d\} >$

$D^N =< \{a\}, \emptyset, \emptyset >, E^N =< \{a\}, \emptyset, \{e\} >,\ F^N =< \{a\}, \emptyset, \{c\} >, \wr = \{A^N, B^N, C^N, D^N, E^N, F^N, \emptyset_1^N, X_1^N\}$

Let $K^N =< \{c\}, \{f\}, \{a\} >, (K^N)^{C_2} =< \{a\}, \{f\}, \{c\} >$

$Se_{11}(K^N) = \cup_1 \{S^N \in \wr, S^N \subseteq_1 (K^N)^{C_2}\} =< \{a\}, \emptyset, \{c\} >$

$Se_{12}(K^N) = \cup_1 \{S^N \in \wr, S^N \subseteq_2 (K^N)^{C_2}\} =< \{a\}, \{e, f\}, \{c\} >$

$Se_{22}(K^N) = \cup_2 \{S^N \in \wr, S^N \subseteq_2 (K^N)^{C_2}\} =< \{a\}, \{e, f\}, \{c\} >$

$Se_{21}(K^N) = \cup_2 \{S^N \in \wr, S^N \subseteq_1 (K^N)^{C_2}\} =< \{a\}, \emptyset, \{c\} >$.

A set of properties of the stable exterior and its simple proof will be examined, based on the properties of stable interior and some mathematical tactics that were tackled in our previous research. We will prove some of them.

Proposition 10: Let (X, $\wr$) be a SNCT-space, $L^N, K^N\ are\ a\ NC - sets$ of any type, Then the properties hold:

a) $Se_{ij}({\emptyset_1}^N) =_i {X_1}^N, Se_{ij}\left({X_1}^N\right) =_i {\emptyset_1}^N$ $\quad$ for $i = 1,2$

b) $Se_{ij}(K^N) \subseteq_1 (K^N)^{C_2}$

c) $L^N \subseteq_i K^N, Then\ Se_{ij}(K^N) \subseteq_i Se_{ij}(L^N)$ $\quad$ for $i = 1,2$

d) $Se_{ij}(L^N \cup_i K^N) =_i Se_{ij}(L^N) \cap_i Se_{ij}(K^N)$ $\quad$ for $i = 1,2$

e) $Se_{ij}(Se_{ij}(L^N)) =_i Si_{ij}(Si_{ij}((K^N)^{C_2}))^{C_2}$ $\quad$ for $i = 1,2$

Proof: We will prove (c, d) when $i = 1,\ j = 1$

c) Let L^N, K^Nbe a NC- set of any type such that $L^N \subseteq_1 K^N, So (K^N)^{C_2} \subseteq_1 (L^N)^{C_2}$

$Si_{11}((K^N)^{C_2}) \subseteq_1 Si_{11}((L^N)^{C_2})$ [by properties of $Si_{ij}(L^N)$].

Thus $Se_{ij}(K^N) \subseteq_1 Se_{ij}(L^N)$

d) $Se_{11}(L^N \cup_i K^N) =_i Si_{11}((L^N \cup_i K^N)^{C_2}) =_i Si_{11}((L^N)^{C_2} \cap_i (K^N)^{C_2}) =_i$

$Si_{ij}((L^N)^{C_2}) \cap_i Si_{ij}((K^N)^{C_2}) =_i Se_{ij}(L^N) \cap_i Se_{ij}(K^N)$ $\quad$ for $i = 1,2$

e) $Se_{ij}(Se_{ij}(L^N)) =_i Se_{ij}(Si_{ij}((K^N)^{C_2}) =_i Si_{ij}(Si_{ij}((K^N)^{C_2}))^{C_2}$

International Journal of Engineering and Information Systems (IJEAIS)
ISSN: 2643-640X
Vol. 8 Issue 10 October - 2024, Pages: 124-130

Through the above theorem, part (b), we showed the relationship between the NC-set and the exterior of NC-set, in other words, they are separate.

Definition 11: Let (X, Ƨ) be a ƧNCT-space and L^N *be a* NC-set , then the Confused crisp set of L^N denoted by $ꭗ_{ij}(L^N)$ and define as: $ꭗ_{ij}(L^N) =_i Si_{ij}(L^N) \cup_i Se_{ij}(L^N), i, j = 1,2$

Example 12: Let X={n, m, z},(X,Ƨ) be a ƧNCT − space ,$Ƨ = \{A^N, B^N, C^N, D^N, \emptyset_1^N, X_1^N\}$ such that:

$A^N =< \{n\}, \emptyset, \{m\} >, B^N =< \{m\}, \emptyset, \{n\} >, C^N =< \emptyset, \emptyset, \{n, m\} >, D^N =< \{n, m\}, \emptyset, \emptyset >$

Let $K^N =< \{n\}, \{z\}, \{m\} >, (K^N)^{C_2} =< \{m\}, \{z\}, \{n\} >$

$ꭗ_{11}(K^N) =< \{n, m\}, \emptyset, \emptyset >$,$ꭗ_{22}(K^N) = \emptyset_1^N, ꭗ_{12}(K^N) = \emptyset_1^N,\ ꭗ_{21}(K^N) =< \{n, m\}, \emptyset, \emptyset >$

We will highlight a few of the numerous properties of the neutrosophic crisp confused set of L^N, which are intimately related to the external and interior conceptions that are founded on it and depend on these properties for their proofs.

Proposition 13: Let (X, Ƨ) be a ƧNCT-space , L^N, K^N *are a NC − sets* of any type, Then properties hold:

a) $ꭗ_{ij}(L^N) =_i \left(\left(Si_{ij}((L^N)^{C_2})\right)^{C_2} - Si_{ij}(L^N)\right)^{C_2}$, for $i = 1,2$

b) $ꭗ_{ij}(L^N) =_i ꭗ_{ij}((L^N)^{C_2})$ for $i = 1,2$

c) $Si_{ij}(L^N) =_i L^N - \left(ꭗ_{ij}(L^N)\right)^{C_2}$ for $i = 1,2$

d) $ꭗ_{ij}\ (L^N) \subseteq_i ꭗ_{ij}\left(Si_{ij}(L^N)\right)$ for $i = 1,2$

e) $ꭗ_{ij}(L^N) \subseteq_i ꭗ_{ij}((Se_{ij}(L^N))^{C_2})$ for $i = 1,2$

f) $ꭗ_{ij}(L^N) \cap_i ꭗ_{ij}(K^N) \subseteq_i ꭗ_{ij}(L^N \cup_i K^N)$ for $i = 1,2$

g) $ꭗ_{ij}(L^N) \cap_i ꭗ_{ij}(K^N) \subseteq_i ꭗ_{ij}(L^N \cap_i K^N)$ for $i = 1,2$

Proof: We will prove when $i = 2, j = 2$

a) $(\left(Si_{22}((L^N)^{C_2})\right)^{C_2} - Si_{22}(L^N))^{C_2} =_i ((Si_{22}((L^N)^{C_2}))^{C_2} \cap_i (Si_{22}(L^N))^{C_2})^{C_2} =_i Si_{22}(L^N) \cup_i Si_{22}((L^N)^{C_2})$

b) $ꭗ_{ij}((L^N)^{C_2}) =_i (Si_{22}((L^N)^{C_2}))^{C_2} \cup_i Si_{22}((L^N)^{C_2}) =_i Se_{22}(L^N) \cup_i ((Si_{22}((L^N)^{C_2})^{C_2}) =_i (Si_{22}(L^N) \cup_i Si_{22}((L^N)^{C_2})$

$=_i ꭗ_{ij}(L^N)$

c) $L^N - \left(ꭗ_{22}(L^N)\right)^{C_2} =_i L^N - [(Si_{22}(L^N) \cup_i Si_{22}((L^N)^{C_2})]^{C_2} =_i L^N - (\left(Si_{22}(L^N)\right)^{C_2} \cap_i \left(Si_{22}((L^N)^{C_2})\right)^{C_2}) =_i Si_{22}(L^N)$

The proof is similar if the definition of $(Si_{11}(L^N), Si_{12}(L^N), Si_{21}(L^N))$ is a dopted. We can see from definition of confused $ꭗ_{ij}({X_1}^N) =_i ꭗ_{ij}({\emptyset_1}^N) =_i {X_1}^N$,since$ꭗ_{ij}({X_1}^N) =_i Si_{ij}({X_1}^N) \cup_i Se_{ij}({X_1}^N) =_i {X_1}^N$,and$ꭗ_{ij}({\emptyset_1}^N) =_i Si_{ij}({\emptyset_1}^N) \cup_i Se_{ij}({\emptyset_1}^N) =_i {X_1}^N$.

It is also simple to state that X$=_i Si_{ij}(L^N) \cup_i Se_{ij}(L^N) \cup_i \left(ꭗ_{ij}(L^N)\right)^{C_2}$, Since $Se_{ij}(L^N) =_i Si_{ij}((L^N)^{C_2})$.Again Since $L^N \cap_i (L^N)^{C_2} =_i {\emptyset_1}^N$,it follows that $Se_{ij}(L^N) \cap_i Si_{ij}(L^N) =_i {\emptyset_1}^N$

Again by $ꭗ_{ij}(L^N), ꭗ_{ij}(L^N) =_i Si_{ij}(L^N) \cup_i Se_{ij}(L^N)$

Thus $\left(ꭗ_{ij}(L^N)\right)^{C_2} =_i \left[(Si_{ij}(L^N) \cup_i Se_{ij}(L^N)\right]^{C_2}$.

It follows that $\left(ꭗ_{ij}(L^N)\right)^{C_2} \cap_i Si_{ij}(L^N) =_i {\emptyset_1}^N$, $\left(ꭗ_{ij}(L^N)\right)^{C_2} \cap_i Se_{ij}(L^N) =_i {\emptyset_1}^N$

and X$=_i Si_{ij}(L^N) \cup_i Se_{ij}(L^N) \cup_i \left(ꭗ_{ij}(L^N)\right)^{C_2}$

Since $Si_{ij}(L^N)$ *and* $Se_{ij}(L^N)$ are ƧNC*O − set*, we get that $ꭗ_{ij}(L^N)$is a ƧNC*O − set*.

Lemma 14: Let (X, Ƨ) be a ƧNCT-space , L^N *be a* NC-set Then

a) $L^N$is a ƧNC*O − set* if and only if $Si_{ij}(L^N) \cap_i (L^N)^{C_2} =_i \emptyset_1^N$

International Journal of Engineering and Information Systems (IJEAIS)
ISSN: 2643-640X
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b) $L^N$is a SNC$C-set$ if and only if $(L^N)\cap_i Se_{ij}(L^N)=_i \emptyset_1^N$

Proof

a) $L^N$is a SNC$O-set \leftrightarrow Si_{ij}(L^N)=_i L^N$

But we have $L^N\cap_i (L^N)^{C_2}=_i \emptyset_1^N \to Si_{ij}(L^N)\cap_i (L^N)^{C_2}=_i \emptyset_1^N$, hence $Si_{ij}(L^N)\cap_i (L^N)^{C_2}=_i \emptyset_1^N$

b) $L^N$is a SNC$C-set \leftrightarrow (L^N)^{C_2}$ is a SNC$O \leftrightarrow Si_{ij}((L^N)^{C_2})=_i (L^N)^{C_2}$

But we have $L^N\cap_i (L^N)^{C_2}=_i \emptyset_1^N \to L^N\cap_i Si_{ij}((L^N)^{C_2})=_i \emptyset_1^N$, therefor $(L^N)\cap_i Se_{ij}(L^N)=_i \emptyset_1^N$.

Proposition 15: Let $(X, \mho)$ be a SNCT-space and L^N *be a* NC-set. Then,

a) If $L^N$is SNC$O-set$ then $\left(\eth_{ij}\ (L^N)\right)^{C_2}=_i \left(Si_{ij}((L^N)^{C_2})\right)^{C_2}-L^N$

b) $\left(Si_{ij}(L^N)\cup_i Se_{ij}(L^N)\right)^{C_2}=_i \emptyset_1^N$if $L^N$is SNC$O-set$ as well as SNC$C-set$. For i,j=1,2

Proof: We will prove when i=j=1

a) We have $\eth_{11}\ (L^N)=_i Si_{11}(L^N)\cup_i Se_{11}(L^N)$

Since $L^N$is SNC$O-set$, $(L^N)=_i Si_{11}(L^N)$.

Hence $\left(\eth_{11}\ (L^N)\right)^{C_2}=_i \left(Si_{11}(L^N)\cup_i Se_{11}(L^N)\right)^{C_2}$

$$=_i \left(Si_{11}(L^N)\right)^{C_2}\cap_i \left(Se_{11}(L^N)\right)^{C_2}$$

$$=_i \left(Si_{11}(L^N)\right)^{C_2}\cap_i \left(Si_{11}((L^N)^{C_2})\right)^{C_2}$$

$$=_i \left(Si_{11}((L^N)^{C_2})\right)^{C_2}-Si_{11}(L^N)$$

$$=_i \left(Si_{11}((L^N)^{C_2})\right)^{C_2}-L^N$$

b) *let* $\left(Si_{11}(L^N)\cup_i Se_{11}(L^N)\right)^{C_2}=_i \emptyset_1^N$. Then $\left(Si_{11}((L^N)^{C_2})\right)^{C_2}-Si_{11}(L^N)=_i \emptyset_1^N$

$$\to \left(Si_{11}((L^N)^{C_2})\right)^{C_2}\subseteq_i Si_{11}(L^N)$$

$$\to \left(Si_{11}((L^N)^{C_2})\right)^{C_2}\subseteq_i L^N \qquad (\because Si_{11}(L^N)\subseteq_i L^N)$$

$$\to (L^N)^{C_2}\subseteq_i Si_{11}((L^N)^{C_2})=_i Se_{11}(L^N)$$

$$\to L^N\cap_i Se_{11}(L^N)=_i \emptyset_1^N,$$

So by lemma(14) $L^N$is closed.

Again $\left(Si_{11}(L^N)\cup_i Se_{11}(L^N)\right)^{C_2}=_i \emptyset_1^N \to \left(Si_{11}((L^N)^{C_2})\right)^{C_2}-Si_{11}(L^N)=_i \emptyset_1^N$

$$\to \left(Si_{11}((L^N)^{C_2})\right)^{C_2}\subseteq_i Si_{11}(L^N)$$

$$\to L^N\subseteq_i Si_{11}(L^N)$$

$$\to Si_{11}(L^N)\cap_i (L^N)^{C_2}=_i \emptyset_1^N$$

It follows that $L^N$is SNC$O-set$ (by lemma14). Thus we have shown that if $\left(\eth_{11}\ (L^N)\right)^{C_2}=_i \emptyset_1^N$,then $L^N$is SNC$O-set$ as well as SNC$C-set$.

Now $\left(Si_{11}(L^N)\cup_i Se_{11}(L^N)\right)^{C_2}=_i \left(Si_{11}((L^N)^{C_2})\right)^{C_2}-Si_{11}(L^N)$

Since $(L^N)$is clopen $\left(Si_{11}(L^N)\cup_i Se_{11}(L^N)\right)^{C_2}=_i L^N-L^N=_i \emptyset_1^N$.

This preposition is also true in the case of $(Si_{22}(L^N))$, but not true in the case of $(Si_{12}(L^N), Si_{21}(L^N))$, and this is illustrated by the following example.

International Journal of Engineering and Information Systems (IJEAIS)
ISSN: 2643-640X
Vol. 8 Issue 10 October - 2024, Pages: 124-130

Example 16: Let X=$\{o,p,q,r,s\}$,(X, $\mho$)be a SNCT − space $where$

$\mho = \{A^N, B^N, C^N, D^N, E^N, F^N, \emptyset_1^N, X_1^N\}$ such that:

$A^N =< \{o\},\{p,q\},\{s\} >, B^N =< \{o\},\{p,r,s\},\{q\} >, C^N =< \{o\},\emptyset,\emptyset >$

$D^N =< \{o\},\{p\},\{s,q\} >, E^N =< \{o\},\emptyset,\{q\} >, F^N =< \{o\},\emptyset,\{s\} >$

$G^N =< \{o\},\{p\},\{q\} >, H^N =< \{o\},\emptyset,\{s,q\} >$

$Si_{21}(D^N)$=$< \{o\},\emptyset,\{s,q\} >$.Thus show that $D^N$is SNC$O - set\ Si_{21}(D^N) \neq D^N$.

This applies to all the following propositions.

Proposition 17: Let (X, $\mho$) be a SNCT-space , $L^N is\ a\ NC - set$ of any type , the following are equivalent:

a) $Se_{ij}(L^N) =_i \emptyset_1^N$

b) $\forall K^N \in \mho, L^N \cap_i K^N \neq_i {\emptyset_1}^N$.

c) $\left(\mathrm{X}_{ij}\,(L^N)\right)^{C_2} =_i (Si_{ij}(\,L^N))^{C_2}$.

Proof: a$\Leftrightarrow b$) Let $Se_{ij}(L^N) =_i \emptyset_1^N$, if possible there exist

$$K^N \in \mho \ni L^N \cap_i K^N =_i {\emptyset_1}^N$$

$$\rightarrow L^N \subseteq_i (K^N)^{C_2} \rightarrow \quad (Si_{ij}((L^N)^{C_2}))^{C_2} \subseteq_i (Si_{ij}(K^N))^{C_2}$$

$$=_i (K^N)^{C_2}, But\ Si_{ij}((L^N)^{C_2}) =_i {\emptyset_1}^N,$$

$(Si_{ij}((L^N)^{C_2}))^{C_2} =_i {X_1}^N \rightarrow {X_1}^N \subseteq_i (K^N)^{C_2} \rightarrow K^N =_i {\emptyset_1}^N$ which is contradiction.

Conversely, $Si_{ij}((L^N)^{C_2}) =_i \cup_i \{K^N, K^N is\ open\ K^N \subseteq_i (L^N)^{C_2}\}$. Thus $L^N \cap K^N =_i {\emptyset_1}^N$.

a$\Leftrightarrow c$) Let$Se_{ij}\,(\,L^N) =_i Si_{ij}((L^N)^{C_2}) = {\emptyset_1}^N$

$$\rightarrow \left(Si_{ij}((L^N)^{C_2})\right)^{C_2} =_i {X_1}^N, but \left(\mathrm{X}_{ij}(L^N)\right)^{C_2} =_i \left(Si_{ij}(L^N) \cup_i Si_{ij}((L^N)^{C_2})\right)^{C_2}$$

$$=_i \left(Si_{ij}((L^N)^{C_2})\right)^{C_2} \cap_i (Si_{ij}(\,L^N))^{C_2} =_i {X_1}^N \cap_i (Si_{ij}(\,L^N))^{C_2} =_i (Si_{ij}(\,L^N))^{C_2}$$

Conversely, let $\left(\mathrm{X}_{ij}\,(L^N)\right)^{C_2} =_i \left(Si_{ij}(\,L^N)\right)^{C_2} if\ and\ only\ if\ \mathrm{X}_{ij}\,(L^N) =_i Si_{ij}((\,L^N)^{C_2}) \cup_i Si_{ij}(\,L^N)$

if and only if $\left(Si_{ij}((L^N)^{C_2})\right)^{C_2} \cap_i \mathrm{X}_{ij}\,(L^N) =_i \left(Si_{ij}((L^N)^{C_2})\right)^{C_2} \cap_i (\left(Si_{ij}(\,(L^N)^{C_2}) \cup_i Si_{ij}(\,L^N)\right)$

$$=_i \left(\left(Si_{ij}((L^N)^{C_2})\right)^{C_2} \cap_i Si_{ij}((\,L^N)^{C_2}\right) \cup_i \left(\left(Si_{ij}((L^N)^{C_2})\right)^{C_2} \cap_i Si_{ij}(\,L^N)\right)$$

$$=_i {\emptyset_1}^N \cup_i Si_{ij}(\,L^N) =_i Si_{ij}(\,L^N) \rightarrow Si_{ij}(\,L^N) =_i \left(Si_{ij}((L^N)^{C_2})\right)^{C_2} \cap_i \mathrm{X}_{ij}\,(L^N)$$

$$=_i S_{ij}(\,L^N) \cup_i \left(\mathrm{X}_{ij}\,(L^N)\right)^{C_2} =_i (\left(Si_{ij}((L^N)^{C_2})\right)^{C_2} \cap_i \mathrm{X}_{ij}\,(L^N)) \cup_i \left(\mathrm{X}_{ij}\,(L^N)\right)^{C_2}$$

$$=_i (\left(Si_{ij}((L^N)^{C_2})\right)^{C_2} \cup_i \left(\mathrm{X}_{ij}\,(L^N)\right)^{C_2}) \cap_i (\ \mathrm{X}_{ij}\,(L^N) \cup_i \left(\mathrm{X}_{ij}\,(L^N)\right)^{C_2})$$

$$=_i \left(Si_{ij}((L^N)^{C_2})\right)^{C_2} \cap_i {X_1}^N =_i \left(Si_{ij}((L^N)^{C_2})\right)^{C_2}$$

$\left(Si_{ij}((L^N)^{C_2})\right)^{C_2} =_i Si_{ij}(\,L^N) \cup_i \left(\mathrm{X}_{ij}\,(L^N)\right)^{C_2} =_i Si_{ij}(\,L^N) \cup_i (Si_{ij}(\,L^N))^{C_2} =_i {X_1}^N \rightarrow Si_{ij}((L^N)^{C_2}) =_i \emptyset_1^N. Thus\ Se_{ij}(L^N) =_i \emptyset_1^N$

Proposition 18: Let (X, $\mho$) be a SNCT-space , $L^N is\ a\ NC - set$ of any type, if $Si_{ij}(\,L^N) = {\emptyset_1}^N then, Se_{ij}(\,(L^N)^{C_2}) =_i {\emptyset_1}^N$

Proof: Let $Si_{ij}(\,L^N) =_i {\emptyset_1}^N but\ Si_{ij}(\,L^N) =_i \left((Si_{ij}(((L^N)^{C_2})^{C_2}))^{C_2}\right)^{C_2} \rightarrow \left((Si_{ij}(((L^N)^{C_2})^{C_2}))^{C_2}\right)^{C_2} =_i {\emptyset_1}^N$

So $(Si_{ij}(((L^N)^{C_2})^{C_2}))^{C_2} =_i X_1{}^N. Therefor, Se_{ij}((L^N)^{C_2}) =_i \emptyset_1{}^N$.

Proposition 19: Let (X, Ꙅ) be a SNCT-space, L^N $a\ NC - sets$ of any type, Then the following properties equivalent:

a) $Se_{ij}\left(Se_{ij}(L^N)\right) =_i \emptyset_1{}^N$,

b) $\left(Si_{ij}((L^N)^{C_2})\right)^{C_2}$ contains no SNCO-set.

Proof: a→b let $Se_{ij}\left(Se_{ij}(L^N)\right) =_i \emptyset_1{}^N \to Si_{ij}\left(\left(Si_{ij}((L^N)^{C_2})\right)^{C_2}\right) =_i \emptyset_1{}^N$

Then there is no SNC$O - set$ set $K^N \in$ Ꙅ such that $K^N \subseteq_i (Si_{ij}((L^N)^{C_2}))^{C_2}$.Thus $(Si_{ij}((L^N)^{C_2}))^{C_2}$contain no SNC$O - set$ set.

b→a let $\left(Si_{ij}((L^N)^{C_2})\right)^{C_2}$contain no SNC$O - set$,∀ $K^N \in$ Ꙅ such that $K^N \nsubseteq_i \left(Si_{ij}((L^N)^{C_2})\right)^{C_2}$,

this mean $K^N \cap_i Si_{ij}((L^N)^{C_2}) \neq_i \emptyset_1{}^N$

$\to Si_{ij}\left[\left(Si_{ij}((L^N)^{C_2})\right)^{C_2}\right] =_i \emptyset_1{}^N$ (by preposition 17(a)).

Proposition 20: Let (X, Ꙅ) be a SNCT-space, $L^N is\ a\ NC - sets$ of any type, if $Si_{ij}\left(\left(Si_{ij}((L^N)^{C_2})\right)^{C_2}\right) =_i \emptyset_1{}^N then,, \forall K^N \in$ Ꙅ $, Si_{ij}((K^N \cap_i L^N)^{C_2}) \neq_i \emptyset_1{}^N$

Proof: Let $Si_{ij}\left(\left(Si_{ij}((L^N)^{C_2})\right)^{C_2}\right) =_i \emptyset_1{}^N$if possible $Si_{ij}((K^N \cap_i L^N)^{C_2}) =_i \emptyset_1{}^N$, then $K^N \subseteq_i \left(Si_{ij}((K^N \cap_i L^N)^{C_2})\right)^{C_2} \to K^N \subseteq_i Si_{ij}\left(\left(Si_{ij}((K^N \cap_i L^N)^{C_2})\right)^{C_2}\right) \subseteq_i Si_{ij}\left(\left(Si_{ij}((L^N)^{C_2})\right)^{C_2}\right) =_i \emptyset_1{}^N$,then $K^N =_i \emptyset_1{}^N$, which is contradiction.

4. Conclosion

All theorems whose proofs include open set equal to its internal points are true in only two cases: $(Si_{11}(L^N), Si_{22}(L^N))$, that is, when adopting the definition of the stable interior on the subset of the first type with the union of the first type and the subset of the second type with the union of the second type. In case when the definition of (($Si_{12}(L^N)$, $Si_{21}(L^N)$) is based on a subset of the first type with a union of the second type or a subset of the second type with a union of the first type is not true. This is because in the last two cases the open set is not necessarily equal to its internal points. The foremost question is, what is the effect of ((stable neutrosophic crisp topological-space) concept on some well-known mathematical concepts such as local function [9] and proximity, and the relationships that are affected by them? The answer is in the process of linking the ideal with SNCT-space, and modifying the neutrosophic in terms of the conditions of the neutrosophic. The substitution of (soft fuzzy set) with the ordinary sets that are included in the structures of NC- sets gave it a new concept in Al-Nafi'i. Here it is worth noting the definition of stable interior based on it as a new research preface.

5. References

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