International Journal of Engineering and Information Systems (IJEAIS)
ISSN: 2643-640X
Vol. 8 Issue 10 October - 2024, Pages: 131-137

New Types of Condensed in Stable Neutrosophic topological space

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Abstract— In this study, three new ideas were constructed in the stable neutrosophic topological space (SNCT-space) based on concepts of stable interior and stable exterior. The effects of each concept on the others were demonstrated by the features and results we found, and the study was bolstered with illustrative examples.

Keywords— Neutrosophic Crisp Set, Stable Neutrosophic Topological Space, $SNC_1 - Condensed\ Set$, $SNC_2 - Condensed$, SNC- Condensed Set .

1. INTRODUCTION

The topic of neutrosophic and neutrosophic crisp, which was sparked by the scientist Smarandache in 1999 [1,2], is one of the scientific paradoxes of the 20th century that has gained widespread recognition in the various sciences and its specific specializations. These two paths developed quickly, particularly in topological spaces [3-8], as the researchers Hadi, M. H., & Al-Swidi, L. A. A. (The Neutrosophic Axial Set theory) were the path of their researc [9].

The course of there research (On Some Types of Neutrosophic Topological Groupswith Respect to Neutrosophic Alpha Open Sets) were the researcher (Imran, Q. H., Al-Obaidi, A. H. M., & Smarandache, F) [10]. was proposed the neuttrosophic as n-valued in 2013 [11], and the final study along this line was conducted by Smarandache, F [12-14].

In this paper we define the following new concepts; $SNC_1 - condensed\ set$, $SNC_2 - condensed$and SNC- $condensed\ set$, using the concepts of stable interior of and stable exterior only. So it may be advocated that these new concepts are totally basic ones.

2. BASIC CONCEPTS

Definition 2.1 [1]: There are Three type of neutrosophic crisp Set ($NC - sets$):
Type 1: The NC-set of the first class, where their members are satisfied $C_1 \cap C_2 = \emptyset, C_1 \cap C_3 = \emptyset\ , C_2 \cap C_3 = \emptyset$
Type 2: The NC- set $C^N$of the second class ,where their members are satisfied

$$C_1 \cap C_2 = \emptyset\ \ , C_1 \cap C_3 = \emptyset\ , C_2 \cap C_3 = \emptyset\ and\ \ C_1 \cup C_2 \cup C_3 = X$$

Type 3: The NC-set $C^N$of the third class,where their members are satisfied $C_1 \cap C_2 \cap C_3 = \emptyset\ and\ \ C_1 \cup C_2 \cup C_3 = X$

Definition2. 2 [1]: take X be a non-empty fixed set, there are four types of NC- empty sets, they are:

$$\emptyset_1^N =< \emptyset, \emptyset, X >, \emptyset_2^N =< \emptyset, X, \emptyset >, \emptyset_3^N =< \emptyset, X, X >, \emptyset_4^N =< \emptyset, \emptyset, \emptyset >.$$

And four type of NC-universal sets: $X_1^N =< X, \emptyset, \emptyset >$, $X_2^N =< X, X, \emptyset >$, $X_3^N =< X, \emptyset, X >$, $X_4^N =< X, X, X >$

Definition 2.3 [1]: Let C^N be a NC-set .Then the complement $C^N$are three type$s$:

$$(C^N)^{C1} =< C_1^C, C_2^C, C_3^C >, (C^N)^{C2} =< C_3, C_2, C_1 >, (C^N)^{C3} =< C_3, C_2^C, C_1 >$$

Definition 2.4 [1]: Let $L^N and\ K^N$ be two NC- sets .Then there are forms of the relation of subsets between two NC-sets

1. $L^N \subseteq_1 K^N \leftrightarrow L_1 \subseteq K_1, L_2 \subseteq K_2, L_3 \supseteq K_3$
2. $L^N \subseteq_2 K^N \leftrightarrow L_1 \subseteq K_1, L_2 \supseteq K_2, L_3 \supseteq K_3$

Definition 2.5[1]: Let $L^N andK^N$ be two NC- sets,

1. The union between two NC-sets as $L^N \cup_1 K^N =< L_1 \cup K_1, L_2 \cup K_2, L_3 \cap K_3 >$ and
$$L^N \cup_2 K^N\ =< L_1 \cup K_1, L_2 \cap K_2, L_3 \cap K_3 >$$
2. The intersection between two NC- sets as $L^N \cap_1 K^N =< L_1 \cap K_1, L_2 \cap K_2, L_3 \cup K_3 >$ and
$$L^N \cap_2 K^N =< L_1 \cap K_1, L_2 \cup K_2, L_3 \cup K_3 >$$

Definition 2.6 [13]: Let X be a fixed set that is not empty, a (SNCT-space) is a family $\mathfrak{Z}$ satisfies the following condition:

1. $\emptyset_1^N, X_1^N \in \mathfrak{Z}$
2. $\forall \mathbb{A}^N, \mathbb{B}^N \in \mathfrak{Z}, \exists\ \mathbb{K}^N \in \mathfrak{Z}, \ni\ \mathbb{K}^N \subseteq_1 \mathbb{A}^N \cap_1 \mathbb{B}^N$
3. $\forall\ \mathbb{A}_i^N \in \mathfrak{Z}, \exists\ \mathbb{F}^N \in \mathfrak{Z}\ \ni\ \mathbb{F}^N \subseteq_1 {}_{i=1}^{n}\cup_2 \mathbb{A}_i^N$

Then$(X, \mathfrak{Z})$is a (SNCT-space). For any $\mathbb{A}^N \in \mathfrak{Z}$ is a stable neutrosophic crisp open set and its denoted by $(SNCO - set)$, the complement of type 2 for $(SNCO - set)$ is stable neutrosophic crisp closed set and denoted by $(SNCC - set)$.

Definition 2.7 [13]: Let$(X, \mathfrak{Z})$ be a (SNCT-space), $\mathbb{A}^N$is a NC- set, then the stable interior of $\mathbb{A}^N$ denoted by $Si_{ij}(\mathbb{A}^N)$ and define as: $Si_{ij}(\mathbb{A}^N) = \cup_i \{\mathbb{S}^N \in \mathfrak{Z}, \mathbb{S}^N \subseteq_j \mathbb{A}^N\},\ i,j = 1,2.$

It can be noted that the index i is an indication of the type of union and the index j is an indication of the type of the subsets.

Definition 2.8 [14]: Let (X, $\mathfrak{Z}$) be a SNCT-space and L^N *be a* NC-set Then, the exterior of L^Ndenoted by $Se_{ij}(L^N)$ and define as:

$$Se_{ij}(L^N) = Si_{ij}((L^N)^{C_2}) \qquad i,j = 1,2$$

Generally, the idea of the exterior is not closed under the process of intersection, and the interior is not closed under the process of union in any topological space [8].

Definition 2.8 [14]: Take (X, $\mathfrak{Z}$) be a SNCT-space and L^N *be a* NC-set, then the Confused crisp set of L^N denoted by $\mathcal{X}_{ij}(L^N)$ and define as: $\mathcal{X}_{ij}(L^N) =_i Si_{ij}(L^N) \cup_i Se_{ij}(L^N), i,j = 1,2$

3. SNC_1 −CONDENSED SET AND SNC_2 −CONDENSED

The tool adopted in this research is NC-sets, where a topological space was defined out of the general perspective of the conventional topological spaces. The procedures of research are about a new definition of internal spaces, which we called stable exterior. Its algebraic and topological structures were shown. Moreover, the necessary and sufficient conditions were described for the union of stable interior to be closed. Also, many properties were discussed for this concept.

lemma 3.1: Let (X, $\mathfrak{Z}$) be a SNCT-space , $\text{Ṿ}^N, \text{Ḩ}^N$ *are a NC − sets* of any type, Then the following relation hold for $i,j = 1,2$

1. $Se_{ij}[Se_{ij}(\text{Ṿ}^N)] =_i Se_{ij}[Se_{ij}(Se_{ij}[Se_{ij}(\text{Ṿ}^N)])]$
2. $\left(Se_{ij}(Se_{ij}(Se_{ij}\left(Si_{ij}(\text{Ṿ}^N)\right)))\right)^{C_2} =_i \left(Se_{ij}\left(Si_{ij}(\text{Ṿ}^N)\right)\right)^{C_2}$

Definition 3.2: Let (X, $\mathfrak{Z}$) be a SNCT-space , Ṿ^N *be a NC − set* of any type, then for $i,j = 1,2$

-$\text{Ṿ}^N$is $SNC_1 - condensed$ set if $Se_{ij}\left(Se_{ij}(\text{Ṿ}^N)\right) =_i Si_{ij}(\text{Ṿ}^N)$

-$\text{Ṿ}^N$is $SNC_2 - condensed$ set if $\left(Se_{ij}\left(Si_{ij}(\text{Ṿ}^N)\right)\right)^{C_2} =_i \left(Se_{ij}((\text{Ṿ}^N))\right)^{C_2}$

-$\text{Ṿ}^N$is $SNC - condensed$ set if $Se_{ij}\left(Se_{ij}(\text{Ṿ}^N)\right) =_i Si_{ij}(\text{Ṿ}^N)$ *and* $\left(Se_{ij}\left(Si_{ij}(\text{Ṿ}^N)\right)\right)^{C_2} =_i \left(Se_{ij}((\text{Ṿ}^N))\right)^{C_2}$

Example 3.3: Let (X, $\mathfrak{Z}$) be a SNCT-space , $\mathfrak{Z} = \{\mathbb{A}^N, \mathbb{B}^N, \mathbb{C}^N, \mathbb{D}^N, \emptyset_1^N, X_1^N\}$, X={e, f, g, h} such that:

$\mathbb{A}^N = < \emptyset, \{e\}, \{f\} >$

$\mathbb{B}^N = < \{g\}, \{f\}, \emptyset >$

$\mathbb{C}^N = < \emptyset, \emptyset, \{f\} >$

$\mathbb{D}^N = < \{g\}, \emptyset, \emptyset >$

$\left(Se_{ij}\left(Si_{ij}(\mathbb{B}^N)\right)\right)^{C_2} =_i \left(Se_{ij}((\mathbb{B}^N))\right)^{C_2} =_i X_1^N$

$\mathbb{B}^N$ is sub condensed set

Example 3.4: Consider $(X, \mathfrak{Z})$ is (SNCT-space). X={c, v, b, n, m} , $\mathbb{S} = \{\mathbb{A}^N, \mathbb{B}^N, \mathbb{C}^N, \mathbb{D}^N, \mathbb{E}^N, \mathbb{F}^N, \mathbb{G}^N, \mathbb{H}^N, \emptyset_1^N, X_1^N\}$

$\mathbb{A}^N = < \{c\}, \{v, b\}, \{m\} >$

$\mathbb{B}^N = < \{c\}, \{v, n, m\}, \{b\} >$

$\mathbb{C}^N = < \{c\}, \emptyset, \emptyset >$

International Journal of Engineering and Information Systems (IJEAIS)
ISSN: 2643-640X
Vol. 8 Issue 10 October - 2024, Pages: 131-137

$\mathbb{D}^N =<\{ c \},\{v\},\{m, b\}>$

$\mathbb{E}^N =<\{ c \},\emptyset,\{b\}>$

$\mathbb{F}^N =<\{ c \},\emptyset,\{m\}>$

$\mathbb{G}^N =<\{c\},\{v\},\{b\}>$

$\mathbb{H}^N =<\{ c \},\emptyset,\{m, b\}>$

Now let $\mathrm{M}^N =<\{m, b\},\{v\},\{ c \}>$

$Se_{11}(Se_{11}(\mathrm{M}^N)) =_i Si_{11}(\mathrm{M}^N)$

$\mathbb{D}^N$is supercondensed

Preposition 3.5: The complement of type 2 of $SNC_1 - condensed\ set$ is $SNC_2 - condensed\ set$ and The complement of type 2 of $SNC_2 - condensed\ set$ is $SNC_1 - condensed\ set$.

Proof: Let $\mathrm{V}^N$is $SNC_1 - condensed\ set$

$$\rightarrow Se_{ij}\left(Se_{ij}(\mathrm{V}^N)\right) =_i Si_{ij}(\mathrm{V}^N)$$

$$\left(Se_{ij}\left(Se_{ij}(\mathrm{V}^N)\right)\right)^{C_2} =_i \left(Si_{ij}(\mathrm{V}^N)\right)^{C_2}, \text{ i.e },$$

$$\left(Se_{ij}\left(Si_{ij}((\mathrm{V}^N)^{C_2})\right)\right)^{C_2} =_i \left(Si_{ij}(\mathrm{V}^N)\right)^{C_2}$$

$$\left(Se_{ij}\left(Si_{ij}(((\mathrm{V}^N)^{C_2})^{C_2})\right)\right)^{C_2} =_i \left(Si_{ij}((\mathrm{V}^N)^{C_2})\right)^{C_2}$$

$$\left(Se_{ij}\left(Si_{ij}(\mathrm{V}^N)\right)\right)^{C_2} =_i \left(Se_{ij}((\mathrm{V}^N))\right)^{C_2}$$

Now let $\mathrm{V}^N$is $SNC_2 - condensed\ set \rightarrow \left(Se_{ij}\left(Si_{ij}(\mathrm{V}^N)\right)\right)^{C_2} =_i \left(Se_{ij}((\mathrm{V}^N))\right)^{C_2}$

by take the complement of type 2 for both side we get, $Se_{ij}\left(Si_{ij}(\mathrm{V}^N)\right) =_i Se_{ij}((\mathrm{V}^N))$

$Se_{ij}\left(Si_{ij}((\mathrm{V}^N)^{C_2})\right) =_i Se_{ij}((\mathrm{V}^N)^{C_2}) \rightarrow Se_{ij}\left(Se_{ij}(\mathrm{V}^N)\right) =_i Si_{ij}(\mathrm{V}^N)$

Hence, $\mathrm{V}^N$is $SNC_2 - condensed\ set$

Corollary 3.6: The complement of type 2 of SNC-condensed set is SNC-condensed set

Proof: If $\mathrm{V}^N$is a SNC-condensed set, then V^N is a $SNC_1 - condensed\ set$ and $SNC_2 - condensed\ set$, by proposition 3.5 , $(\mathrm{V}^N)^{C_2}$ is a $SNC_1 - condensed\ set$ and $SNC_2 - condensed\ set$.Hence $(\mathrm{V}^N)^{C_2}$ is a SNC-condensed set.

Preposition 3.7: A NC-set is $SNC_1 - condensed\ set$ if and only if $Se_{ij}\left(Se_{ij}(\mathrm{V}^N)\right) \subseteq_i \mathrm{V}^N$, A NC-set is $SNC_2 - condensed\ set$ if and only if $\mathrm{V}^N \subseteq_i \left(Se_{ij}\left(Si_{ij}(\mathrm{V}^N)\right)\right)^{C_2}$

Proof: Let A be a $SNC_1 - condensed\ set$, Then we have $Se_{ij}\left(Se_{ij}(\mathrm{V}^N)\right) =_i Si_{ij}(\mathrm{V}^N) \subseteq_i \mathrm{V}^N$

This is a relation that we need. Conversely, we assume the relation $Se_{ij}\left(Se_{ij}(\mathrm{V}^N)\right) \subseteq_i \mathrm{V}^N$.

Taking the stable interior of both sides $Si_{ij}\left[Se_{ij}\left(Se_{ij}(\mathrm{V}^N)\right)\right] \subseteq_i Si_{ij}(\mathrm{V}^N)$, we get $Se_{ij}\left(Se_{ij}(\mathrm{V}^N)\right) \subseteq_i Si_{ij}(\mathrm{V}^N)$,

Considering the self-evident relation $Si_{ij}(\mathrm{V}^N) \subseteq_i Se_{ij}\left(Se_{ij}(\mathrm{V}^N)\right)$

We obtain $Si_{ij}(\mathrm{V}^N) =_i Se_{ij}\left(Se_{ij}(\mathrm{V}^N)\right)$.

This relation implies that A is a $SNC_1 - condensed\ set$. The second half of this theorem can be proved in the similar manner.

Corollary3.8: A set V^N of SNCT-space$(X, \mathcal{Z})$ is SNC-condensed if and only if $Se_{ij}\left(Se_{ij}(V^N)\right) \subseteq_i V^N \subseteq_i \left(Se_{ij}\left(Si_{ij}(V^N)\right)\right)^{C_2}$.

This corollary is evident from the definition of the condensed set.

Remark 3.9:

1. $\emptyset_1^N$ *and* X_1^N are satisfied $Se_{ij}\left(Se_{ij}(V^N)\right) =_i V^N$ and $\left(Se_{ij}\left(Si_{ij}(V^N)\right)\right)^{C_2} =_i V^N$.
2. If V^N satisfies $Se_{ij}\left(Se_{ij}(V^N)\right) =_i V^N$, then The complement of type 2 of V^N satisfies $\left(Se_{ij}\left(Si_{ij}(V^N)\right)\right)^{C_2} =_i V^N$, If V^N satisfies$\left(Se_{ij}\left(Si_{ij}(V^N)\right)\right)^{C_2} =_i V^N$, then The complement of type 2 of V^N satisfies $Se_{ij}\left(Se_{ij}(V^N)\right) =_i V^N$.
3. $If\ V^N, H^N\ are\ satisfied\ Se_{ij}\left(Se_{ij}(V^N)\right) =_i V^N, Se_{ij}\left(Se_{ij}(, H^N)\right) =_i H^N$, tnen $Se_{ij}\left(Se_{ij}(V^N \cap_i H^N)\right) =_i V^N \cap_i H^N$, If V^N, H^N are satisfied $\left(Se_{ij}\left(Si_{ij}(V^N)\right)\right)^{C_2} =_i V^N$,
$$\left(Se_{ij}\left(Si_{ij}(H^N)\right)\right)^{C_2} =_i H^N \text{ tnen } \left(Se_{ij}\left(Si_{ij}(V^N \cup_i H^N)\right)\right)^{C_2} =_i V^N \cup_i H^N$$
4. For any NC-set V^N of SNCT-space (X, $\mathcal{Z}$), $Se_{ij}\left(Se_{ij}(V^N)\right)$ satisfies q$Se_{ij}\left(Se_{ij}\left(Se_{ij}\left(Se_{ij}(V^N)\right)\right)\right)$ and
$$\left(Se_{ij}\left(Si_{ij}(V^N)\right)\right)^{C_2} \text{satisfies} \left(Se_{ij}\left(Si_{ij}\left(\left(Se_{ij}\left(Si_{ij}(V^N)\right)\right)^{C_2}\right)\right)\right)^{C_2}.$$

proof

2. let $Se_{ij}\left(Se_{ij}(V^N)\right) =_i V^N$by take the complement for both side $\left(Se_{ij}\left(Se_{ij}(V^N)\right)\right)^{C_2} \subseteq_i {V^N}^{C_2}$ substituting for V^N by ${V^N}^{C_2}$, we get $\left(Se_{ij}\left(Si_{ij}(V^N)\right)\right)^{C_2} =_i V^N$

   therefor ${V^N}^{C_2}$is r.closed

   now let $\left(Se_{ij}\left(Si_{ij}(V^N)\right)\right)^{C_2} =_i V^N$by take the complement for both side

   $Se_{ij}\left(Si_{ij}(V^N)\right) =_i {V^N}^{C_2}$ substituting for V^N by ${V^N}^{C_2}$,

   we get $Se_{ij}\left(Se_{ij}(V^N)\right) =_i V^N$
3. let $V^N, H^N$are satisfied $\rightarrow Se_{ij}\left(Se_{ij}(V^N)\right) =_i V^N\ and\ Se_{ij}\left(Se_{ij}(H^N)\right) =_i H^N$
$$\left(Se_{ij}(V^N \cap_i H^N)\right)^{C_2} \subseteq_i \left(Se_{ij}(V^N)\right)^{C_2} \cap_i \left(Se_{ij}(H^N)\right)^{C_2}$$
   take the interior for both side $Si_{ij}\left[\left(Se_{ij}(V^N \cap_i H^N)\right)^{C_2}\right] \subseteq_i Si_{ij}\left[\left(Se_{ij}(V^N)\right)^{C_2} \cap_i \left(Se_{ij}(H^N)\right)^{C_2}\right]$

   $=_i Se_{ij}\left(Se_{ij}(V^N)\right) \cap_i Se_{ij}\left(Se_{ij}(H^N)\right) =_i V^N \cap_i H^N$ ….. 1

   Also $V^N \cap_i H^N \subseteq_i \left(Se_{ij}(V^N \cap_i H^N)\right)^{C_2}$

   Then $Si_{ij}(V^N) \cap_i Si_{ij}(H^N) \subseteq_i Se_{ij}\left(Se_{ij}(V^N \cap_i H^N)\right)$

   But $Si_{ij}(V^N) \cap_i Si_{ij}(H^N) =_i Si_{ij}\left[Se_{ij}\left(Se_{ij}(V^N)\right)\right] \cap_i Si_{ij}\left[Se_{ij}\left(Se_{ij}(H^N)\right)\right]$

$=_i Se_{ij}\left(Se_{ij}(\mathrm{V}^N)\right) \cap_i Se_{ij}\left(Se_{ij}(\mathrm{H}^N)\right)$

$=_i \mathrm{V}^N \cap_i \mathrm{H}^N$

Which imply that $\mathrm{V}^N \cap_i \mathrm{H}^N \subseteq_i Se_{ij}\left(Se_{ij}(\mathrm{V}^N \cap_i \mathrm{H}^N)\right)$ ……..2

From 1 and 2 we get the result

The prove of the second part is similar manner.

4. Directly by lemma 1

Preposition 3.10: LetV^N $be\ NC-set$ of any type then

i. $Se_{ij}\left(Se_{ij}(\mathrm{V}^N)\right) =_i \mathrm{V}^N$if and only if the set is $SNC_1-condensed\ set$ and open.

ii. $\left(Se_{ij}\left(Si_{ij}(\mathrm{V}^N)\right)\right)^{C_2} =_i \mathrm{V}^N$ if and only if the set is $SNC_1-condensed\ set$ and closed.

Proof. i. let V^N satisfies the following relation $Se_{ij}\left(Se_{ij}(\mathrm{V}^N)\right) =_i \mathrm{V}^N$ ……. 3

Using this relation $Si_{ij}\left[Se_{ij}\left(Se_{ij}(\mathrm{V}^N)\right)\right] =_i \left(Si_{ij}(\mathrm{V}^N)\right)$ we get $Si_{ij}(\mathrm{V}^N) =_i \mathrm{V}^N$

This implies that V^N is a open set and From eq. (3), we get $Se_{ij}\left(Se_{ij}(\mathrm{V}^N)\right) =_i \left(Si_{ij}(\mathrm{V}^N)\right)$

This relation implies that V^N is a $SNC_1-condensed\ set$.Let a set V^Nbe $SNC_1-condensed\ set$ and open.

Then we have $Se_{ij}\left(Se_{ij}(\mathrm{V}^N)\right) =_i \left(Si_{ij}(\mathrm{V}^N)\right) =_i \mathrm{V}^N$.

The second part of the proposition can be proved in the similar manner.

Corollary 3.11: $Se_{ij}\left(Se_{ij}(\mathrm{V}^N)\right) =_i \mathrm{V}^N$if and only if the set is a SNC- condensed and open and $\left(Se_{ij}\left(Si_{ij}(\mathrm{V}^N)\right)\right)^{C_2} =_i \mathrm{V}^N$ if and only if the set is a SNC- condensed and closed.

Proof:

Directly by proposition 1.12

Theorem 3.12: Let V^N is a NC-set then

a) $\mathrm{V}^N$is SNC-condensed if and only if there is a NC-set I^N such that$Se_{ij}\left(Se_{ij}(\mathrm{I}^N)\right) =_i \mathrm{I}^N and$ $\mathrm{I}^N \subseteq_i \mathrm{V}^N \subseteq_i \left(Se_{ij}((\mathrm{I}^N))\right)^{C_2}$.

b) $\mathrm{V}^N$is SNC-condensed if and only if there is a NC- set I such that $\left(Se_{ij}\left(Si_{ij}(\mathrm{I}^N)\right)\right)^{C_2} =_i \mathrm{I}^N\ and\ Si_{ij}(\mathrm{I}^N) \subseteq_i \mathrm{V}^N \subseteq_i \mathrm{I}^N$.

Proof. Let us assume V^N is SNC-condensed. We take $\mathrm{I}^N =_i Se_{ij}\left(Se_{ij}(\mathrm{V}^N)\right)$.

Then $\mathrm{I}^N =_i Se_{ij}\left(Se_{ij}(\mathrm{V}^N)\right) =_i Si_{ij}(\mathrm{V}^N) \subseteq_i \mathrm{V}^N$.

And $\left(Se_{ij}((\mathrm{I}^N))\right)^{C_2} =_i \left(Se_{ij}\left[Se_{ij}\left(Se_{ij}(\mathrm{V}^N)\right)\right]\right)^{C_2} \supseteq_i \left(Se_{ij}\left(Si_{ij}(\mathrm{V}^N)\right)\right)^{C_2} \supseteq_i \mathrm{V}^N$.

So I^Nsatisfies $\mathrm{I} \subseteq_i \mathrm{V}^N \subseteq_i \left(Se_{ij}((\mathrm{I}^N))\right)^{C_2}$.

If we assume that there is a NC- set I^N which satisfies $\mathrm{I}^N \subseteq_i \mathrm{V}^N \subseteq_i \left(Se_{ij}((\mathrm{I}^N))\right)^{C_2}$,

then $Se_{ij}\left(Se_{ij}(\mathrm{V}^N)\right) \subseteq_i Se_{ij}\left(Se_{ij}(\mathrm{I}^N)\right) =_i \mathrm{I}^N$. So $\mathrm{I}^N =_i Se_{ij}\left(Se_{ij}(\mathrm{V}^N)\right)$.

On the other hand $\left(Se_{ij}\left(Si_{ij}(\mathrm{V}^N)\right)\right)^{C_2} \subseteq_i \left(Se_{ij}\left[Se_{ij}\left(Se_{ij}(\text{Į}^N)\right)\right]\right)^{C_2} =_i \left(Se_{ij}((\text{Į}^N))\right)^{C_2}$

$\left(Se_{ij}\left(Si_{ij}(\mathrm{V}^N)\right)\right)^{C_2} \supseteq_i \left(Se_{ij}\left(Si_{ij}(\text{Į}^N)\right)\right)^{C_2} =_i \left(Se_{ij}((\text{Į}^N))\right)^{C_2}$

So $\left(Se_{ij}\left(Si_{ij}(\mathrm{V}^N)\right)\right)^{C_2} =_i \left(Se_{ij}((\text{Į}^N))\right)^{C_2}$

Then we have $Se_{ij}\left(Se_{ij}(\mathrm{V}^N)\right) \subseteq_i \mathrm{V}^N \subseteq_i \left(Se_{ij}\left(Si_{ij}(\mathrm{V}^N)\right)\right)^{C_2}$.

The remaining part of the theorem is dual to the proved part.

Theorem 3.13. A set V^N is a SNC-condensed if and only if the complement of confused of V^N coincides with

$$([\mathrm{X}_{ij}\ (\mathrm{V}^N)])^{C_2} =_i \left(Se_{ij}\left(Si_{ij}(\mathrm{V}^N)\right)\right)^{C_2} \cap_i \left(Se_{ij}\left(Si_{ij}((\mathrm{V}^N)^{C_2})\right)\right)^{C_2}.$$

Proof. We assume that V^N is a SNC-condensed set. By the corollary to Theorem 3.6, $(\mathrm{V}^N)^{C_2}$ is a SNC- condensed set. So we have the following equalities

$$\left(Se_{ij}\left(Si_{ij}(\mathrm{V}^N)\right)\right)^{C_2} =_i \left(Se_{ij}((\mathrm{V}^N))\right)^{C_2}$$

$$\left(Se_{ij}\left(Si_{ij}((\mathrm{V}^N)^{C_2})\right)\right)^{C_2} =_i \left(Se_{ij}(((\mathrm{V}^N)^{C_2}))\right)^{C_2}$$

Therefore $\left(Se_{ij}\left(Si_{ij}(\mathrm{V}^N)\right)\right)^{C_2} \cap_i \left(Se_{ij}\left(Se_{ij}(\mathrm{V}^N)\right)\right)^{C_2}$

$=_i \left(Se_{ij}((\mathrm{V}^N))\right)^{C_2} \cap_i \left(Si_{ij}((\mathrm{V}^N))\right)^{C_2}$

$=_i ([\mathrm{X}_{ij}\ (\mathrm{V}^N)])^{C_2}$

Next we prove the sufficiency. we assume $([\mathrm{X}_{ij}\ (\mathrm{V}^N)])^{C_2} =_i \left(Se_{ij}\left(Se_{ij}(\mathrm{V}^N)\right)\right)^{C_2} \cap_i \left(Se_{ij}\left(Si_{ij}(\mathrm{V}^N)\right)\right)^{C_2}$

Then we have $\left(Se_{ij}((\mathrm{V}^N))\right)^{C_2} =_i Si_{ij}(\mathrm{V}^N) \cup_i ([\mathrm{X}_{ij}\ (\mathrm{V}^N)])^{C_2}$

$$=_i Si_{ij}(\mathrm{V}^N) \cup_i \left[\left(Se_{ij}\left(Se_{ij}(\mathrm{V}^N)\right)\right)^{C_2} \cap_i \left(Se_{ij}\left(Si_{ij}(\mathrm{V}^N)\right)\right)^{C_2}\right]$$

$$\subseteq_i Si_{ij}(\mathrm{V}^N) \cup_i \left(Se_{ij}\left(Si_{ij}(\mathrm{V}^N)\right)\right)^{C_2}$$

$$=_i \left(Se_{ij}\left(Si_{ij}(\mathrm{V}^N)\right)\right)^{C_2}$$

Considering the evident relation $\left(Se_{ij}\left(Si_{ij}(\mathrm{V}^N)\right)\right)^{C_2} \subseteq_i \left(Se_{ij}((\mathrm{V}^N))\right)^{C_2}$

We have $\left(Se_{ij}\left(Si_{ij}(\mathrm{V}^N)\right)\right)^{C_2} =_i \left(Se_{ij}((\mathrm{V}^N))\right)^{C_2}$ … (*)

The same relation holds for $(\mathrm{V}^N)^{C_2}$ because the complement of type 2 of confused of $(\mathrm{V}^N)^{C_2}$ coincides with that of V^N.

So we have $\left[Si_{ij}(((\mathrm{V}^N)^{C_2})^{C_2})\right]^{C_2} =_i \left(Se_{ij}\left(Se_{ij}(\mathrm{V}^N)\right)\right)^{C_2}$.

Considering $\left[Si_{ij}(((\mathrm{V}^N)^{C_2})^{C_2})\right]^{C_2} =_i \left(Se_{ij}((\mathrm{V}^N))\right)^{C_2}$

we get $\left(Se_{ij}\left(Se_{ij}(\mathrm{V}^N)\right)\right)^{C_2} =_i \left(Se_{ij}((\mathrm{V}^N))\right)^{C_2}$

Taking the complement of both sides we obtain $Si_{ij}(\mathrm{V}^N)$. This relation and relation (*) imply that the set A is a SNC- condensed.

4. Conclusion

In general, it can be said that every open set is $SNC_1 - condensed\ set$ and every closed set is $SNC_2 - condensed\ set$ in the cases of $(Si_{11}(L^N) and\ Si_{22}(L^N))$, but this is not true in the cases of $(Si_{12}(L^N)\ \ Si_{21}(L^N))$. All theorems whose proofs include open set equal to its interior points are true in only two cases: $(Si_{11}(L^N), Si_{22}(L^N))$, that is, when adopting the definition of the stable interior on the subset of the first type with the union of the first type and the subset of the second type with the union of the second type. In case when the definition of (($Si_{12}(L^N)\ \ Si_{21}(L^N)$) is based on a subset of the first type with a union of the second type or a subset of the second type with a union of the first type is not true. This is because in the last two cases the open set is not necessarily equal to its interior points

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