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New Concepts in Stable Neutrosophic Crisp Topological space

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Abstract— In applied fields, neutrosophic is a significant topic. This study's central hypothesis is split into two sections. Building a new concept on top of the confused set is the first step; we named it stable border and listed its key characteristics that are directly connected to the stable interior and stable exterior. The idea's second section discusses the three groups that make up the nitro family. We named the first type ($c1-groop$), second type ($c2-groop$), and third type ($c3-groop$), which are distinguished by their respective attributes, and we provided several instances that supported the findings of the study.

Keywords—Stable Neutrosophic Topological Spaces, Stable Neutrosophic Crisp Interior Set, Stable Neutrosophic Crisp Exterior Set, Confused Crisp Set.

1. INTRODUCTION

In general terms, every problem applied or not needs to identify its source and methodology for assessment. There is another issue if this is not stated, and practically all researchers can relate to this idea. As a result, every issue needs to be properly identified and assessed. From this, the concept of neutrosophic or neutrosophic as failure was developed, which involves breaking the evaluation process down into three separate but occasionally connected parts. Samrandash was the first to do this[1,2].
The neurosophistication is considered a generalization of the science that was built in 1965[3,4], and the acceleration of these two concepts took place in various fields and various sciences. There is almost no science in which nitrosophy is not indulged, and the applied fields are very broad in both categories, and this is the basic foundation of the importance of the subject[5,6,7]. As there are three types of netrosophic groups, and for more information, see [8,9,10].
A group of researchers combining the Neutrosophic and Soft groups[11]and extended from three to (n, n≥ 3) [12,13]. In this study, we categorize the Netrosophic groups into three groups and provide instances and confirmation of some of their qualities. The symbol for the group () will be ().

2. STABLE NEUTROSOPHIC CRISP TOPOLOGICAL SPACE

Lemma 1.1: Let (X, Ƨ) be a SNCT-space , $\text{V}^N, \text{Ꜧ}^N$ *are a NC − sets* of any type, Then the following relation hold for $i, j = 1,2$

1. $Se_{ij}[Se_{ij}(\text{V}^N)] =_i Se_{ij}[Se_{ij}(Se_{ij}[Se_{ij}(\text{V}^N)])]$
2. $\left(Se_{ij}(Se_{ij}(Se_{ij}\left(Si_{ij}(\text{V}^N)\right)))\right)^{C_2} =_i \left(Se_{ij}\left(Si_{ij}(\text{V}^N)\right)\right)^{C_2}$

Lemma 1.2: Let (X, Ƨ) be a SNCT-space , $\text{V}^N, \text{Ꜧ}^N$ *are a NC − sets* of any type.Then the following relation hold for $i, j = 1,2$

1. $Se_{ij}\left[Se_{ij}\left(Si_{ij}(\text{V}^N)\right)\right] \cap_i Se_{ij}\left[Se_{ij}\left(Si_{ij}(\text{Ꜧ}^N)\right)\right] =_i Se_{ij}\left[Se_{ij}\left(Si_{ij}(\text{V}^N \cap_i \text{Ꜧ}^N)\right)\right]$
2. $\left(Se_{ij}\left[Se_{ij}\left(Se_{ij}(\text{V}^N)\right)\right]\right)^{C_2} \cup_i \left(Se_{ij}\left[Se_{ij}\left(Se_{ij}(\text{Ꜧ}^N)\right)\right]\right)^{C_2} =_i \left(Se_{ij}\left[Se_{ij}\left(Se_{ij}(\text{V}^N \cup_i \text{Ꜧ}^N)\right)\right]\right)^{C_2}$

Definition 1.3: Let (X, Ƨ) be a SNCT-space , V^N *be a NC − set* of any type, the exterior of confused is called the stable border of $\text{V}^N$and denoted by $\text{Б}(\text{V}^N)$.

In other word $\text{Б}(\text{V}^N) =_i Se_{ij}[\text{Ӿ}_{ij}(\text{V}^N)] =_i Se_{ij}[Se_{ij}(\text{V}^N)] \cap_i Se_{ij}[Si_{ij}(\text{V}^N)]$

Example1.4 : Let X={n, m, z},(X,Ƨ) be a SNCT − space, $Ƨ = \{A^N, B^N, C^N, D^N, \emptyset_1^N, X_1^N\}$ such that:

$A^N =< \{n\}, \emptyset, \{m\} >$,

$B^N =< \{m\}, \emptyset, \{n\} >$,

$C^N =< \emptyset, \emptyset, \{n, m\} >$,

$D^N =< \{n, m\}, \emptyset, \emptyset >$

Let $K^N =< \{n\}, \{z\}, \{m\} >$,

$(K^N)^{C_2} =< \{m\}, \{z\}, \{n\} >$

$\text{Ӿ}_{11}(K^N) =< \{n, m\}, \emptyset, \emptyset >$,

$\text{Ӿ}_{22}(K^N) = \emptyset_1^N$,

$\text{Ӿ}_{12}(K^N) = \emptyset_1^N$,

$\text{Ӿ}_{21}(K^N) =< \{n, m\}, \emptyset, \emptyset >$

$\text{Б}_{11}(K^N) =< \emptyset, \emptyset, \{n, m\} >$,

$\text{Б}_{22}(K^N) = X_1^N$,

$\text{Б}_{12}(K^N) = X_1^N$, $\text{Б}_{21}(K^N) =< \emptyset, \emptyset, \{n, m\} >$

Proposition 1.5: Let V^N *be a NC − set* of any type, then $Se_{ij}[Se_{ij}(\text{Б}(\text{V}^N))] =_i \text{Б}(\text{V}^N)$.

Proof. We must prove $\text{Б}(\text{V}^N)$ is open and $SNC_1 - condensed\ set$

First from definition of stable border it clear that $\text{Б}(\text{V}^N)$ is open

Now to prove $\text{Б}(\text{V}^N)$is $SNC_1 - condensed\ set$

$$Se_{ij}[Se_{ij}(\text{Б}(\text{V}^N))] =_i Se_{ij}[Se_{ij}(Se_{ij}[\text{Ӿ}_{ij}(\text{V}^N)])]$$

$$=_i Se_{ij}[\text{Ӿ}_{ij}(\text{V}^N)] \text{ (by lemma 1)}$$

$$=_i \text{Б}(\text{V}^N) =_i Si_{ij}[\text{Б}(\text{V}^N)] \quad \text{since Б}(\text{V}^N) \text{ is open.}$$

Remark 1.6: For any NC-set V^N of SNCT-space(X, Ƨ) , $\text{Б}(\text{Б}(\text{V}^N)) =_i \emptyset_1^N$.

Proof. By above proposition $Se_{ij}[Se_{ij}(\text{Б}(\text{V}^N))] =_i \text{Б}(\text{V}^N)$.

By proposition( ) $\text{Б}(\text{V}^N)$is $SNC_2 - condensed\ set$

$$Se_{ij}[Se_{ij}(\text{Б}(\text{V}^N))] =_i Si_{ij}(\text{Б}(\text{V}^N)) \subseteq_i \left[\left[Se_{ij}[Si_{ij}(\text{Б}(\text{V}^N))]\right]\right]^{C_2} \quad \text{.....1}$$

$$\text{Б}(\text{Б}(\text{V}^N)) =_i Se_{ij}[Se_{ij}(\text{Б}(\text{V}^N))] \cap_i Se_{ij}[Si_{ij}(\text{Б}(\text{V}^N))]$$

$$\subseteq_i \left[\left[Se_{ij}[Si_{ij}(\text{Б}(\text{V}^N))]\right]\right]^{C_2} \cap_i Se_{ij}[Si_{ij}(\text{Б}(\text{V}^N))] =_i \emptyset_1^N$$

Therefor $\text{Б}(\text{Б}(\text{V}^N)) =_i \emptyset_1^N$

Proposition 1.7:

a) $\text{V}^N$is $SNC_1 - condensed\ set$ if and only if

1. $Se_{ij}\left[Se_{ij}\left(Si_{ij}(\text{V}^N)\right)\right] =_i Si_{ij}(\text{V}^N)$
2. $\text{Б}(\text{V}^N) =_i \emptyset_1^N$

b) $\text{V}^N$is $SNC_2 - condensed\ set$ if and only if

1. $Se_{ij}[Se_{ij}(Se_{ij}(\text{V}^N))] =_i Se_{ij}(\text{V}^N)$
2. $\text{Б}(\text{V}^N) =_i \emptyset_1^N$.

Proof

a) Let V^N is $SNC_1 - condensed\ set$. Then $Se_{ij}[Se_{ij}(V^N)] =_i Si_{ij}[V^N]$ …….2

So we get $Se_{ij}[Se_{ij}(Se_{ij}[Se_{ij}(V^N)])] =_i Se_{ij}[Se_{ij}(Si_{ij}[V^N])]$

The left side of above equality becomes $Se_{ij}[Se_{ij}(Se_{ij}[Se_{ij}(V^N)])] =_i Si_{ij}[V^N]$

(by eq. (2) and lemma). Then we get, the relation $Se_{ij}[Se_{ij}(Si_{ij}[V^N])] =_i Si_{ij}[V^N]$.

Using (2) we get $Se_{ij}[Se_{ij}(V^N)] =_i Si_{ij}$

$[V^N] \subseteq_i \left[\left[Se_{ij}[Si_{ij}(V^N)]\right]\right]^{C_2}$ $then,$ $Б(V^N) =_i Se_{ij}[Se_{ij}(V^N)] \cap_i Se_{ij}[Si_{ij}(Б(V^N))]$.

Next we prove the sufficiency. Suppose that satisfies the following two conditions;

$Se_{ij}[Se_{ij}(Si_{ij}[V^N])] =_i Si_{ij}[V^N]$ ……. 3

$Se_{ij}[Se_{ij}(V^N)] \subseteq_i \left[\left[Se_{ij}[Si_{ij}(V^N)]\right]\right]^{C_2}$ (this is equivalent to $Б(V^N) =_i \emptyset_1^N$)

Take the interior $Se_{ij}[Se_{ij}(V^N)] \subseteq_i Se_{ij}[Se_{ij}(Si_{ij}[V^N])]$

Combining this relation and relation (3) we have $Se_{ij}[Se_{ij}(V^N)] \subseteq_i Si_{ij}(V^N)$

Considering the self-evident relation $Si_{ij}(V^N) \subseteq_i Se_{ij}[Se_{ij}(V^N)]$ we get $Si_{ij}(V^N) =_i Se_{ij}[Se_{ij}(V^N)]$.

This relation implies that V^N is a $SNC_1 - condensed\ set$.

b) Let V^N is $SNC_2 - condensed\ set$. Then $\left(Se_{ij}\left(Si_{ij}(V^N)\right)\right)^{C_2} =_i \left(Se_{ij}((V^N))\right)^{C_2}$

$\left[Se_{ij}\left[Se_{ij}\left[Se_{ij}[Si_{ij}(V^N)]\right]\right]\right]^{C_2} =_i \left[Se_{ij}\left[Se_{ij}\left[Se_{ij}[(V^N)]\right]\right]\right]^{C_2}$……….3

$\left(Se_{ij}(Se_{ij}(Se_{ij}\left(Si_{ij}(V^N)\right)))\right)^{C_2} =_i \left(Se_{ij}\left(Si_{ij}(V^N)\right)\right)^{C_2}$ ………4

By (3) and (4) $\left(Se_{ij}\left(Si_{ij}(V^N)\right)\right)^{C_2} =_i \left[Se_{ij}\left[Se_{ij}\left[Se_{ij}[(V^N)]\right]\right]\right]^{C_2}$

So $\left[Se_{ij}\left[Se_{ij}\left[Se_{ij}[(V^N)]\right]\right]\right]^{C_2} =_i \left(Se_{ij}((V^N))\right)^{C_2}$ by take the complement of both side

We get $Se_{ij}\left[Se_{ij}\left[Se_{ij}[(V^N)]\right]\right] =_i Se_{ij}((V^N))$

And $Б(V^N) =_i Se_{ij}[Se_{ij}(V^N)] \cap_i Se_{ij}[Si_{ij}(V^N)] \subseteq_i \left(Se_{ij}\left(Si_{ij}(V^N)\right)\right)^{C_2} \cap_i Se_{ij}\left(Si_{ij}(V^N)\right) =_i \emptyset_1^N$

Now suppose $Se_{ij}\left[Se_{ij}\left(Se_{ij}(V^N)\right)\right] =_i \left(Se_{ij}((V^N))\right)^{C_2}$ and $Б(V^N) =_i \emptyset_1^N$

$Se_{ij}[Se_{ij}(V^N)] \cap_i Se_{ij}[Si_{ij}(V^N)] =_i \emptyset_1^N \rightarrow Se_{ij}[Se_{ij}(V^N)] \subseteq_i \left(Se_{ij}\left(Si_{ij}(V^N)\right)\right)^{C_2}$

$\left[Se_{ij}\left[Se_{ij}\left[Se_{ij}[(V^N)]\right]\right]\right]^{C_2} \subseteq_i \left(Se_{ij}\left(Si_{ij}(V^N)\right)\right)^{C_2} \rightarrow \left(Se_{ij}((V^N))\right)^{C_2} \subseteq_i \left(Se_{ij}\left(Si_{ij}(V^N)\right)\right)^{C_2}$

But $\left(Se_{ij}\left(Si_{ij}(V^N)\right)\right)^{C_2} \subseteq_i \left(Se_{ij}((V^N))\right)^{C_2}$.

Thus $\left(Se_{ij}\left(Si_{ij}(V^N)\right)\right)^{C_2} =_i \left(Se_{ij}((V^N))\right)^{C_2}$.

Corollary 1.7: V^N is a SNC-condensed set if and only if

a) $Si_{ij}(V^N)$ satisfies $Se_{ij}[Se_{ij}(Si_{ij}[V^N])] =_i Si_{ij}(V^N)$

b) $Se_{ij}(V^N)$ satisfies $Se_{ij}[Se_{ij}(Se_{ij}[V^N])] =_i Se_{ij}(V^N)$

c) Б$(V^N) =_i \emptyset_1^N$.

Definition 1.8: Any NC-set can be classified into one of the following three group:

1. $c1 - groop = \left\{V^N, Se_{ij}(Se_{ij}[V^N]) \subseteq_i \left[\left[Se_{ij}[Si_{ij}(V^N)]\right]\right]^{C_2}\right\}$
2. $c2 - groop = \left\{V^N, \left[\left[Se_{ij}[Si_{ij}(V^N)]\right]\right]^{C_2} \subsetneq Se_{ij}(Se_{ij}[V^N])\right\}$
3. $c3 - groop = \left\{V^N, Se_{ij}(Se_{ij}[V^N]) and \left[\left[Se_{ij}[Si_{ij}(V^N)]\right]\right]^{C_2} are\ non - comparable\right\}$

Example 1.9: Reconsider example (1.4)**.** Let Q==$< \{z\}, \emptyset, \emptyset >$

$Se_{ij}(Se_{ij}[Q^N]) =< \{n, m\}, \emptyset, \emptyset >$ and $\left[\left[Se_{ij}[Si_{ij}(Q^N)]\right]\right]^{C_2} =< \{m\}, \emptyset, \{n\} >$

We note that $\left[\left[Se_{ij}[Si_{ij}(Q^N)]\right]\right]^{C_2} \subsetneq Se_{ij}(Se_{ij}[Q^N])$. Q$\in$ $c\mathbf{2} - \boldsymbol{groop}$**.**

Example 1.10: Let X=$\{k, g, l, f, s\}$, (X, Ƹ)be a SNCT − space where Ƹ $= \{A^N, B^N, C^N, D^N, K^N, L^N, M^N, N^N, \emptyset_1^N, X_1^N\}$ such that:

$A^N =< \{k, g, l, s\}, \emptyset, \{f\} >$,

$B^N =< \{l, s\}, \{f\}, \emptyset >$,

$C^N =< \{s\}, \emptyset, \emptyset >$,

$D^N =< \{s\}, \emptyset, \{f\} >$,

$K^N =< \emptyset, \emptyset, \{s, f\} >$

Now let

$F^N =< \emptyset, \emptyset, \{s\} >$,

$F^{N^{C_2}} =< \{s\}, \emptyset, \emptyset >$

$Si_{11}\left(F^{N^{C_2}}\right) =< \{s\}, \emptyset, \emptyset >$,

$\left[Si_{11}\left(F^{N^{C_2}}\right)\right]^{C_2} =< \emptyset, \emptyset, \{s\} >$,

$\left[\left[Si_{11}\left(F^{N^{C_2}}\right)\right]^{C_2}\right] = Se_{ij}(Se_{ij}[F^N]) =< \emptyset, \emptyset, \{s, f\} >$

$Si_{ij}(F^N) =< \emptyset, \emptyset, \{s, f\} >$,

$[Si_{11}(F^N)]^{C_2} =< \{s, f\}, \emptyset, \emptyset >$,

$Si_{11}[Si_{11}(F^N)]^{C_2} =< \{s\}, \emptyset, \emptyset > [Si_{11}[Si_{11}(F^N)]^{C_2}]^{C_2}$

$= \left[\left[Se_{ij}[Si_{ij}(F^N)]\right]\right]^{C_2} =< \emptyset, \emptyset, \{s\} >, Se_{ij}(Se_{ij}[F^N]) \subseteq_i \left[\left[Se_{ij}[Si_{ij}(F^N)]\right]\right]^{C_2}$.

Thus $F^N \in c\mathbf{1} - \boldsymbol{groop}$.

Remake 1.11

1. A set belongs to $c\mathbf{1} - \boldsymbol{groop}$ if and only if the stable border of the set is empty.
2. a $SNC_1 - condensed\ set$ and a $SNC_2 - condensed\ set$ belong to $c\mathbf{1} - \boldsymbol{groop}$.

Proof: 1. Assume that V^N belongs to $c\mathbf{1}-\boldsymbol{groop} \to Se_{ij}(\,Se_{ij}\,[V^N]) \subseteq_i \left[\left[Se_{ij}[Si_{ij}(\,V^N)]\right]\right]^{C_2}$

From definition of stable border of V^N, $Б(\,V^N) =_i Se_{ij}[Se_{ij}(V^N)] \cap_i Se_{ij}[Si_{ij}(V^N)] =_i \emptyset_1^N$.

Conversely, let $Б(\,V^N) =_i Se_{ij}[Se_{ij}(V^N)] \cap_i Se_{ij}[Si_{ij}(V^N)] =_i \emptyset_1^N$

Thus $Se_{ij}(\,Se_{ij}\,[V^N]) \subseteq_i \left[\left[Se_{ij}[Si_{ij}(\,V^N)]\right]\right]^{C_2}$.

Therefore V^N belongs to $c\mathbf{1}-\boldsymbol{groop}$.

2.Assume that $V^N$is a $SNC_1 - condensed\ set \to Se_{ij}(\,Se_{ij}\,[V^N]) =_i Si_{ij}\,[V^N]$

But we have $Si_{ij}\,[V^N] \subseteq_i \left[\left[Se_{ij}[Si_{ij}(\,V^N)]\right]\right]^{C_2}$. Thus $Se_{ij}(\,Se_{ij}\,[V^N]) \subseteq_i \left[\left[Se_{ij}[Si_{ij}(\,V^N)]\right]\right]^{C_2}$.

Hence V^N belongs to $c\mathbf{1}-\boldsymbol{groop}$.

Assume that $V^N$is a $SNC_2 - condensed\ set \to \left(Se_{ij}\left(Si_{ij}(V^N)\right)\right)^{C_2} =_i \left(Se_{ij}((V^N))\right)^{C_2}$

But we have $Se_{ij}(\,Se_{ij}\,[V^N]) \subseteq_i \left(Se_{ij}((V^N))\right)^{C_2}$. So $Se_{ij}(\,Se_{ij}\,[V^N]) \subseteq_i \left(Se_{ij}\left(Si_{ij}(V^N)\right)\right)^{C_2}$.

Hence V^N belongs to $c\mathbf{1}-\boldsymbol{groop}$.

Proposition 1.12: We assume that V^N belongs to $\mathbf{1}-\boldsymbol{groop}$.

Then the following relation holds $\left[Se_{ij}(\,Se_{ij}\,[V^N])\right] \subseteq_i \left[\left[Se_{ij}[Si_{ij}(\,V^N)]\right]\right]^{C_2}$.

Taking the complement of both sides we obtain $Se_{ij}[Si_{ij}(\,V^N)] \subseteq_i \left[\left[Se_{ij}[Se_{ij}(\,V^N)]\right]\right]^{C_2}$

Using the property of the complement we get the relation $Se_{ij}[Se_{ij}(\,(\,V^N)^{C_2})] \subseteq_i \left[\left[Se_{ij}(\,Si_{ij}\,[(V^N)^{C_2}])\right]\right]^{C_2}$

This implies that $(V^N)^{C_2}$also belongs to $c\mathbf{1}$.

We can prove similarly the fact that the complement of a set of $c\mathbf{2}$ belongs to $c\mathbf{2}$. Next we prove about $c\mathbf{3}-\boldsymbol{groop}$.

In this case we must say that if $Se_{ij}(\,Se_{ij}\,[V^N])and \left[\left[Se_{ij}[Si_{ij}(\,V^N)]\right]\right]^{C_2}$ $are\ non-comparable$

Then $Se_{ij}(\,Se_{ij}\,[(V^N)^{C_2}])and \left[\left[Se_{ij}[Si_{ij}((V^N)^{C_2})]\right]\right]^{C_2}$ $are\ non-comparable$ are also noncomparable.

We use reduction absurdum for proving this proposition.

Let $Se_{ij}(\,Se_{ij}\,[(V^N)^{C_2}])and \left[\left[Se_{ij}[Si_{ij}(\,(V^N)^{C_2})]\right]\right]^{C_2}$ $are\ comparable$.

Then $(V^N)^{C_2}$ belongs to $c\mathbf{1}$ or $\mathbf{2}$. So $((V^N)^{C_2})^{C_2} = V^N$ must belong to $c\mathbf{1}-\boldsymbol{groop}$ or $c\mathbf{2}-\boldsymbol{groop}$

This is a contradiction.

Proposition 1.13: If a set V^N belongs to $c\mathbf{1}$ .Then

$Se_{ij}(\,Se_{ij}\,[V^N]) =_i Se_{ij}[Se_{ij}(\,Si_{ij}\,[V^N])]$ and $\left[\left[Se_{ij}[Si_{ij}(\,V^N)]\right]\right]^{C_2} =_i \left[Se_{ij}\left[Se_{ij}[Se_{ij}(\,V^N)]\right]\right]^{C_2}$

On the other hand, if one of these two equalities holds then the set V^N belongs to $c\mathbf{1}-\boldsymbol{groop}$.

Proof. Suppose that V^N belongs to $c\mathbf{1}-\boldsymbol{groop}$.

Then the following relation holds. $\left[Se_{ij}(\,Se_{ij}\,[V^N])\right] \subseteq_i \left[\left[Se_{ij}[Si_{ij}(\,V^N)]\right]\right]^{C_2}$.

Taking the interior of both sides we have $\left[Se_{ij}\left(Se_{ij}\,[\mathrm{V}^N]\right)\right] \subseteq_i Se_{ij}\left[Se_{ij}\left(Si_{ij}\,[\mathrm{V}^N]\right)\right]$

Considering the self-evident relation

$Se_{ij}\left[Se_{ij}\left(Si_{ij}\,[\mathrm{V}^N]\right)\right] \subseteq_i \left[Se_{ij}\left(Se_{ij}\,[\mathrm{V}^N]\right)\right]$, we get $\left[Se_{ij}\left(Se_{ij}\,[\mathrm{V}^N]\right)\right] =_i Se_{ij}\left[Se_{ij}\left(Si_{ij}\,[\mathrm{V}^N]\right)\right]$.

In the similar way we have $\left[\left[Se_{ij}\left[Si_{ij}(\mathrm{V}^N)\right]\right]\right]^{C_2} =_i \left[Se_{ij}\left[Se_{ij}\left[Se_{ij}(\mathrm{V}^N)\right]\right]\right]^{C_2}$

Next, we prove the fact that if $\left[\left[Se_{ij}\left[Si_{ij}(\mathrm{V}^N)\right]\right]\right]^{C_2} =_i \left[Se_{ij}\left[Se_{ij}\left[Se_{ij}(\mathrm{V}^N)\right]\right]\right]^{C_2}$ then the set V^N belongs to $c\mathbf{1}-\boldsymbol{groop}$.

Clearly, $\left[Se_{ij}\left(Se_{ij}\,[\mathrm{V}^N]\right)\right] =_i Se_{ij}\left[Se_{ij}\left(Si_{ij}\,[\mathrm{V}^N]\right)\right] \subseteq_i \left[\left[Se_{ij}\left[Si_{ij}(\mathrm{V}^N)\right]\right]\right]^{C_2}$

This implies that V^N belongs to $c\mathbf{1}-\boldsymbol{groop}$.

We can prove the proposition for $\left[\left[Se_{ij}\left[Si_{ij}(\mathrm{V}^N)\right]\right]\right]^{C_2} =_i \left[Se_{ij}\left[Se_{ij}\left[Se_{ij}(\mathrm{V}^N)\right]\right]\right]^{C_2}$ in the similar manner.

Proposition1.14

Any sets $\mathrm{V}^N, \text{Ḩ}^N$ of $c\mathbf{1}-\boldsymbol{groop}$ satisfy the following relations,

1. $Se_{ij}\left(Se_{ij}\,[\mathrm{V}^N]\right) \cap_i Se_{ij}\left(Se_{ij}\,[\text{Ḩ}^N]\right) =_i Se_{ij}\left(Se_{ij}\,[\mathrm{V}^N \cap_i \text{Ḩ}^N]\right)$
2. $\left[\left[Se_{ij}\left[Si_{ij}(\mathrm{V}^N)\right]\right]\right]^{C_2} \cup_i \left[\left[Se_{ij}\left[Si_{ij}(\text{Ḩ}^N)\right]\right]\right]^{C_2} =_i \left[\left[Se_{ij}\left[Si_{ij}(\mathrm{V}^N \cup_i \text{Ḩ}^N)\right]\right]\right]^{C_2}$.

Proof

1. $Se_{ij}\left(Se_{ij}\,[\mathrm{V}^N \cap_i \text{Ḩ}^N]\right) \subseteq_i Si_{ij}\left[\left[\left[Se_{ij}(\mathrm{V}^N)\right]\right]^{C_2} \cap_i \left[\left[Se_{ij}(\mathrm{V}^N)\right]\right]^{C_2}\right]$

$$=_i Se_{ij}\left(Se_{ij}\,[\mathrm{V}^N]\right) \cap_i Se_{ij}\left(Se_{ij}\,[\text{Ḩ}^N]\right)$$

On the other hand $Se_{ij}\left(Se_{ij}\,[\mathrm{V}^N]\right) \cap_i Se_{ij}\left(Se_{ij}\,[\text{Ḩ}^N]\right) =_i Se_{ij}\left(Se_{ij}\,(Si_{ij}[\mathrm{V}^N])\right) \cap_i Se_{ij}\left(Se_{ij}\,(Si_{ij}[\text{Ḩ}^N])\right)$

$$=_i Se_{ij}\left(Se_{ij}\,(Si_{ij}[\mathrm{V}^N \cap_i \text{Ḩ}^N])\right) \subseteq_i Se_{ij}\left(Se_{ij}\,[\mathrm{V}^N \cap_i \text{Ḩ}^N]\right)$$ (Lemma1. 2)

2. $\mathrm{V}^N, \text{Ḩ}^N \subseteq_i \mathrm{V}^N \cup_i \text{Ḩ}^N \rightarrow \left[\left[Se_{ij}\left[Si_{ij}(\mathrm{V}^N)\right]\right]\right]^{C_2} \cup_i \left[\left[Se_{ij}\left[Si_{ij}(\text{Ḩ}^N)\right]\right]\right]^{C_2} \subseteq_i \left[\left[Se_{ij}\left[Si_{ij}(\mathrm{V}^N \cup_i \text{Ḩ}^N)\right]\right]\right]^{C_2}$

$$Se_{ij}\left(Se_{ij}\,[\mathrm{V}^N]\right) \cup_i Se_{ij}\left(Se_{ij}\,[\text{Ḩ}^N]\right) \subseteq_i \left[Se_{ij}\left[Se_{ij}\left[Se_{ij}(\mathrm{V}^N)\right]\right]\right]^{C_2} \cup_i \left[Se_{ij}\left[Se_{ij}\left[Se_{ij}(\text{Ḩ}^N)\right]\right]\right]^{C_2}$$

$$=_i \left[Se_{ij}\left[Se_{ij}\left[Se_{ij}(\mathrm{V}^N \cup_i \text{Ḩ}^N)\right]\right]\right]^{C_2}$$ by lemma1.2

$$\left[\left[Se_{ij}\left[Si_{ij}(\mathrm{V}^N \cup_i \text{Ḩ}^N)\right]\right]\right]^{C_2} \subseteq_i \left[\left[Se_{ij}\left[Si_{ij}(\mathrm{V}^N \cup_i \text{Ḩ}^N)\right]\right]\right]^{C_2}$$

$$=_i \left[\left[Se_{ij}\left[Si_{ij}(\mathrm{V}^N)\right]\right]\right]^{C_2} \cup_i \left[\left[Se_{ij}\left[Si_{ij}(\text{Ḩ}^N)\right]\right]\right]^{C_2}$$

$$\left[\left[Se_{ij}\left[Si_{ij}(\mathrm{V}^N \cup_i \text{Ḩ}^N)\right]\right]\right]^{C_2} =_i \left[\left[Se_{ij}\left[Si_{ij}(\mathrm{V}^N)\right]\right]\right]^{C_2} \cup_i \left[\left[Se_{ij}\left[Si_{ij}(\text{Ḩ}^N)\right]\right]\right]^{C_2}.$$

Theorem 1.15: The intersection and the union of any two sets of $c\mathbf{1}-\boldsymbol{groop}$ belong to $c\mathbf{1}-\boldsymbol{groop}$.

Proof. Let $\mathrm{V}^N, \text{Ḩ}^N$ belong to $c\mathbf{1}-\boldsymbol{groop}$.

Then $Se_{ij}\left(Se_{ij}\,[\mathrm{V}^N \cap_i \text{Ḩ}^N]\right) =_i Se_{ij}\left(Se_{ij}\,[\mathrm{V}^N]\right) \cap_i Se_{ij}\left(Se_{ij}\,[\text{Ḩ}^N]\right)$

$$=_i Se_{ij}\left(Se_{ij}\,(Si_{ij}[\mathrm{V}^N])\right) \cap_i Se_{ij}\left(Se_{ij}\,(Si_{ij}[\text{Ḩ}^N])\right)$$. (Lemma1.2)

So, $\mathrm{V}^N \cap_i \text{Ḩ}^N$ belongs to $c\mathbf{1}$ b. The second part of the theorem is dual to the first one.

3. Conclosion

Classifying space into groups makes it easier for us to study the properties of this space in addition to the properties of its groups. Therefore, this work focused on classifying the neutrosophic groups into three sections based on the conditions they meet, which helped to clarify their properties and the most important relationships that link these groups together, as well as clarifying the relationship of these groups to the previous topological concepts within the new topological structure.

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