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Parameter Distribution to Improve the Reliability of Complex Network

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Abstract— A more effective algorithm for enumerating minimal path sets for assessing a complex network's global reliability is introduced and examined in this paper. The graph's adjacency matrix is used to generate the minimal path set. When compared to current approaches, this algorithm appears to be promising in terms of computation time and computer memory requirements. The algorithms are based on the elementary concept of graph theory.

Keywords— Minimal path sets, Reliability of complex network, Path tracing method, Hazard Rate and Mean Time to Failure.

1. INTRODUCTION

Reliability engineers often need to work with networks having elements connected in parallel and series, and to calculate their reliability. To this end, when a network consists of a combination of series and parallel segments, engineers often apply very convoluted block reliability formulas and use software calculation packages. As the underlying statistical theory behind the formulas is not always well understood, errors or misapplications may occur.

Much of our probability analysis has dealt with the method of independent trials. Such processes are the basis of classical probability and a significant part of statistics [1]. The current research discussed two significant theorems for these procedures: The law of large numbers and the central limit theorem. We have shown that the general result is where a sequence of chance tests forms an independent testing procedure. The general result is the same for any experiment and happens with the same probability [2]. Furthermore, the findings of preliminary tests have no bearing on our predictions for the results of the future experiment.

A single experiment result distribution is adequate to create a tree and a tree measure for a set of n experiments. Using this tree measure can answer some probability questions regarding these experiments [3]. Modern probability theory examines potential causes for which the experience informs forecasts for future trials of previous findings. All the past results may affect our forecasts for the next experiment while observing a sequence of chance experiments.

This may be the case, for example, in forecasting the grades of a student on a sequence of examinations in a course. It would be quite challenging to demonstrate that general outcomes have so much generality [4]. According to Markov's research, an important new category of chance phenomena began in 1907. The result of a given experiment will predict the development of the subsequent investigation in this sequence [5].

There are many researchers focused on the study of fundamental reliability concepts [6]. Like the probability, mutually exclusive events, conditional probability, probability density function p.d.f., cumulative distribution function c.d.f., failure rate, hazard rate, and mean time to failure (MTTF) [7].

Because these concepts help in calculating the reliability of the networks [8]. Researchers have tried to understand the relationship between these concepts to get a clear idea of how to calculate the reliability of networks, and there are researchers focused on the study of types of failure rate which will be mentioned. In the latter, the researchers studied the statistical distributions, which can be used in the evaluation of the reliability of networks [9].

2. FUNDAMENTAL CONCEPTS ASSOCIATED WITH RELIABILITY

Definition 1.1 [10]: Probability Density Function: If X is a continuous random variable, then the probability density function, $p.d.f$, of X is a function, $f(x)$, such that for two numbers, a and b with $a \leq b$

$$P(a \leq X \leq b) = \int_a^b f(x)\,dx \text{ and } f(x) \geq 0, \text{ for all } x.$$

Definition 1.2 [10]: Cumulative Distribution Function: is a function, $F(x)$, of a random variable X, and is defined for a number x by

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(s)ds$$

The $c.d.f.$ is used to measure the probability that the item in question will fail before the associated time value, t, and is also called unreliability.

Definition 1.3 [11]: Reliability Function: It is the probability that the network survives from some specified period of time. This may be expressed in terms of random variable T the time to network failure.

$$R(t) = 1 - F(x) = 1 - \int_{-\infty}^{x} f(u)du = \int_{t}^{\infty} f(u)du$$

Definition 1.4 [11]: Hazard Rate: is defined as the limit of the failure rate as the interval approaches zero,

$$h(t) = \lim_{\Delta t \to 0} \frac{F(t+\Delta t) - F(t)}{\Delta t R(t)} = \frac{f(t)}{R(t)}$$

Definition 2.5 [11]: Mean Time to Failure: A basic measure of reliability for nonrepairable networks, average failure free operating time, during a particular measurement period under stated conditions,

$$MTTF = E(t) = \int_0^\infty t f(t)dt = \int_0^\infty R(t)dt$$

3. Fundamental Concepts Associated with Reliability

The function $f(t), F(t), R(t)$ and $h(t)$ can be transformed with one another. For example:

$F(t) + R(t) = 1 \leftrightarrow F(t) = 1 - R(t)$

$F(t) = \int_{-\infty}^{t} f(s)ds$

$\frac{dF(t)}{dt} = f(t)$

$\frac{dF(t)}{dt} = -\frac{dR(t)}{dt}$

$f(t) = -\frac{dR(t)}{dt} \quad \ldots *$

But $h(t) = \frac{f(t)}{R(t)} \to f(t) = h(t).R(t) \quad \ldots **$

From (*) and (**), we get:

$h(t).R(t) = -\frac{dR(t)}{dt}$

$R(t) = e^{-\int_0^t h(t)\,dt}$

Finally, by differentiation, we can write f(t) in terms of h(t),

$f(t) = h(t).e^{-\int_0^t h(t)\,dt}$

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4. THE RELIABILITY OF COMPLEX NETWORK BY USING PATH TRACING METHOD (PTM)

4.1. Complex Network

It is often challenging to figure out which parts of a complex network are connected in parallel and which in series. The network in Fig. 2 is an excellent example of a complex network [12].

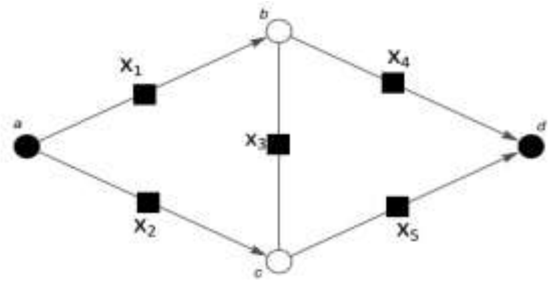

Fig. 1. Complex Network

4.2. Minimal Path Sets of Complex Network

we'll use an algorithm to find the minimal path sets for the complex network we just talked about. We'll then show how this method can be used to find the minimal path sets for any complex network. We will give the steps of the algorithm, which will help in finding the minimal path sets for the given system:

Step1. Create the connection matrix (P) in the form of the logic graph.
Step2. To the connection matrix *[P]*, append a $p \times p$ diagonal unity matrix *[U]*.
Step3. Delete any column or row that is empty of variables, along with the column that corresponds to n_1 an the row that corresponds to n.
Step4. Find the system success determinant S using the remaining rows and columns. At this point, every algebraic variable is transformed into its corresponding Boolean variable.
Step5. Using Boolean sum and product operations, expand the determinant S.
Step6. We are dealing with a polynomial multiplication of n, where n is the number of partial systems in the total system.

$$P = \begin{array}{c} node \\ a \\ b \\ c \\ d \end{array} \begin{vmatrix} a & b & c & d \\ 1 & x_1 & x_2 & 0 \\ 0 & 1 & x_3 & x_4 \\ 0 & x_3 & 1 & x_5 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

$$P^2 = \begin{vmatrix} x_1 & x_2 & 0 \\ 1 & x_3 & x_4 \\ x_3 & 1 & x_5 \end{vmatrix}$$

$$P^3 = \begin{vmatrix} x_1 & x_2 & 0 \\ 1 & x_3 & x_4 \\ x_3 & 1 & x_5 \end{vmatrix} \begin{matrix} x_1 & x_2 \\ 1 & x_3 \\ x_3 & 1 \end{matrix}$$

$$P^4 = x_1x_3x_5 + x_2x_3x_4 - x_1x_4 + x_2x_5$$

It's clear that the minimal path sets of network in Fig. 1 are:

$MP_1^S = \{x_1x_4\}$, $MP_2^S = \{x_2x_5\}$, $MP_3^S = \{x_1x_3x_5\}$, $MP_4^S = \{x_2x_3x_4\}$

4.3. Path tracing method (PTM)

In this method, the following procedures are used to determine a complex network's reliability [12]:

1) List the network's MPS (tie-set).
2) The success of all segments in MPS ways determines the network's success (tie-set).
3) The series' elements are connected, as evidenced by this.

4) The effectiveness of the network is produced by each set of minimal paths.

5) It indicates a parallel connection between the minimal tie sets.

6) Use a parallel and series reduction to draw an equal network and determine the network's dependability. The network's dependability is determined by the likelihood of merging these shortest MPS. The union of MPS can be calculated using the generic equation shown below. Pi:

$$P_r(P_1 + P_2 + \cdots + P_n) = P_r\left(\sum_{i=1}^{n} P_i\right)$$

$$= \sum_{i=1}^{n} P_r(P_i) - \sum_{i<j=2}^{n} P_r(P_i\,.\,P_j) + \sum_{i<j<k=3}^{n} P_r(P_i\,.\,P_j.\,P_k) - \cdots + (-1^{n+1})\, P_r(P_i.\,P_j \ldots P_k)$$

The reliability that at least one MPS will work for n independent MPS. As a result, we end up with.

$$P_r\left(\sum_{i=1}^{n} P_i\right) = 1 - \big[[1 - P_r(Mp_1)] \times [1 - P_r(Mp_2)] \times \ldots \times [1 - P_r(Mp_n)]\big]$$

4.3.1: Path tracing method (PTM) of complex network

To compute the network reliability depending on the sets of all minimal paths (S)

$MP_1^S = \{x_1x_4\},\ \ MP_2^S = \{x_2x_5\},\ \ MP_3^S = \{x_1x_3x_5\},\ \ MP_4^S = \{x_2x_3x_4\}$

Then the structure function is:

$$S = 1 - \big[[1 - P_r(x_1x_4)] \times [1 - P_r(x_2x_5)] \times [1 - P_r(x_1x_3x_5)] \times [1 - P_r(x_2x_3x_4)]\big]$$

$$\begin{aligned}
R_s &= 1 - \big[[1 - (x_1x_4)] \times [1 - (x_2x_5)] \times [1 - (x_1x_3x_5)] \times [1 - (x_2x_3x_4)]\big] \\
&= 1 - \big[[1 - (x_1x_4) - (x_2x_5) + (x_1x_2x_4x_5)] \times [1 - (x_1x_3x_5)] \times [1 - (x_2x_3x_4)]\big] \\
&= 1 - [1 - (x_1x_3x_5) - (x_1x_4) + (x_1x_3x_4x_5) - (x_2x_5) + (x_1x_2x_3x_5) + (x_1x_2x_4x_5)] \\
&\quad -(x_1x_2x_3x_4x_5) \times [1 - (x_2x_3x_4)] \\
&= 1 - [1 - (x_2x_3x_4) - (x_1x_3x_5) + (x_1x_2x_3x_4x_5) - (x_1x_4) + (x_1x_2x_3x_4) + (x_1x_3x_4x_5) \\
&\quad -(x_1x_2x_3x_4x_5) - (x_2x_5) + (x_2x_3x_4x_5) + (x_1x_2x_3x_5) - (x_1x_2x_3x_4x_5) + (x_1x_2x_4x_5) \\
&\quad -(x_1x_2x_3x_4x_5) - (x_1x_2x_3x_4x_5) + (x_1x_2x_3x_4x_5)] \\
&= (x_2x_3x_4) + (x_1x_3x_5) - (x_1x_2x_3x_4x_5) + (x_1x_4) - (x_1x_2x_3x_4) - (x_1x_3x_4x_5) \\
&\quad +(x_1x_2x_3x_4x_5) + (x_2x_5) - (x_2x_3x_4x_5) - (x_1x_2x_3x_5) + (x_1x_2x_3x_4x_5) - (x_1x_2x_4x_5) \\
&\quad +(x_1x_2x_3x_4x_5) + (x_1x_2x_3x_4x_5) - (x_1x_2x_3x_4x_5) \\
&= (x_1x_4) + (x_2x_5) + (x_2x_3x_4) + (x_1x_3x_5) - (x_1x_2x_3x_4) - (x_1x_3x_4x_5) - (x_2x_3x_4x_5) \\
&\quad -(x_1x_2x_3x_5) - (x_1x_2x_4x_5) + 2(x_1x_2x_3x_4x_5)
\end{aligned}$$

So, the reliability of complex network in Fig.1 is:

$$\begin{aligned}
R_s &= (x_1x_4) + (x_2x_5) + (x_2x_3x_4) + (x_1x_3x_5) - (x_1x_2x_3x_4) - (x_1x_3x_4x_5) - (x_2x_3x_4x_5) \\
&\quad -(x_1x_2x_3x_5) - (x_1x_2x_4x_5) + 2(x_1x_2x_3x_4x_5)
\end{aligned}$$

If $x_i \sim Exp \leftrightarrow f(t) = \lambda e^{-\lambda t}, \forall x_i, i = 1,2,3,4,5$

We get:

$R(t) = \int_t^{\infty} \lambda e^{-\lambda t} dt = 0 - \left(-e^{-\lambda t}\right) = e^{-\lambda t},$

Now,

$R_s(t) = e^{-(\lambda_1+\lambda_4)t} + e^{-(\lambda_2+\lambda_5)t} + e^{-(\lambda_2+\lambda_3+\lambda_4)t} + e^{-(\lambda_1+\lambda_3+\lambda_5)t} - e^{-(\lambda_1+\lambda_2+\lambda_3+\lambda_4)t} - e^{-(\lambda_1+\lambda_3+\lambda_4+\lambda_5)t}$

$$-e^{-(\lambda_2+\lambda_3+\lambda_4+\lambda_5)t}-e^{-(\lambda_1+\lambda_2+\lambda_3+\lambda_5)t}-e^{-(\lambda_1+\lambda_2+\lambda_4+\lambda_5)t}+2e^{-(\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5)t}$$

If $\lambda_1 = 0.50$, $\lambda_2 = 0.61$, $\lambda_3 = 0.72$, $\lambda_4 = 0.69$, $\lambda_5 = 0.75$, then we get:

$$R_s(t) = 2e^{-3.07t} - e^{-2.57t} - e^{-2.51t} - e^{-2.46t} - e^{-2.38t} - e^{-2.34t} + e^{-2.02t} + e^{-1.77t} + e^{-1.19t} + e^{-1.16t}$$

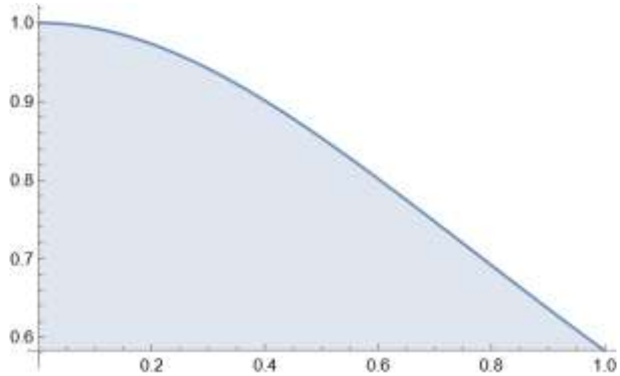

Fig. 2. Complex Network

4.3.2: Hazard Rate and Mean Time to Failure of Complex Network

The hazard rate function and MTTF for the complex network in Fig. 1 can also be found in this section and are expressed as:

$$f(t) = -\frac{dR(t)}{dt} = -\frac{d}{dt}(e^{-(\lambda_1+\lambda_4)t}+e^{-(\lambda_2+\lambda_5)t}+e^{-(\lambda_2+\lambda_3+\lambda_4)t}+e^{-(\lambda_1+\lambda_3+\lambda_5)t}-e^{-(\lambda_1+\lambda_2+\lambda_3+\lambda_4)t}$$
$$-e^{-(\lambda_1+\lambda_3+\lambda_4+\lambda_5)t}-e^{-(\lambda_2+\lambda_3+\lambda_4+\lambda_5)t}-e^{-(\lambda_1+\lambda_2+\lambda_3+\lambda_5)t}$$
$$-e^{-(\lambda_1+\lambda_2+\lambda_4+\lambda_5)t}+2e^{-(\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5)t})$$

$$f(t) = -e^{t(-\lambda_1-\lambda_4)}(-\lambda_1-\lambda_4)-e^{t(-\lambda_2-\lambda_3-\lambda_4)}(-\lambda_2-\lambda_3-\lambda_4)+e^{t(-\lambda_1-\lambda_2-\lambda_3-\lambda_4)}(-\lambda_1-\lambda_2-\lambda_3-\lambda_4)$$
$$-e^{t(-\lambda_2-\lambda_5)}(-\lambda_2-\lambda_5)-e^{t(-\lambda_1-\lambda_3-\lambda_5)}(-\lambda_1-\lambda_3-\lambda_5)+e^{t(-\lambda_1-\lambda_2-\lambda_3-\lambda_5)}(-\lambda_1-\lambda_2-\lambda_3-\lambda_5)$$
$$+e^{t(-\lambda_1-\lambda_2-\lambda_4-\lambda_5)}(-\lambda_1-\lambda_2-\lambda_4-\lambda_5)+e^{t(-\lambda_1-\lambda_3-\lambda_4-\lambda_5)}(-\lambda_1-\lambda_3-\lambda_4-\lambda_5)$$
$$+e^{t(-\lambda_2-\lambda_3-\lambda_4-\lambda_5)}(-\lambda_2-\lambda_3-\lambda_4-\lambda_5)-2e^{t(-\lambda_1-\lambda_2-\lambda_3-\lambda_4-\lambda_5)}(-\lambda_1-\lambda_2-\lambda_3-\lambda_4-\lambda_5)$$

So, the hazard rate function can be conclusion as:

$$h(t) = \frac{\begin{array}{c}-e^{t(-\lambda_1-\lambda_4)}(-\lambda_1-\lambda_4)-e^{t(-\lambda_2-\lambda_3-\lambda_4)}(-\lambda_2-\lambda_3-\lambda_4)-e^{t(-\lambda_2-\lambda_5)}(-\lambda_2-\lambda_5)\\ +e^{t(-\lambda_1-\lambda_2-\lambda_3-\lambda_4)}(-\lambda_1-\lambda_2-\lambda_3-\lambda_4)-e^{t(-\lambda_1-\lambda_3-\lambda_5)}(-\lambda_1-\lambda_3-\lambda_5)\\ +e^{t(-\lambda_1-\lambda_2-\lambda_3-\lambda_5)}(-\lambda_1-\lambda_2-\lambda_3-\lambda_5)+e^{t(-\lambda_1-\lambda_2-\lambda_4-\lambda_5)}(-\lambda_1-\lambda_2-\lambda_4-\lambda_5)\\ +e^{t(-\lambda_1-\lambda_3-\lambda_4-\lambda_5)}(-\lambda_1-\lambda_3-\lambda_4-\lambda_5)+e^{t(-\lambda_2-\lambda_3-\lambda_4-\lambda_5)}(-\lambda_2-\lambda_3-\lambda_4-\lambda_5)\\ -2e^{t(-\lambda_1-\lambda_2-\lambda_3-\lambda_4-\lambda_5)}(-\lambda_1-\lambda_2-\lambda_3-\lambda_4-\lambda_5)\end{array}}{\begin{array}{c}e^{-(\lambda_1+\lambda_4)t}+e^{-(\lambda_2+\lambda_5)t}+e^{-(\lambda_2+\lambda_3+\lambda_4)t}+e^{-(\lambda_1+\lambda_3+\lambda_5)t}-e^{-(\lambda_1+\lambda_2+\lambda_3+\lambda_4)t}\\ -e^{-(\lambda_1+\lambda_3+\lambda_4+\lambda_5)t}-e^{-(\lambda_2+\lambda_3+\lambda_4+\lambda_5)t}-e^{-(\lambda_1+\lambda_2+\lambda_3+\lambda_5)t}-e^{-(\lambda_1+\lambda_2+\lambda_4+\lambda_5)t}\\ +2e^{-(\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5)t}\end{array}}$$

$$h(t) = \lambda_1+\lambda_4+\lambda_2+\lambda_5+\lambda_1+\lambda_3+\lambda_5-\lambda_1-\lambda_2-\lambda_3-\lambda_4-\lambda_1-\lambda_3-\lambda_4-\lambda_5-\lambda_2-\lambda_3-\lambda_4$$
$$-\lambda_5-\lambda_1-\lambda_2-\lambda_4-\lambda_5+2\lambda_1+2\lambda_2+2\lambda_3+2\lambda_4+2\lambda_5$$

$$h(t) = \lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5 = 3.27$$

Now, we will calculate the MTTF

$$\text{MTTF} = \int_0^\infty R(t)dt = \int_0^\infty [e^{-(\lambda_1+\lambda_4)t}+e^{-(\lambda_2+\lambda_5)t}+e^{-(\lambda_2+\lambda_3+\lambda_4)t}+e^{-(\lambda_1+\lambda_3+\lambda_5)t}$$
$$-e^{-(\lambda_1+\lambda_2+\lambda_3+\lambda_4)t}-e^{-(\lambda_1+\lambda_3+\lambda_4+\lambda_5)t}-e^{-(\lambda_2+\lambda_3+\lambda_4+\lambda_5)t}$$

$$-e^{-(\lambda_1+\lambda_2+\lambda_3+\lambda_5)t} - e^{-(\lambda_1+\lambda_2+\lambda_4+\lambda_5)t} + 2e^{-(\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5)t}]$$

$$\mathrm{MTTF} = \frac{1}{\lambda_1+\lambda_4} + \frac{1}{\lambda_2+\lambda_3+\lambda_4} - \frac{1}{\lambda_1+\lambda_2+\lambda_3+\lambda_4} + \frac{1}{\lambda_2+\lambda_5} + \frac{1}{\lambda_1+\lambda_3+\lambda_5} - \frac{1}{\lambda_1+\lambda_2+\lambda_3+\lambda_5}$$

$$-\frac{1}{\lambda_1+\lambda_2+\lambda_4+\lambda_5} - \frac{1}{\lambda_1+\lambda_3+\lambda_4+\lambda_5} - \frac{1}{\lambda_2+\lambda_3+\lambda_4+\lambda_5} + \frac{2}{\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5}$$

$$\mathrm{MTTF} = 1.276$$

5. Conclosion

Essential concepts of reliability are examined. These include the following: probability, conditional probability, mutually exclusive events, hazard rate, failure rate, cumulative distribution function, probability density function, and mean time to failure (MTTF). Due to the fact that these ideas aid in determining the dependability of networks. The focus has been on examining the various failure rate types as we have attempted to comprehend the connections between these ideas in order to gain a clear understanding of how to compute the reliability of networks. A study is conducted on the statistical distributions that can be utilized to assess the dependability of networks.

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