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Applications AT transform in Differential Equations

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Abstract: *In this paper we used the AT transform to solve differential equations with constant coefficients. It has applications in a variety of fields including engineering (automatic control) and physics (nuclear physics).We proved some important properties that we used for this purpose.*

Keyword- AT integral transform, inverse of AT transform, Differential equations

1. Introduction

In solving differential equations, particular ones with constant coefficients, integral transformations are an effective mathematical technique. In order to solve differential equations, they convert them into algebraic equations, which are typically simpler. The solution to the original differential problem is obtained by applying the inverse transformation after the algebraic equation has been solved. Differential equations with constant coefficients can be simplified and solved with the use of integral transformation .In disciplines like physics, engineering, and others where differential equations are frequently encountered, these techniques are extensively employed. The inverse transformation returns the solution to the original domain once the transformed problem has been solved in the algebraic domain**.**

2.preliminary Results

Let f(ŧ) be a given function which is defined for all $ŧ \geq 1$ and $\mathfrak{v}$ parameter, then

$$AT\{f(ŧ)\} = \int_1^{\infty} ŧ^{-\frac{\mathfrak{v}+1}{\mathfrak{v}}} f(ŧ)dŧ$$

Considering $f = f(\ln ŧ)$ so, we define AT transform as $AT\big(f(\ln ŧ)\big) = \int_1^{\infty} ŧ^{-\frac{\mathfrak{v}+1}{\mathfrak{v}}} f(\ln ŧ)dŧ$.

3. properties of the AT transform:

3.1 (Linearity property)

The AT integral transform is a linear operator.

Proof:

Suppose that f(ln ŧ) and g(ln ŧ) are the function and a, b are real constant, then

$$AT\big(a\,f(ln\,ŧ) + b\,g(ln\,ŧ)\big) = \int_1^{\infty} (a\,f(ln\,ŧ) + b\,g(ln\,ŧ))\,ŧ^{-\frac{\mathfrak{v}+1}{\mathfrak{v}}} dŧ$$

$$= a\int_1^{\infty} f(ln\,ŧ)\cdot ŧ^{-\frac{\mathfrak{v}+1}{\mathfrak{v}}} dŧ + b\int_1^{\infty} g(ln\,ŧ)\cdot ŧ^{-\frac{\mathfrak{v}+1}{\mathfrak{v}}} dŧ$$

$$\therefore AT\big(af(ln\,ŧ) + bg(ln\,ŧ)\big) = aAT\big(f(ln\,ŧ)\big) + bAT\big(g(ln\,ŧ)\big) \ldots \quad (1)$$

3.2 (Inverse operator)

The inverse AT transform is also linear operator.

Proof:

If we take the inverse integral transform of the equation above (1) on both sides, we get $af(ln\,ŧ) + bg(ln\,ŧ) = (AT)^{-1}(aAT\big(f(ln\,ŧ)\big) + bAT\big(g(ln\,ŧ)\big)$

$af(ln\,ŧ) + bg(ln\,ŧ) = a(AT)^{-1}\left(AT\big(f(ln\,ŧ)\big) + b(AT)^{-1}(AT\big(g(ln\,ŧ)\big)\right)$

$af(ln\,ŧ) + bg(ln\,ŧ) = a(AT)^{-1}\big(F(ṽ)) + b(AT)^{-1}(G(ṽ))\big)$

$af(ln\,ŧ) + bg(ln\,ŧ) = (AT)^{-1}(aF(ṽ) + bG(ṽ))$

Where $AT\big(f(ln\,ŧ)\big) = F(ṽ)$ *and* $AT\big(g(ln\,ŧ)\big) = G(ṽ)$, respectively.

$\therefore F(ṽ) = \frac{1}{ṽ}E(ṽ)$.

3.3 The AT Integral Transform of Derivatives

This section gives the AT integral transform of the function $\psi(\ln ŧ)$ of derivatives.

Theorem (3.3.1):

If $\psi, \psi', \dots, \psi^{n-1}$ are continuous on $[1, \infty)$ and if $\psi^{n}\ (\ln ŧ)$ is piecewise continuous on $[1, \infty)$,then

$AT\left(\psi^{(n)}(ln\,ŧ)\right) = \left(\frac{1}{ṽ}\right)^{n} F(ṽ) - \left(\frac{1}{ṽ}\right)^{n-1}\psi(0) - \left(\frac{1}{ṽ}\right)^{n-2}\psi'(0) - \cdots - \psi^{(n-1)}(0)$

Where $F(ṽ) = AT\big(\psi(ln\,ŧ)\big)$

Proof:

1. $AT\big(\psi'(ln\,ŧ)\big) = \int_1^{\infty} \frac{\psi'(ln\,ŧ)}{ŧ} \cdot ŧ^{-\frac{ṽ+1}{ṽ}+1} dŧ$

using integrating by part

$= \lim_{b\to\infty}\left[ŧ^{-\frac{ṽ+1}{ṽ}}\psi(ln\,ŧ)\right]_1^{b} + \frac{1}{ṽ}\lim_{b\to\infty}\int_1^{b}\psi(ln\,ŧ)\cdot ŧ^{-\frac{ṽ+1}{ṽ}}dŧ$

$= \frac{1}{ṽ}F(ṽ) - \psi(0)$

2. $AT\big(\psi'(ln\,t)\big) = \int_1^{\infty} t^{-\frac{ṽ+1}{ṽ}+1}\cdot\frac{\psi''(ln\,t)}{t}dt$

$= \lim_{b\to\infty}\left[t^{-\frac{ṽ+1}{ṽ}+1}\psi'(ln\,t)\right]_1^{b} + \frac{1}{ṽ}\lim_{b\to\infty}\int_1^{b} t^{-\frac{ṽ+1}{ṽ}}\psi'(ln\,t)dt$

$= -\psi'(0) + \frac{1}{ṽ}\int_1^{b} t^{-\frac{ṽ+1}{ṽ}}\psi'(ln\,t)dt$

$= -\psi(0) + \frac{1}{ṽ}AT\big(\psi'(ln\,t)\big)$

Substituting the expression of $\psi'(\ln ŧ)$ into the equation above, we get

$AT\big(\psi''(ln\,ŧ)\big) = \frac{1}{ṽ^2}F(ṽ) - \frac{1}{ṽ}\psi(0) - \psi'(0)$.

3- $AT\big(\psi'''(ln\,ŧ)\big) = \int_1^{\infty} ŧ^{-\frac{ṽ+1}{ṽ}+1}\cdot\frac{\psi'''(ln\,ŧ)}{ŧ}dŧ$

$= \lim_{b\to\infty}\left[ŧ^{-\frac{ṽ+1}{ṽ}+1}\psi''(ln\,ŧ)\right]_1^{b} + \frac{1}{ṽ}\lim_{b\to\infty}\int_1^{b} ŧ^{-\frac{ṽ+1}{ṽ}}\psi''(ln\,ŧ)dŧ$

$= -\psi''(0) + \frac{1}{ṽ}AT\big(\psi''(ln\,ŧ)\big)$

$= \frac{1}{v^3}F(v) - \left(\frac{1}{v^2}\right)\psi(0) - \frac{1}{v}\psi'(0) - \psi''(0)$.

4- $AT\left(\psi^{IV}(ln\, t)\right) = \int_1^{\infty} t^{-\frac{v+1}{v}+1} \cdot \frac{\psi^{(u)}(ln\, t)}{t}\, dt$

$= \left[\lim_{b\to\infty} t^{-\frac{v+1}{v}+1}\psi'''(ln\, t)\right]_1^b + \frac{1}{v}\lim_{b\to\infty}\int_1^b t^{-\frac{v+1}{v}}\psi'''(ln\, t)dt$

$= -\psi'''(0) + \frac{1}{v}AT\left(\psi'''(ln\, t)\right)$

$= \frac{1}{v^4}F(v) - \frac{1}{v^3}\psi(0) - \frac{1}{v^2}\psi'(0) - \frac{1}{v}\psi''(0) - \psi'''(0)$.

By induction we get

$$AT\left(\psi^{(n)}(ln\, t)\right) = \left(\frac{1}{v}\right)^n F(v) - \left(\frac{1}{v}\right)^{n-1}\psi(0) - \left(\frac{1}{v}\right)^{n-2}\psi(0) - \cdots - \psi^{(n-1)}(0),$$

Where $F(v) = AT\left(\psi(ln\, t)\right)$.

4- Applications

In order to solve linear ordinary differential equations with constant coefficients equations, such as those involving electrical circuits, nuclear physics, pharmacokinetics, population growth, and decay problems, the AT transform was applied.

Example (4.1):Simple Electrical circuit

The simple electrical circuit consists of resistance R in (ohms), a capacitor C in (farads), an inductor L ,in henrys, an electromotive Power of voltage , E in (volts), and switch is also connected in the circuit, as shown in figure (1). he current in the circuit is measured in (Amperes) and the charge on the capacitor in (coulombs).

Then by Kirchhoff's Law, we have :

$L\frac{dI}{dt} + RI + \frac{Q}{C} = E$ ... (2)

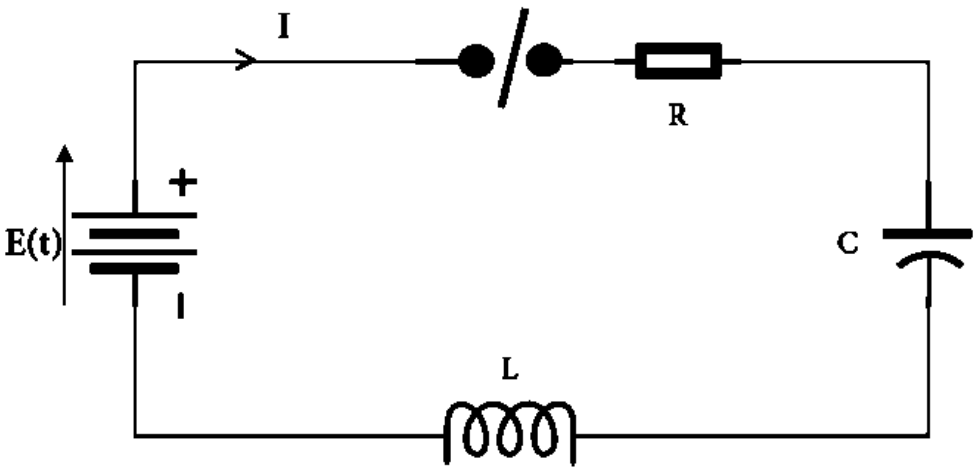

Figure (1): Electric circuit [6]

An Inductance of 0.5 Henry resistor of 25×10^{-1} ohms and a capacitor of $\frac{1}{3}$ farad are connected in series with an emf of 70 volts, $t \geq 0$ the charge on the capacitor and current in the circuit is $zero$, $find\ the\ charge\ and\ current\ at\ any\ time\ t > 0$..

Solution :

By substituting the given values into the equation (2), we obtain

$0.5\frac{d^2Q}{dt^2} + 2.5\frac{dQ}{dt} + 3Q = 70 \ldots \left(since\ I = \frac{dQ}{dt}\right)$

Then $\frac{d^2Q}{dt^2} + 5\frac{dQ}{dt} + 6Q = 140$

To solve the ODE let us consider that Q = ψ (lnt) so, the above equation becomes

$\psi'' + 5\psi' + 6\psi = 140,\ \psi = \psi(ln\,t),\ \psi(0) = \psi'(0) = 0$

Take (AT) integral transform of a above equation

$-\psi'(0) + \frac{1}{v}\psi(0) + \frac{1}{v^2}AT(\psi) - 5\big(\psi(0)\big) + \frac{5}{v}AT(\psi) + 6AT(\psi) = AT(140)$

$\left(\frac{1}{v^2} + \frac{5}{v} + 6\right)AT(\psi) = 140v$

$\left(\frac{1+5v+6v^2}{v^2}\right)AT\big(\psi(ln\,t)\big) = 140v$

$AT(\psi) = \frac{140v^3}{1+5v+6v^2} \Longrightarrow AT(\psi) = \frac{140v^3}{(1+2v)(1+3v)}$

Take $\frac{v}{(1+2v)(1+3v)} = \frac{A}{1+2v} + \frac{B}{1+3v} \Longrightarrow \frac{A+3Av+B+2Bv}{(1+2v)(1+3v)} \Longrightarrow A + B = 0, 3A + 2B = 1$

$A = 1, B = -1$

$\frac{v}{(1+2v)(1+3v)} = \frac{1}{1+2v} - \frac{1}{1+3v}$

$\frac{140v^3}{(1+2v)(1+3v)} = 140 \cdot \left(\frac{v^2}{1+2v} - \frac{v^2}{1+3v}\right) = 140 \cdot \left(\frac{v^2+v-v}{1+2v} - \frac{v^2+v-v}{1+3v}\right)$

$= 140 \cdot \left(\frac{v^2+v}{1+2v} - \frac{v}{1+2v} - \frac{v^2+v}{1+3v} + \frac{v}{1+3v}\right)$

$= \frac{140}{2}\left(\frac{2v(v+1)}{1+2v}\right) - 140\frac{v}{1+2v} - 140\left(\frac{2}{3}\right) \cdot \left(\frac{3v(v+1)}{2(1+3v)}\right) + 140\frac{v}{1+3v}\Big)$

By taking $(AT)^{-1}$

$= 70\,(t^{-2} + 1) - 140\,t^{-2} - \frac{280}{3}\left(t^{-3} + \frac{1}{2}\right) + 140t^{-3}$

$= 70\,t^{-2} + 70 - 140\,t^{-2} - \frac{280}{3}t^{-3} - \frac{140}{3} + 140t^{-3}$

$= -70\,t^{-2} + \frac{140}{3}\,t^{-3} + \frac{70}{3}$

$Q = \frac{140}{3t^3} - \frac{70}{t^2} + \frac{70}{3}$

And $I = \frac{dQ}{dt} = \frac{-140}{t^4} + \frac{140}{t^3}$

Example (4.2): Nuclear Physics

Using an AT transform gets a real form of radioactive decay. The study of the analytical components of nuclear physics is made easy by it.

The concepts presented in the following example come from nuclear physics. Consider the following linear differential equation of first order

$\frac{dy}{dt} = -\mathfrak{B}\psi$ ... (3)

represents the number of un decayed atoms remaining in a sample of radioactive isotope at time t and $\mathfrak{B}$ is the decay constant, [2,6].

We can write equation (3) as:

$\frac{d\psi}{dt} + \mathfrak{B}\psi = 0$ … (4)

Let $\psi(0) = \psi_0$, $\psi = \psi(\ln t)$, $0 \leq \ln t < \infty$, $1 \leq t < \infty$.

Taking (AT) transform on both sides,

$AT\left(\frac{d\psi}{dt}\right) + AT(\mathfrak{B}\psi) = AT(0)$

$-\psi(0) + \frac{1}{v}AT(\psi) + \mathfrak{B}AT(\psi) = 0$

$$\left(\frac{1}{v} + \mathfrak{B}\right)AT(\psi) = \psi_0$$

$\left(\frac{1+v\mathfrak{B}}{v}\right)AT\big(\psi(ln\, t)\big) = \psi_0$

$AT(\psi) = \frac{v\psi_0}{1+\mathfrak{B}v} = \frac{v}{1+\mathfrak{B}v}\psi_0$

Take inverse of (AT) to both sides , we get

$\psi(lnt) = t^{-\mathfrak{B}}\psi_0$

Example (4.3): problem of pharmacokinetics

Solution of the pharmacokinetics problem [8]

$\frac{dp}{dt} + \beta p = \frac{\alpha}{VOL}, t > 0$ ... (5)

Let $p = p(ln\, t)$, Then equation (5) become

$\frac{dp(ln\, t)}{dt} + \beta(p(ln\, t) = \frac{\alpha}{VOL}$ ... *(6)*

With $p(0) = 0$,

$Here\ p(t)$: $drug\ concentration\ in\ the\ blood\ at\ any\ time\ t$,

β: $elimination\ velocity\ constant$,

α: $infusion\ rate\ (in\ mg/min)$,

VOL: $volume\ in\ which\ drug\ is\ distributed$.

Now, AT transform of both sides of equation give:

$AT\left(\frac{dp(ln\, t)}{dt}\right) + AT\big(\beta(ln\, t)\big) = AT\left(\frac{\alpha}{VOL}\right)$

$-P(0)+\frac{1}{\mathfrak{v}}AT\big(p(ln\,\mathfrak{t})\big)+\beta AT\big(p(ln\,\mathfrak{t})\big)=\frac{\alpha}{VOL}\cdot\mathfrak{v}$

$\left(\frac{1}{\mathfrak{v}}+\beta\right)AT\big(p(ln\,\mathfrak{t})\big)=\frac{\alpha}{VOL}\cdot\mathfrak{v}$

$\left(\frac{1+\mathfrak{v}\beta}{\mathfrak{v}}\right)AT\big(p(ln\,\mathfrak{t})\big)=\left(\frac{\alpha}{VOL}\right)\cdot\mathfrak{v}$

$AT(p)=\frac{\alpha}{VOL}\cdot\frac{\mathfrak{v}^2}{1+\beta\mathfrak{v}}=\frac{\alpha}{VOL}\left(\frac{\mathfrak{v}^2+\mathfrak{v}-\mathfrak{v}}{1+\beta\mathfrak{v}}\right)$

$=\frac{\alpha}{VOL}\left(\frac{\mathfrak{v}(\mathfrak{v}+1)}{1+\beta\mathfrak{v}}-\frac{\mathfrak{v}}{1+\beta\mathfrak{v}}\right)$

$=\frac{\alpha}{VOL}\cdot\frac{-\beta+1}{-\beta}\left(\frac{-\beta}{-\beta+1}\cdot\frac{\mathfrak{v}(1+\mathfrak{v})}{1+\beta\mathfrak{v}}\right)-\frac{\alpha}{VOL}\left(\frac{\mathfrak{v}}{1+\beta\mathfrak{v}}\right)$

By taking $(AT)^{-1}$of both sides, we get

$p(lnt)=\frac{\alpha}{VOL}\left(\frac{-\beta+1}{-\beta}\left(\mathfrak{t}^{-\beta}-\frac{1}{-\beta+1}\right)-\frac{\alpha}{VOL}\mathfrak{t}^{-\beta}\right.$

$=\frac{\alpha}{VOL}\left(\left(\frac{\beta-1}{\beta}\mathfrak{t}^{-\beta}+\frac{1}{\beta}\right)-\mathfrak{t}^{-\beta}\right)$

$=\frac{\alpha}{VOL}\left(\left(\frac{\beta-1}{\beta}-1\right)\mathfrak{t}^{-\beta}+\frac{1}{\beta}\right)$

$p(lnt)=\frac{\alpha}{VOL}\left(\frac{-1}{\beta}\mathfrak{t}^{-\beta}+\frac{1}{\beta}\right)=\frac{\alpha}{VOL}\left(-\frac{1}{\beta}\mathfrak{t}^{-\beta}+\frac{1}{\beta}\right)$

This is the necessary drug concentration in the body at any given time during a continuous intravenous injection of the drug.

Example (4.4): problem of population Growth

A city's population increases at a rate that is proportionate to the total number of the people already living there .Calculate how many people were originally living in the city if after two years, the population has tripled and, after four years ,it is 40,000.

Estimate the initial population of the city, [1,6,7,11,12].

The above application can be expressed mathematically as:

$\frac{dg}{d\mathfrak{t}}=\mathfrak{D}\,g$ ... (7)

Let $g=g(\ln\mathfrak{t}),\ 0<\ln\mathfrak{t}<\infty$ and $1<\mathfrak{t}<\infty$

The equation (3.30) becomes :

$\frac{d\,g(\ln\mathfrak{t})}{d\mathfrak{t}}=\mathfrak{D}\,g(\ln\mathfrak{t})$ ... (8)

where the population of the city at any given time is counted it is denoted by $g\,(\ln\mathfrak{t})$ and $\mathfrak{D}$ is the proportionality constant.

The initial population number of the city at time $\mathfrak{t}=0$ is denoted by g_0.

Now, applying AT transform to equation (8), we get

$AT\frac{dg(\ln\mathfrak{t})}{d\mathfrak{t}}=\mathfrak{D}AT(g)$

$-g(0)+\frac{1}{\mathfrak{v}}AT(g)=\mathfrak{D}AT(g)$

$\left(\frac{1}{v} - \mathfrak{D}\right) \mathrm{AT}(g) = g_0$

$\left(\frac{1-\mathfrak{D}v}{v}\right) \mathrm{AT}(G) = g_0 \Longrightarrow \mathrm{AT}(g) = \frac{g_0 v}{(1-\mathfrak{D}v)}$

operating inverse AT transform on both sides equation, we have

$g(\ln t) = g_0 t^{\mathfrak{D}}$

or $g(t) = g_0 e^{\mathfrak{D}t}$ (9)

Now, at $t = 2$ (two years), $g = 3g_0$

$3g_0 = g_0 e^{2D}$

$2\mathfrak{D} = \ln 3 \Longrightarrow \mathfrak{D} = \frac{\ln 3}{2} \Longrightarrow \mathfrak{D} = 0.549$

Now, using at $t = 4$ (four years), $G = 40.000$ in equation (9), we obtain:

$40.000 = G_0\ (9.025) \Longrightarrow\ G_0 \approx 18181.9$

which required the initial population number of the city.

Example (4.5): problem of Decay

It is Known that radioactive material decays at a rate that is proportionate to its concentration, calculate the radioactive substance's half-life if there are 100 milligram of it initially and after six hours it is seen that the radioactive substance has lost thirty percent of its initial mass,[1,6,7,11,12].

This is problem can be expressed mathematically as :

$\frac{d\mathcal{M}}{dt} = -k\mathcal{M}$ ... (10)

Let $\mathcal{M} = \mathcal{M}(\ln t), 0 < \ln t < \infty\ , 1 < t < \infty$

The equation (10) becomes

$\frac{d\mathcal{M}(\ln t)}{dt} = -k\mathcal{M}(\ln t)$ ... (11)

where $\mathcal{M}$ is the amount of radioactive material at time t and k is the proportionality constant. Assume that at time $t = 0$, the initial amount of the radioactive material is represented by $\mathcal{M}_0$

Now, applying AT transform on both sides of (11) we have

$AT\left(\frac{d\mathcal{M}(ln\, t)}{d\, ln\, t}\right) = -k\, AT(\mathcal{M})$

$-\mathcal{M}(0) + \frac{1}{v} AT(\mathcal{M}) = -k\, AT(\mathcal{M})$

$\left(\frac{1}{v} + k\right) AT(\mathcal{M}) = \mathcal{M}(0)$

At $t = 0, \mathcal{M} = \mathcal{M}_0 = 100$ *(12)*

$AT(\mathcal{M}) = \frac{v}{1+kv} \cdot 100$ ... *(13)*

taking inverse to equation (13) , we get

$\mathcal{M}(ln\, t) = 100\, t^{-k}$

Or

$M\,(t) = 100e^{-kt}$ (14)

Now, at the time $t = 6$, there is 30% loss of the radioactive substance original mass 100 mg, so:

$\mathcal{M} = 100 - 30 = 70$, *using in equation (15) , we get*

$70 = 100\, e^{-kt}$

$0.7 = e^{-6k}$

$ln(0.7) = -6k$

$k = \frac{-ln\, 0.7}{-6} = 0.05944$

We required t when $\mathcal{M} = \frac{\mathcal{M}_0}{2} = \frac{100}{2} = 50$, *so from equation (16)*

$50 = 100\, e^{-0.05944t}$

$\frac{1}{2} = e^{-0.05944t}$

$ln(0.5) = -0.05944t$

$t = 11.661$ hours

Which is the required half-time of the radioactive substance.

Example (4.6): Automatic control of missiles

An example of an application of ordinary differential equations is automatic control of a moving missile that follows a moving object, like an aircraft. In this application are used ordinary differential equation to model the missile's motion and guide its trajectory. By solving these equations control systems can adjust the missile's flight path in real time to hit a target accurately or evade threats.

Suppose that a missile M is searching for a hostile aircraft if, at time t, the hostile aircraft turns via an angle φ(t), thes, in order for the missile M to catch and destroy the hostile aircraft, it must likewise turn through the same angle φ(t). The robotic control of the ordinary missile problems [4,5,9,11] can be represented by the second-order Linear differential equation, which is expressed as

$I\varphi''(t) + K\varphi(t) = K\alpha t$

Or

$\varphi''(t) + \frac{K}{I}\varphi(t) = \frac{k}{I}\alpha t$ (17)

Here φ(t) represents the aircraft's turning angle at any given time t, $\varphi''(t)$ represents the angular acceleration, αt is the assumed intended turning angle of the missile, where α is the missile's angular velocity, I represents the moment of inertia, and $k > 0$ is a constant [3,10]

Additionally, it is assumed that the beginning angular velocity is $\varphi'(0) = 0$,and the initiate turning angle is $\varphi(0) = 0$,

The AT of (13) provides

$-\varphi'(0) - \frac{1}{v}\varphi(0) + \frac{1}{v^2}AT(\varphi) + \frac{k}{I}AT(\varphi) = \frac{k}{I}\alpha\, AT(t)$ *(18)*

Set $\varphi'(0) = \varphi(0) = 0$ *, and simply (18), we obtain*

$$\left(\frac{1}{v^2} + \frac{k}{I}\right) AT(\varphi) = \frac{k}{I}\alpha \cdot \frac{v}{1-v}$$

$$\left(\frac{I+kv^2}{Iv^2}\right) AT(\varphi) = \frac{k}{I}\alpha \cdot \frac{v}{1-v}$$

$$AT(\varphi) = k\alpha \cdot \frac{v^3}{(1-v)(I+kv^2)}$$

Take $\frac{v^2}{(1-v)(I+kv^2)}$

$$\frac{A}{1-v} + \frac{Bv+C}{I+kv^2} \Rightarrow \frac{AI+Akv^2+Bv+C-Bv^2-Cv}{(1-v)(I+kv^2)}$$

$$AI + C = 0,\ B - C = 0,\ AK - B = 1$$

$$\Rightarrow A = \frac{1}{I+k},\ B = \frac{-I}{I+k},\ C = \frac{-I}{I+K}$$

Now, $\frac{v^2}{(1-v)(I+kv^2)} = \frac{1}{I+k} \cdot \frac{1}{1-v} - \frac{I}{I+k}\left(\frac{v+1}{I+kv^2}\right)$

$$= k.\alpha \cdot \frac{v^3}{(1-v)(I+kv^2)} = \frac{k.\alpha}{I+k} \cdot \frac{v}{1-v} - \frac{k\alpha I}{I+k} \cdot \frac{v+v^2}{I+kv^2}$$

$$= \frac{k.\alpha}{I+k} \cdot \frac{v}{1-v} - \frac{k\alpha}{I+k}\left[\frac{v}{1+\frac{k}{I}v^2} + \frac{v^2}{1+\frac{k}{I}v^2}\right]$$

Taking inverse AT, we get

$$\varphi(ln\, t) = \frac{k.\alpha}{I+k}\left[t - cos\sqrt{\frac{k}{I}}ln\, t - \sqrt{\frac{I}{k}}sin\sqrt{\frac{k}{I}}ln\, t\right]$$

Is the required turn angle of the missile.

Conclusion

The AT transformation has been used to solve some engineering and physical applications. The transformation laws have been studied for differential equations with constant coefficients.

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