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# Supra Topology and Rough Approximations Via Graphs in Generalized Approximation Space

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***Abstract :*** *In this paper, we introduce a new space called A-space, derived from the admixture vertex edges system on a graph G . We define the family of open sets in the A-space, which forms a supra topology on G . Furthermore, we present the definitions of key operators such as interior, closure, and boundary within this framework. We extend our work to introduce the concept of a generalized approximation space on graphs, defining the lower and upper approximations. Additionally, we discuss methods for calculating the accuracy of sub graphs, demonstrating the applicability of these structures in graph theory.*

**Keywords : graph , supra topology, lower and upper approximations, accuracy.**

## 1. INTRODUCTION :

Mostly used in discrete mathematics, graph theory is an imp ortant and fascinating branch of mathematics for two reasons . The graph has a pleasing mathematical appearance. Despit e being basic relation graphs, they can be used to represent d iverse mathematical graphs, such as harmonic objects and to pographic space. Secondly, graphs will be very useful in pra ctice when numerous concepts are empirically represented b y them. What topological graph theory is about [1, 2, 3, 4, 5, 8, and 9]

Rephrase are a branch of mathematics with a wide range of theoretical and practical applications. In order to bridge the gap between topology and applications, we believe that topological graph structure will be crucial. We consult Harary [6] for all graph theory jargon and nomenclature, and Moller [7] for all topology language and notation . A few fundamental ideas of graph theory [10] are introduced. An undirected graph $\mathfrak{U} = (\mathbb{V}(\mathfrak{U}), \mathfrak{Z}(\mathfrak{U}))$ is a graph where edges have no direction. It consists of a set of vertices $\mathbb{V}(\mathfrak{U})$ and a set of edges $\mathfrak{Z}(\mathfrak{U})$ , where each edge is an unordered pair (u,v) of vertices. A loop is an edge in a graph that connects a vertex to itself.. A star graph $S_n$ is a tree with one central vertex connected to all other vertices, which are leaves. It has n vertices ($n \geq 2$) and n-1 edges. An antisymmetric graph, let $\mathfrak{U} = (\mathbb{V}(\mathfrak{U}), \mathfrak{Z}(\mathfrak{U}))$ be a graph if $(\breve{\omega}, \tilde{u}) \in \mathfrak{Z}(\mathfrak{U})$ and$(\tilde{u}, \breve{\omega}) \in \mathfrak{Z}(\mathfrak{U})$ implies
$\tilde{u} = \breve{\omega}$ Then $\mathfrak{U}$ is called antisymmetric graph.

An approximation space is a mathematical structure used in rough set theory, defined as a pair (S,R) , where: S is a non-empty set called the universe of discourse. R is an equivalence relation on S , which partitions S into disjoint equivalence classes. The approximation space is used to analyze uncertainty by approximating subsets of S through lower and upper approximations based on the relation R. A Generalized Approximation Space is a structure (G,T), where G is a universal set, and T is a generalized structure such as a relation supra topology used to define lower and upper approximations of subsets of G , enabling flexible and broad applications beyond traditional equivalence based approximation spaces.

## 2. A-space, I-space and N-space

**Definition 2.1.** The undirected graph $\mathfrak{U} = (\mathbb{V}(\mathfrak{U}), \mathfrak{Z}(\mathfrak{U}))$ is amathematical structure cnsisting of two sets: the first $\mathbb{V}(\mathfrak{U})$ is a non-empty set represents points (called vertices). The second set $\mathfrak{Z}(\mathfrak{U})$) represents binary relations between these points (called edges).

**Definition 2.2.** Let $\mathfrak{Z}(\mathfrak{U})$ be a non-empty set and let $\mathcal{T}_{\mathfrak{h}_a}$be a collection of open subsets of $\mathfrak{U}$. The pair $(\mathfrak{U}, \mathcal{T}_{\mathfrak{h}_a})$ is called supra topological space if the folloowing conditions are met : 1⟩ $\phi$ , $\mathfrak{Z}(\mathfrak{U}) \in \mathcal{T}_{\mathfrak{h}_a}$ . 2⟩ If $\{S_i : i \in I\} \in \mathcal{T}_{\mathfrak{h}_a}$ then $\cup_i S_i \in \mathcal{T}_{\mathfrak{h}_a}$.

**Definition 2.3.** Let $\mathfrak{U} = (\mathbb{V}(\mathfrak{U}), \mathfrak{Z}(\mathfrak{U}))$ be a graph and a vertex $\breve{\omega} \in \mathbb{V}(\mathfrak{U})$ then:

a) The incidence vertex edges set of $\breve{\omega}$ is denoted by INVE($\breve{\omega}$) and defined by: INVE$(\breve{\omega}) = \{\ddot{\imath} \in \mathfrak{Z}(\mathfrak{U}) : \ddot{\imath} = (\breve{\omega}, u)$ for some $u \in \mathbb{V}(\mathfrak{U})\}$ .

b) The non-incidence vertex edges set of $\breve{\omega}$ is denoted by NINVE($\breve{\omega}$) and defined by: NINVE($\breve{\omega}$) $= \{\ddot{\imath} \in \mathfrak{Z}(\mathfrak{U}) : \ddot{\imath} = (u, r)$and $u , r \neq \breve{\omega}$ for some $u, r \in \mathbb{V}(\mathfrak{U})\}$.

**Definition 2.4.** Let $\mathfrak{U} = (\mathbb{V}(\mathfrak{U}), \mathfrak{Z}(\mathfrak{U}))$ be a graph , then the incidence vertex edges system (resp. non-incidence vertex edges system) of a vertex $\breve{\omega} \in \mathbb{V}(\mathfrak{U})$ is denoted by INVES($\breve{\omega}$) (resp. NINVES($\breve{\omega}$)) and defined by : INVES($\breve{\omega}$) = {INVE($\breve{\omega}$)} (resp. NINVES($\breve{\omega}$) = {NINVE($\breve{\omega}$)}) .

**Example 2.5.** Let $\mathfrak{U} = (\mathbb{V}(\mathfrak{U}), \mathfrak{Z}(\mathfrak{U}))$ be an graph such that $\mathbb{V}(\mathfrak{U}) = \{\breve{\omega}_1, \breve{\omega}_2, \breve{\omega}_3, \breve{\omega}_4, \breve{\omega}_5\}$ ,

$\mathfrak{Z}(\mathfrak{U}) = \{\ddot{\iota}_1 = (\ddot{\omega}_1, \ddot{\omega}_1), \ddot{\iota}_2 = (\ddot{\omega}_1, \ddot{\omega}_2), \ddot{\iota}_3 = (\ddot{\omega}_2, \ddot{\omega}_4), \ddot{\iota}_4 = (\ddot{\omega}_2, \ddot{\omega}_5), \ddot{\iota}_5 = (\ddot{\omega}_3, \ddot{\omega}_4), \ddot{\iota}_6 = (\ddot{\omega}_3, \ddot{\omega}_5), \ddot{\iota}_7 = (\ddot{\omega}_4, \ddot{\omega}_5)\}$

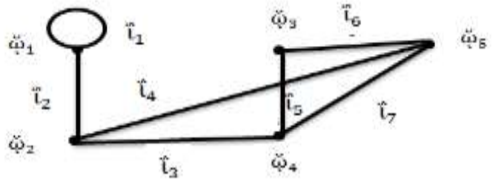

Figure 2.1. graph $\mathfrak{U}$ given in Example (2.5).

Then $INVE(\ddot{\omega}_1) = \{\ddot{\iota}_1, \ddot{\iota}_2\}$, $INVE(\ddot{\omega}_2) = \{\ddot{\iota}_2, \ddot{\iota}_3, \ddot{\iota}_4\}$, $INVE(\ddot{\omega}_3) = \{\ddot{\iota}_5, \ddot{\iota}_6\}$, $INVE(\ddot{\omega}_4) = \{\ddot{\iota}_3, \ddot{\iota}_5, \ddot{\iota}_7\}$, $INVE(\ddot{\omega}_5) = \{\ddot{\iota}_4, \ddot{\iota}_6, \ddot{\iota}_7\}$ . and
$INVES(\ddot{\omega}_1) = \{\{\ddot{\iota}_1, \ddot{\iota}_2\}\}$, $INVES(\ddot{\omega}_2) = \{\{\ddot{\iota}_2, \ddot{\iota}_3, \ddot{\iota}_4\}\}$, $INVES(\ddot{\omega}_3) = \{\{\ddot{\iota}_5, \ddot{\iota}_6\}\}$,
$INVES(\ddot{\omega}_4) = \{\{\ddot{\iota}_3, \ddot{\iota}_5, \ddot{\iota}_7\}\}$, $INVES(\ddot{\omega}_5) = \{\{\ddot{\iota}_4, \ddot{\iota}_6, \ddot{\iota}_7\}\}$ .
Also, we have
$NINVE(\ddot{\omega}_1) = \{\ddot{\iota}_3, \ddot{\iota}_4, \ddot{\iota}_5, \ddot{\iota}_6, \ddot{\iota}_7\}$, $NINVE(\ddot{\omega}_2) = \{\ddot{\iota}_1, \ddot{\iota}_5, \ddot{\iota}_6, \ddot{\iota}_7\}$, $NINVE(\ddot{\omega}_3) = \{\ddot{\iota}_1, \ddot{\iota}_2, \ddot{\iota}_3, \ddot{\iota}_4, \ddot{\iota}_7\}$, $NINVE(\ddot{\omega}_4) = \{\ddot{\iota}_1, \ddot{\iota}_2, \ddot{\iota}_4, \ddot{\iota}_6\}$, $NINVE(\ddot{\omega}_5) = \{\ddot{\iota}_1, \ddot{\iota}_2, \ddot{\iota}_3, \ddot{\iota}_4\}$ . and
$NINVES(\ddot{\omega}_1) = \{\{\ddot{\iota}_3, \ddot{\iota}_4, \ddot{\iota}_5, \ddot{\iota}_6, \ddot{\iota}_7\}\}$,
$NINVES(\ddot{\omega}_2) = \{\{\ddot{\iota}_1, \ddot{\iota}_5, \ddot{\iota}_6, \ddot{\iota}_7\}\}$, $NINVES(\ddot{\omega}_3) = \{\{\ddot{\iota}_1, \ddot{\iota}_2, \ddot{\iota}_3, \ddot{\iota}_4, \ddot{\iota}_7\}\}$, $NINVES(\ddot{\omega}_4) = \{\{\ddot{\iota}_1, \ddot{\iota}_2, \ddot{\iota}_4, \ddot{\iota}_6\}\}$, $NINVES(\ddot{\omega}_5) = \{\{\ddot{\iota}_1, \ddot{\iota}_2, \ddot{\iota}_3, \ddot{\iota}_5\}\}$.

**Definition 2.6.** Let $\mathfrak{U} = (\mathbb{V}(\mathfrak{U}), \mathfrak{Z}(\mathfrak{U}))$ be a graph The Admixture vertex edges system of a vertex $\ddot{\omega} \in \mathbb{V}(\mathfrak{U})$ is denoted by $AVES(\ddot{\omega})$ and defined by : $AVES(\ddot{\omega}) = \{INVES(\ddot{\omega}), NINVES(\ddot{\omega})\}$ .

**Definition 2.7.** Let $\mathfrak{U} = (\mathbb{V}(\mathfrak{U}), \mathfrak{Z}(\mathfrak{U}))$ be a graph the admixture vertex edges of a vertex $\ddot{\omega} \in \mathbb{V}(\mathfrak{U})$ is denoted by $AVE(\ddot{\omega})$ such that $AVE(\ddot{\omega}) \in AVES(\ddot{\omega})$ .

**Example 2.8.** According to Example(2.4) , the Admixture vertex edges systems are given by :
$AVES(\ddot{\omega}_1) = \{\{\ddot{\iota}_1, \ddot{\iota}_2\}, \{\ddot{\iota}_3, \ddot{\iota}_4, \ddot{\iota}_5, \ddot{\iota}_6, \ddot{\iota}_7\}\}$,
$AVES(\ddot{\omega}_2) = \{\{\ddot{\iota}_2, \ddot{\iota}_3, \ddot{\iota}_4\}, \{\ddot{\iota}_1, \ddot{\iota}_5, \ddot{\iota}_6, \ddot{\iota}_7\}\}$
$AVES(\ddot{\omega}_3) = \{\{\ddot{\iota}_5, \ddot{\iota}_6\}, \{\ddot{\iota}_1, \ddot{\iota}_2, \ddot{\iota}_3, \ddot{\iota}_4, \ddot{\iota}_7\}\}$,
$AVES(\ddot{\omega}_4) = \{\{\ddot{\iota}_3, \ddot{\iota}_5, \ddot{\iota}_7\}, \{\ddot{\iota}_1, \ddot{\iota}_2, \ddot{\iota}_4, \ddot{\iota}_6\}\}$,
$AVES(\ddot{\omega}_5) = \{\{\ddot{\iota}_4, \ddot{\iota}_6, \ddot{\iota}_7\}, \{\ddot{\iota}_1, \ddot{\iota}_2, \ddot{\iota}_3, \ddot{\iota}_5\}\}$ .

**Definition 2.9.** Let $\mathfrak{U} = (\mathbb{V}(\mathfrak{U}), \mathfrak{Z}(\mathfrak{U}))$ be a graph. and suppose that $\mathfrak{h}_a: \mathbb{V}(\mathfrak{U}) \to p\left(p(\mathfrak{Z}(\mathfrak{U}))\right)$ is a mapping which assigns for each $\ddot{\omega}$ in $\mathbb{V}(\mathfrak{U})$ its admixture vertex edges system in $p\left(p(\mathfrak{Z}(\mathfrak{U}))\right)$ . The pair $(\mathfrak{U}, \mathfrak{h}_a)$ is called the A-space.

**Example 2.10.** According to Example (2.4) , the mapping $\mathfrak{h}_a$ is given by:
$\mathfrak{h}_a(\ddot{\omega}_1) = \{\{\ddot{\iota}_1, \ddot{\iota}_2\}, \{\ddot{\iota}_3, \ddot{\iota}_4, \ddot{\iota}_5, \ddot{\iota}_6, \ddot{\iota}_7\}\}$,
$\mathfrak{h}_a(\ddot{\omega}_2) = \{\{\ddot{\iota}_2, \ddot{\iota}_3, \ddot{\iota}_4\}, \{\ddot{\iota}_1, \ddot{\iota}_5, \ddot{\iota}_6, \ddot{\iota}_7\}\}$,
$\mathfrak{h}_a(\ddot{\omega}_3) = \{\{\ddot{\iota}_5, \ddot{\iota}_6\}, \{\ddot{\iota}_1, \ddot{\iota}_2, \ddot{\iota}_3, \ddot{\iota}_4, \ddot{\iota}_7\}\}$,
$\mathfrak{h}_a(\ddot{\omega}_4) = \{\{\ddot{\iota}_3, \ddot{\iota}_5, \ddot{\iota}_7\}, \{\ddot{\iota}_1, \ddot{\iota}_2, \ddot{\iota}_4, \ddot{\iota}_6\}\}$,
$\mathfrak{h}_a(\ddot{\omega}_5) = \{\{\ddot{\iota}_4, \ddot{\iota}_6, \ddot{\iota}_7\}, \{\ddot{\iota}_1, \ddot{\iota}_2, \ddot{\iota}_3, \ddot{\iota}_5\}\}$.
Therefore $(\mathfrak{U}, \mathfrak{h}_a)$ is an A-space.

**Definition 2.11.** Let $\mathfrak{U} = \left(\mathbb{V}(\mathfrak{U}), \mathfrak{Z}(\mathfrak{U})\right)$ be a graph and suppose that $\mathfrak{h}_i: \mathbb{V}(\mathfrak{U}) \longrightarrow p\left(p(\mathfrak{Z}(\mathfrak{U}))\right)$ (resp. $\mathfrak{h}_n: \mathbb{V}(\mathfrak{U}) \longrightarrow p\left(p(\mathfrak{Z}(\mathfrak{U}))\right)$ is a mapping which assigns for each $\ddot{\omega}$ in $\mathbb{V}(\mathfrak{U})$ it's incidence (resp. non incidence) vertex edges system in $p\left(p(\mathfrak{Z}(\mathfrak{U}))\right)$. The pair $(\mathfrak{U}, \mathfrak{h}_i)$(resp. $(\mathfrak{U}, \mathfrak{h}_n)$) is called an I-space (resp. N-space).

**Definition 2.12.** Let $(\mathfrak{U}, \mathfrak{h}_a)$ be an A-space , $(\mathfrak{U}, \mathfrak{h}_i)$ be an I-space and $(\mathfrak{U}, \mathfrak{h}_n)$ be an N-space and let $\mathfrak{X} \subseteq \mathfrak{U}$. Then :

(1) The a-derived , i-derived and n-derived of an graph $\mathfrak{X}$ are defined respectively by :

$$[\mathfrak{Z}(\mathfrak{X})]_a^{`} = \left\{ \begin{matrix} \ddot{\iota} \in \mathfrak{Z}(\mathfrak{U}); \forall AVE(\ddot{\omega}) \text{ where } \ddot{\omega} \text{ incidence on } \ddot{\iota} \\ , AVE(\ddot{\omega}) \cap (\mathfrak{Z}(\mathfrak{X}) - \{\ddot{\iota}\}) \neq \phi \end{matrix} \right\}$$

$$[\mathfrak{Z}(\mathfrak{X})]_i^{`} = \left\{ \begin{matrix} \ddot{\iota} \in \mathfrak{Z}(\mathfrak{U}); INVE(\ddot{\omega}) \cap (\mathfrak{Z}(\mathfrak{X}) - \{\ddot{\iota}\}) \neq \phi, \\ \text{where } \ddot{\omega} \text{ incidence on } \ddot{\iota} \end{matrix} \right\}.$$

$$[\mathfrak{Z}(\mathfrak{X})]_n^{`} = \left\{ \begin{matrix} \ddot{\iota} \in \mathfrak{Z}(\mathfrak{U}); NINVE(\ddot{\omega}) \cap (\mathfrak{Z}(\mathfrak{X}) - \{\ddot{\iota}\}) \neq \phi, \\ \text{where } \ddot{\omega} \text{ incidence on } \ddot{\iota} \end{matrix} \right\}.$$

(2) The families of a-closed , i-closed and n-closed of an graph $\mathfrak{X}$ in A-space, I-space and N-space are defined respectively by:

$$\dot{F}_{\mathfrak{h}_a} = \{\mathfrak{Z}(\mathfrak{X}) \subseteq \mathfrak{Z}(\mathfrak{U});\ [\mathfrak{Z}(\mathfrak{X})]_a^{`} \subseteq \mathfrak{Z}(\mathfrak{X})\}.$$

$$\dot{F}_{\mathfrak{h}_i} = \{\mathfrak{Z}(\mathfrak{X}) \subseteq \mathfrak{Z}(\mathfrak{U});\ [\mathfrak{Z}(\mathfrak{X})]_i^{`} \subseteq \mathfrak{Z}(\mathfrak{X})\}.$$

$$\dot{F}_{\mathfrak{h}_n} = \{\mathfrak{Z}(\mathfrak{X}) \subseteq \mathfrak{Z}(\mathfrak{U});\ [\mathfrak{Z}(\mathfrak{X})]_n^{`} \subseteq \mathfrak{Z}(\mathfrak{X})\}.$$

(3) The families of a-open, i-open and n-open of an graph $\mathfrak{X}$ in A-space, I-space and N-space are defined respectively by :

$Ƭ_{\mathfrak{h}_a} = \{\mathfrak{Z}(S) \subseteq \mathfrak{Z}(\mathfrak{U}); \mathfrak{Z}(S) = \mathfrak{Z}(\mathfrak{U}) - \mathfrak{Z}(\mathfrak{X})$ such that $\mathfrak{Z}(\mathfrak{X}) \in \dot{F}_{\mathfrak{h}_a}\}$.

$Ƭ_{\mathfrak{h}_i} = \{\mathfrak{Z}(S) \subseteq \mathfrak{Z}(\mathfrak{U}); \mathfrak{Z}(S) = \mathfrak{Z}(\mathfrak{U}) - \mathfrak{Z}(\mathfrak{X})$ such that $\mathfrak{Z}(\mathfrak{X}) \in \dot{F}_{\mathfrak{h}_i}\}$.

$Ƭ_{\mathfrak{h}_n} = \{\mathfrak{Z}(S) \subseteq \mathfrak{Z}(\mathfrak{U}); \mathfrak{Z}(S) = \mathfrak{Z}(\mathfrak{U}) - \mathfrak{Z}(\mathfrak{X})$ such that $\mathfrak{Z}(\mathfrak{X}) \in \dot{F}_{\mathfrak{h}_n}\}$.

(4) The a-interior, i-interior and n-interior of an graph $\mathfrak{X}$ are defined respectively by :

$$Int_a(\mathfrak{Z}(\mathfrak{X})) = \cup \{\mathfrak{Z}(S) \in Ƭ_{\mathfrak{h}_a}; \mathfrak{Z}(S) \subseteq \mathfrak{Z}(\mathfrak{X})\}.$$

$$Int_i(\mathfrak{Z}(\mathfrak{X})) = \cup \{\mathfrak{Z}(S) \in Ƭ_{\mathfrak{h}_i}; \mathfrak{Z}(S) \subseteq \mathfrak{Z}(\mathfrak{X})\}.$$

$$Int_n(\mathfrak{Z}(\mathfrak{X})) = \cup \{\mathfrak{Z}(S) \in Ƭ_{\mathfrak{h}_n}; \mathfrak{Z}(S) \subseteq \mathfrak{Z}(\mathfrak{X})\}.$$

(5) The a-closure, i-closure and n-closure of an graph $\mathfrak{X}$ are defined respectively by :

$$Cl_a(\mathfrak{Z}(\mathfrak{X})) = \cap \{\mathfrak{Z}(K) \in \dot{F}_{\mathfrak{h}_a}; \mathfrak{Z}(\mathfrak{X}) \subseteq \mathfrak{Z}(K)\}.$$

$$Cl_i(\mathfrak{Z}(\mathfrak{X})) = \cap \{\mathfrak{Z}(K) \in \dot{F}_{\mathfrak{h}_i}; \mathfrak{Z}(\mathfrak{X}) \subseteq \mathfrak{Z}(K)\}.$$

$$Cl_n(\mathfrak{Z}(\mathfrak{X})) = \cap \{\mathfrak{Z}(K) \in \dot{F}_{\mathfrak{h}_n}; \mathfrak{Z}(\mathfrak{X}) \subseteq \mathfrak{Z}(K)\}.$$

(6) The a-boundary, i-boundary and n-boundary of an graph $\mathfrak{X}$ are defined respectively by :

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$$[\mathfrak{Z}(\mathfrak{X})]_a^b = Cl_a(\mathfrak{Z}(\mathfrak{X})) - Int_a(\mathfrak{Z}(\mathfrak{X})).$$
$$[\mathfrak{Z}(\mathfrak{X})]_i^b = Cl_i(\mathfrak{Z}(\mathfrak{X})) - Int_i(\mathfrak{Z}(\mathfrak{X})).$$
$$[\mathfrak{Z}(\mathfrak{X})]_n^b = Cl_n(\mathfrak{Z}(\mathfrak{X})) - Int_n(\mathfrak{Z}(\mathfrak{X})).$$

**Theorem 2.13.** Let $(\mathfrak{U}, \mathfrak{h}_a)$ be an A-space and $\mathfrak{X} \subseteq \mathfrak{U}$, then $\mathfrak{X}$ is an a-open if and only if it contains at least one admixture vertex edges of $\breve{\omega} \in V(\mathfrak{X})$ incidence on $\ddot{\imath}$ for each $\ddot{\imath} \in \mathfrak{Z}(\mathfrak{X})$.

**proof .** Let $(\mathfrak{U}, \mathfrak{h}_a)$ be an A-space and $\mathfrak{X}$ be an a-open graph contained in $\mathfrak{U}$ and $\ddot{\imath} \in \mathfrak{Z}(\mathfrak{X})$ . Suppose that for each admixture vertex edges of $\breve{\omega} \in V(\mathfrak{X})$ incidence on $\ddot{\imath}$, $AVE(\breve{\omega}) \nsubseteq \mathfrak{Z}(\mathfrak{X})$ where $\breve{\omega} \in V(\mathfrak{X})$
incidence on $\ddot{\imath}$ for each $\ddot{\imath} \in \mathfrak{Z}(\mathfrak{X})$. $\Rightarrow AVE(\breve{\omega}) \cap [\mathfrak{Z}(\mathfrak{U}) - \mathfrak{Z}(\mathfrak{X})] \neq \phi$
where $\breve{\omega}$ incidence on $\ddot{\imath} \in \mathfrak{Z}(\mathfrak{X}) \Rightarrow \ddot{\imath} \in [\mathfrak{Z}(\mathfrak{U}) - \mathfrak{Z}(\mathfrak{X})]_a^{`}$ . Since $\mathfrak{Z}(\mathfrak{X})$ is a-open $\therefore \mathfrak{Z}(\mathfrak{U}) - \mathfrak{Z}(\mathfrak{X})$ is a-closed, so
$[\mathfrak{Z}(\mathfrak{U}) - \mathfrak{Z}(\mathfrak{X})]_a^{`} \subseteq [\mathfrak{Z}(\mathfrak{U}) - \mathfrak{Z}(\mathfrak{X})]$
thus $\ddot{\imath} \in [\mathfrak{Z}(\mathfrak{U}) - \mathfrak{Z}(\mathfrak{X})]$. Therefore $\ddot{\imath} \notin \mathfrak{Z}(\mathfrak{X})$ which contradicts with $\ddot{\imath} \in \mathfrak{Z}(\mathfrak{X})$ and
consequently if $\mathfrak{X} \subseteq \mathfrak{U}$ is a-open and $\ddot{\imath} \in \mathfrak{Z}(\mathfrak{X})$, then at least one admixture vertex edges of $\breve{\omega} \in V(\mathfrak{X})$ incidence on $\ddot{\imath}$ for each $\ddot{\imath} \in \mathfrak{Z}(\mathfrak{X})$ which is contained in $\mathfrak{Z}(\mathfrak{X})$. **Conversely,** let $\mathfrak{X}$ contains at least one admixture vertex edges of $\breve{\omega} \in V(\mathfrak{X})$ incidence on $\ddot{\imath}$ for each $\ddot{\imath} \in \mathfrak{Z}(\mathfrak{X})$

Let $\varrho \in [\mathfrak{Z}(\mathfrak{U}) - \mathfrak{Z}(\mathfrak{X})]_a^{`}$
$\Rightarrow AVE(u) \cap [(\mathfrak{Z}(\mathfrak{U}) - \mathfrak{Z}(\mathfrak{X})) - \{\varrho\}] \neq \phi$
where $u$ icidence on $\varrho$ then $\varrho \notin \mathfrak{Z}(\mathfrak{X})$.If $\varrho \in \mathfrak{Z}(\mathfrak{X})$ there would be an admixture vertex edges of $u$, $AVE(u)$, such that $AVE(u) \subseteq \mathfrak{Z}(\mathfrak{X})$ and this would imply that $AVE(u) \cap [(\mathfrak{Z}(\mathfrak{U}) - \mathfrak{Z}(\mathfrak{X})) - \{\varrho\}] = \phi$, $\therefore \varrho \notin \mathfrak{Z}(\mathfrak{X}) \Rightarrow \varrho \in \mathfrak{Z}(\mathfrak{U}) - \mathfrak{Z}(\mathfrak{X})$ thus $[\mathfrak{Z}(\mathfrak{U}) - \mathfrak{Z}(\mathfrak{X})]_a^{`} \subseteq [\mathfrak{Z}(\mathfrak{U}) - \mathfrak{Z}(\mathfrak{X})]$. Hence $\mathfrak{Z}(\mathfrak{U}) - \mathfrak{Z}(\mathfrak{X})$ is a-closed and $\mathfrak{Z}(\mathfrak{X})$ is a-open.

**Theorem 2.14.** Let $(\mathfrak{U}, \mathfrak{h}_i)$ (resp. $(\mathfrak{U}, \mathfrak{h}_n)$) be is an I-space (resp. N-space) and $\mathfrak{X} \subseteq \mathfrak{U}$, then $\mathfrak{X}$ is an i-open (resp. n-open) if and only if it contains at least one incidence vertex edges (resp. non incidence vertex edges) of $\breve{\omega} \in V(\mathfrak{X})$ incidence on $\ddot{\imath}$ for each $\ddot{\imath} \in \mathfrak{Z}(\mathfrak{X})$.

**proof .** we use the same method mentioned above

**proposition 2.15.** Let $ß = (\mathbb{V}(\mathfrak{U}), \mathfrak{Z}(\mathfrak{U}))$ be a generalized approximation space, then the following classes:

$$\mho_{\mathfrak{h}_a} = \left\{ \begin{matrix} \mathfrak{X} \subseteq \mathfrak{U};\ \forall\, \ddot{\imath} \in \mathfrak{Z}(\mathfrak{X})\ \exists\, \breve{\omega} \in V(\mathfrak{X}) \text{ incidence on } \ddot{\imath} \\ \text{such that } AVE(\breve{\omega}) \subseteq \mathfrak{Z}(\mathfrak{X}) \end{matrix} \right\}$$

$$\mho_{\mathfrak{h}_i} = \left\{ \begin{matrix} \mathfrak{X} \subseteq \mathfrak{U};\ \forall\, \ddot{\imath} \in \mathfrak{Z}(\mathfrak{X})\ \exists\, \breve{\omega} \in V(\mathfrak{X}) \text{ incidence on } \ddot{\imath} \\ \text{such that } INVE(\breve{\omega}) \subseteq \mathfrak{Z}(\mathfrak{X}) \end{matrix} \right\}$$

$$(\text{resp. } \mho_{\mathfrak{h}_n} = \left\{ \begin{matrix} \mathfrak{X} \subseteq \mathfrak{U};\ \forall\, \ddot{\imath} \in \mathfrak{Z}(\mathfrak{X})\ \exists\, \breve{\omega} \in V(\mathfrak{X}) \text{ incidence on } \ddot{\imath} \\ \text{such that } NINVE(\breve{\omega}) \subseteq \mathfrak{Z}(\mathfrak{X}) \end{matrix} \right\}).$$

Are supra topologies on $\mathfrak{U}$.

**Theorem 2.16.** Let $ß = (\mathbb{V}(\mathfrak{U}), \mathfrak{Z}(\mathfrak{U}))$ be a generalized approximation space and $e$ is isolated edge then :
(1) $\{e\}$ is i-closed.
(2) If $\{e\} \subsetneq \mathfrak{Z}(\mathfrak{U})$ then $\{e\}$ is not n-closed.

**proof.** Let $e$ is isolated edge in a graph $\mathfrak{U}$. Then:
(1) Since $e = (\breve{\omega}_1, \breve{\omega}_2)$ is isolated edge for every $\ddot{\imath} \in \mathfrak{Z}(\mathfrak{U})$ such that $\ddot{\imath} \neq e$ then $e \notin INVE(\breve{\omega})$ where $\breve{\omega}$ incidence on $\ddot{\imath}$ we get $INVE(\breve{\omega}) \cap (\{e\} - \{\ddot{\imath}\}) = \phi$ hence for every $\ddot{\imath} \in \mathfrak{Z}(\mathfrak{U})$ then $\ddot{\imath} \notin [\{e\}]_i^{`}$, thus $[\{e\}]_i^{`} = \phi \subseteq \{e\}$ there fore $\{e\}$ is i-closed.
(2) Since $\{e\} \subsetneq \mathfrak{Z}(\mathfrak{U})$ then there exist at least a edge $\ddot{\imath}_1 \in \mathfrak{Z}(\mathfrak{U})$ different from $e$ since $e$ is isolated edge then $e \in NINVE(\breve{\omega})$ where $\breve{\omega}$ incidence on $\ddot{\imath}_1$ and $NINVE(\breve{\omega}) \cap (\{e\} - \{\ddot{\imath}_1\}) = \{e\} \neq \phi$ and hence $\ddot{\imath}_1 \in [\{e\}]_n^{`}$. Thus $[\{e\}]_n^{`} \nsubseteq \{e\}$, therefore $\{e\}$ is not n-closed.

**Theorem 2.17.** Let $ß = (\mathbb{V}(\mathfrak{U}), \mathfrak{Z}(\mathfrak{U}))$ be a generalized approximation space. Then if $\mathfrak{U}$ is disconnected graph then $\dot{F}_{\mathfrak{h}_n} = \{\phi, \mathfrak{Z}(\mathfrak{U})\}$.

**proof.** Let $\mathfrak{U}$ be a disconnected graph then there exist at least two disjoint component in $\mathfrak{U}$ say $\mathcal{A}$ and $\mathcal{B}$ that is $\mathcal{A} \cap \mathcal{B} = \phi$. Let $\mathfrak{X}$ be any a proper sub graph in $\mathfrak{U}$ i.e. $\phi \subsetneq \mathfrak{X} \subsetneq \mathfrak{Z}(\mathfrak{U})$. Then:
(i) If $\mathfrak{X} \subseteq \mathcal{A}$ then $\mathfrak{X} \nsubseteq \mathcal{B}$ and for all $\ddot{\imath} \in \mathcal{B}$ then $\mathfrak{X} \subseteq NINVE(\breve{\omega})$ where $\breve{\omega}$ incidence on $\ddot{\imath}$ and $NINVE(\breve{\omega}) \cap (\mathfrak{X} - \{\ddot{\imath}\}) = \mathfrak{X} \neq \phi$ then $\ddot{\imath} \in [\mathfrak{X}]_n^{`}$ and $\ddot{\imath} \notin \mathfrak{X}$ ,there fore $[\mathfrak{X}]_n^{`} \nsubseteq \mathfrak{X}$ then $\mathfrak{X}$ is not n-closed.
(ii) If $\mathfrak{X} \subseteq \mathcal{B}$ then $\mathfrak{X} \nsubseteq \mathcal{A}$ and for all $\ddot{\imath} \in \mathcal{A}$ then $\mathfrak{X} \subseteq NINVE(\breve{\omega})$ where $\breve{\omega}$ incidence on $\ddot{\imath}$ and $NINVE(\breve{\omega}) \cap (\mathfrak{X} - \{\ddot{\imath}\}) = \mathfrak{X} \neq \phi$ then $\ddot{\imath} \in [\mathfrak{X}]_n^{`}$ and $\ddot{\imath} \notin \mathfrak{X}$ ,there fore $[\mathfrak{X}]_n^{`} \nsubseteq \mathfrak{X}$ then $\mathfrak{X}$ is not n-closed.
(iii) If $\mathfrak{X} \cap \mathcal{A} \neq \phi$ and $\mathfrak{X} \cap \mathcal{B} \neq \phi$ then there exist at least $\ddot{\imath} \in \mathcal{A}$ and $\ddot{\imath}_1 \in \mathcal{B}$ such that $\ddot{\imath}, \ddot{\imath}_1 \in \mathfrak{X}$, since $\mathfrak{X} \subsetneq \mathfrak{Z}(\mathfrak{U})$ then there exist $\ddot{\imath}_2 \in \mathfrak{Z}(\mathfrak{U})$ and $\ddot{\imath}_2 \notin \mathfrak{X}$ then $\ddot{\imath}_2$ belong to one of them component of $\mathfrak{U}$, if $\ddot{\imath}_2 \in \mathcal{A}$ then $\ddot{\imath}_1 \in NINVE(\breve{\omega})$ where $\breve{\omega}$ incidence on $\ddot{\imath}_2$ and $NINVE(\breve{\omega}) \cap (\mathfrak{X} - \{\ddot{\imath}_2\})$ contain at least $\ddot{\imath}_1$ and hence $NINVE(\breve{\omega}) \cap (\mathfrak{X} - \{\ddot{\imath}_2\}) \neq \phi$, then $\ddot{\imath}_2 \in [\mathfrak{X}]_n^{`}$ and $\ddot{\imath}_2 \notin \mathfrak{X}$, thus $[\mathfrak{X}]_n^{`} \nsubseteq \mathfrak{X}$ and hence $\mathfrak{X}$ is not n-closed, if $\ddot{\imath}_2 \in \mathcal{B}$ then $\ddot{\imath} \in NINVE(\breve{\omega})$ where $\breve{\omega}$ incidence on $\ddot{\imath}_2$ and $NINVE(\breve{\omega}) \cap (\mathfrak{X} - \{\ddot{\imath}_2\})$ contain at least $\ddot{\imath}$ and hence $NINVE(\breve{\omega}) \cap (\mathfrak{X} - \{\ddot{\imath}_2\}) \neq \phi$, then $\ddot{\imath}_2 \in [\mathfrak{X}]_n^{`}$ and $\ddot{\imath}_2 \notin \mathfrak{X}$, thus $[\mathfrak{X}]_n^{`} \nsubseteq \mathfrak{X}$ and hence $\mathfrak{X}$ is not n-closed ,there for the only n-closed are $\phi, \mathfrak{Z}(\mathfrak{U})$ and hence $\dot{F}_{\mathfrak{h}_n} = \{\phi, \mathfrak{Z}(\mathfrak{U})\}$.

**Theorem 2.18.** Let $ß = (\mathbb{V}(\mathfrak{U}), \mathfrak{Z}(\mathfrak{U}))$ be a generalized approximation space. Then if $\mathfrak{U}$ is star graph then: $\dot{F}_{\mathfrak{h}_i} = \dot{F}_{\mathfrak{h}_n} = p(\mathfrak{Z}(\mathfrak{U}))$.

**proof.**Let $\mathfrak{U}$ is star graph and $\mathfrak{X}$ is subgraph of $\mathfrak{U}$ then there exist center vertex $\breve{\omega}_c$ adjacent with every vertex in $\mathfrak{U}$, let $\ddot{\imath}_n = (\breve{\omega}_n, \breve{\omega}_c) \in \mathfrak{Z}(\mathfrak{U})$ and since $\mathfrak{U}$ is star graph so $INVE(\breve{\omega}_n) = \{\ddot{\imath}_n\}$ but $INVE(\breve{\omega}_n) \cap \mathfrak{X} - \{\ddot{\imath}_n\} = \phi$ hence $\ddot{\imath}_n \notin [\mathfrak{X}]_i^{`}$ thus $[\mathfrak{X}]_i^{`} = \phi$ therefor $\dot{F}_{\mathfrak{h}_i} = p(\mathfrak{Z}(\mathfrak{U}))$. And $NINVE(\breve{\omega}_c) = \phi$ thus $NINVE(\breve{\omega}_c) \cap \mathfrak{X} - \{\ddot{\imath}_n\} = \phi$ hence $\ddot{\imath}_n \notin [\mathfrak{X}]_n^{`}$ thus $[\mathfrak{X}]_n^{`} = \phi$ therefor $\dot{F}_{\mathfrak{h}_n} = p(\mathfrak{Z}(\mathfrak{U}))$. Hence $\dot{F}_{\mathfrak{h}_i} = \dot{F}_{\mathfrak{h}_n} = p(\mathfrak{Z}(\mathfrak{U}))$.

**Theorem 2.19.** Let $ß = (\mathbb{V}(\mathfrak{U}), \mathfrak{Z}(\mathfrak{U}))$ be a generalized approximation space. Then if $\mathfrak{U}$ is antisymmetric graph then:
(1) $\dot{F}_{\mathfrak{h}_n} = \{\phi, \mathfrak{Z}(\mathfrak{U})\}$.
(2) $\dot{F}_{\mathfrak{h}_i} = p(\mathfrak{Z}(\mathfrak{U}))$.

**proof.** Let $\mathfrak{U}$ be an antisymmetric graph then:

(1) Since $\mathfrak{U}$ is antisymmetric graph then $\mathfrak{U}$ is disconnected graph and hence by Theorem (2.17) we get $\dot{F}_{ɧ_n} = \{\phi, \mathfrak{Z}(\mathfrak{U})\}$.
(2) Since $\mathfrak{U}$ is antisymmetric graph then every $\ddot{\iota}_n \in \mathfrak{Z}(\mathfrak{U})$ it can be written $\ddot{\iota}_n = (\breve{\omega}_n, \breve{\omega}_n)$, so $\mathrm{INVE}(\breve{\omega}_n) = \{\ddot{\iota}_n\}$ and assume that $\mathfrak{X}$ is every subgraph of $\mathfrak{U}$ then $\mathrm{INVE}(\breve{\omega}_n) \cap (\mathfrak{X} - \{\ddot{\iota}_n\}) = \phi$ thus $[\mathfrak{X}]_i^{\grave{}} = \phi \subseteq \mathfrak{X}$ hence $\mathfrak{X}$ is i-closed and hence $\dot{F}_{ɧ_i} = p(\mathfrak{Z}(\mathfrak{U}))$.

## 3. Accuracy of the Lower, Upper and Boundary Approximation Spaces.

**Definition 3.1.** Let $ß = (\mathbb{V}(\mathfrak{U}), \mathfrak{Z}(\mathfrak{U}))$ be a generalized approximation space and $Ƭ_{ɧ_i}$, $Ƭ_{ɧ_n}$ and $Ƭ_{ɧ_a}$ be the supra topologies induced by ß and let $\mathfrak{X} \subseteq \mathfrak{U}$. Then:
(1) The i-lower and i-upper approximations of $\mathfrak{X}$ are defined respectively by:

$$L_i(\mathfrak{Z}(\mathfrak{X})) = \mathrm{Int}_i(\mathfrak{Z}(\mathfrak{X})).$$
$$U_i(\mathfrak{Z}(\mathfrak{X})) = \mathrm{Cl}_i(\mathfrak{Z}(\mathfrak{X})).$$

(2) The n-lower and n-upper approximations of $\mathfrak{X}$ are defined respectively by:

$$L_n(\mathfrak{Z}(\mathfrak{X})) = \mathrm{Int}_n(\mathfrak{Z}(\mathfrak{X})).$$
$$U_n(\mathfrak{Z}(\mathfrak{X})) = \mathrm{Cl}_n(\mathfrak{Z}(\mathfrak{X})).$$

(3) The a-lower and a-upper approximations of $\mathfrak{X}$ are defined respectively by:

$$L_a(\mathfrak{Z}(\mathfrak{X})) = \mathrm{Int}_a(\mathfrak{Z}(\mathfrak{X})).$$
$$U_a(\mathfrak{Z}(\mathfrak{X})) = \mathrm{Cl}_a(\mathfrak{Z}(\mathfrak{X})).$$

**Definition 3.2.** Let $ß = (\mathbb{V}(\mathfrak{U}), \mathfrak{Z}(\mathfrak{U}))$ be a generalized approximation space and $Ƭ_{ɧ_i}$, $Ƭ_{ɧ_n}$ and $Ƭ_{ɧ_a}$ be the supra topologies induced by ß and let $\mathfrak{X} \subseteq \mathfrak{U}$. Then:
(1) The i-boundary of $\mathfrak{X}$ are defined by:

$$Bd_i(\mathfrak{Z}(\mathfrak{X})) = U_i(\mathfrak{Z}(\mathfrak{X})) - L_i(\mathfrak{Z}(\mathfrak{X})).$$

(2) The n-boundary of $\mathfrak{X}$ are defined by:

$$Bd_n(\mathfrak{Z}(\mathfrak{X})) = U_n(\mathfrak{Z}(\mathfrak{X})) - L_n(\mathfrak{Z}(\mathfrak{X})).$$

(3) The a-boundary of $\mathfrak{X}$ are defined by:

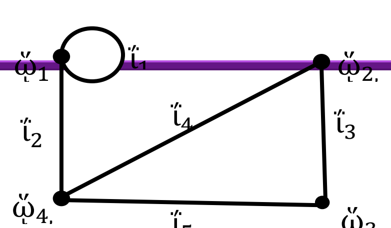
$$Bd_a(\mathfrak{Z}(\mathfrak{X})) = U_a(\mathfrak{Z}(\mathfrak{X})) - L_a(\mathfrak{Z}(\mathfrak{X})).$$

**Definition 3.4.** Let $ß = (\mathbb{V}(\mathfrak{U}), \mathfrak{Z}(\mathfrak{U}))$ be a generalized approximation space. The accuracy of the approximation of a subgraph $\mathfrak{X} \subseteq \mathfrak{U}$ using ($ɧ_i, ɧ_n$ and $ɧ_a$) are defined respectively by: $\mathfrak{f}$

$$\mathfrak{f}_i(\mathfrak{Z}(\mathfrak{X})) = 1 - \frac{|Bd_i(\mathfrak{Z}(\mathfrak{X}))|}{|\mathfrak{Z}(\mathfrak{U})|}.$$
$$\mathfrak{f}_n(\mathfrak{Z}(\mathfrak{X})) = 1 - \frac{|Bd_n(\mathfrak{Z}(\mathfrak{X}))|}{|\mathfrak{Z}(\mathfrak{U})|}.$$
$$\mathfrak{f}_a(\mathfrak{Z}(\mathfrak{X})) = 1 - \frac{|Bd_a(\mathfrak{Z}(\mathfrak{X}))|}{|\mathfrak{Z}(\mathfrak{U})|}.$$

It is obvious that $0 \le \mathfrak{f}_i(\mathfrak{Z}(\mathfrak{X})) \le 1$, $0 \le \mathfrak{f}_n(\mathfrak{Z}(\mathfrak{X})) \le 1$ and $0 \le \mathfrak{f}_a(\mathfrak{Z}(\mathfrak{X})) \le 1$. Moreover, if $\mathfrak{f}_i(\mathfrak{Z}(\mathfrak{X})) = 1$ or $\mathfrak{f}_n(\mathfrak{Z}(\mathfrak{X})) = 1$ or $\mathfrak{f}_a(\mathfrak{Z}(\mathfrak{X})) = 1$ then $\mathfrak{X}$ is called $\mathfrak{X}$-definable ($\mathfrak{X}$-exact) graph otherwise, it is called $\mathfrak{X}$-rough.

**Example 3.5.** Let $\mathfrak{U} = (\mathbb{V}(\mathfrak{U}), \mathfrak{Z}(\mathfrak{U}))$ such that $\mathbb{V}(\mathfrak{U}) = \{\breve{\omega}_1, \breve{\omega}_2, \breve{\omega}_3, \breve{\omega}_4\}$, $\mathfrak{Z}(\mathfrak{U}) = \{\ddot{\iota}_1 = (\breve{\omega}_1, \breve{\omega}_1), \ddot{\iota}_2 = (\breve{\omega}_1, \breve{\omega}_4), \ddot{\iota}_3 = (\breve{\omega}_2, \breve{\omega}_3), \ddot{\iota}_4 = (\breve{\omega}_2, \breve{\omega}_4), \ddot{\iota}_5 = (\breve{\omega}_3, \breve{\omega}_4)\}$.




Then $ɧ_i(\breve{\omega}_1) = \{\{\ddot{\iota}_1, \ddot{\iota}_2\}\}$, $ɧ_i(\breve{\omega}_2) = \{\{\ddot{\iota}_3, \ddot{\iota}_4\}\}$, $ɧ_i(\breve{\omega}_3) = \{\{\ddot{\iota}_3, \ddot{\iota}_5\}\}$, $ɧ_i(\breve{\omega}_4) = \{\{\ddot{\iota}_2, \ddot{\iota}_4, \ddot{\iota}_5\}\}$. And
$ɧ_n(\breve{\omega}_1) = \{\{\ddot{\iota}_3, \ddot{\iota}_4, \ddot{\iota}_5\}\}$, $ɧ_n(\breve{\omega}_2) = \{\{\ddot{\iota}_1, \ddot{\iota}_2, \ddot{\iota}_5\}\}$, $ɧ_n(\breve{\omega}_3) = \{\{\ddot{\iota}_1, \ddot{\iota}_2, \ddot{\iota}_4\}\}$, $ɧ_n(\breve{\omega}_4) = \{\{\ddot{\iota}_1, \ddot{\iota}_3\}\}$. And
$ɧ_a(\breve{\omega}_1) = \{\{\ddot{\iota}_1, \ddot{\iota}_2\}, \{\ddot{\iota}_3, \ddot{\iota}_4, \ddot{\iota}_5\}\}$, $ɧ_a(\breve{\omega}_2) = \{\{\ddot{\iota}_3, \ddot{\iota}_4\}, \{\ddot{\iota}_1, \ddot{\iota}_2, \ddot{\iota}_5\}\}$, $ɧ_a(\breve{\omega}_3) = \{\{\ddot{\iota}_3, \ddot{\iota}_5\}, \{\ddot{\iota}_1, \ddot{\iota}_2, \ddot{\iota}_4\}\}$, $ɧ_a(\breve{\omega}_4) = \{\{\ddot{\iota}_2, \ddot{\iota}_4, \ddot{\iota}_5\}, \{\ddot{\iota}_1, \ddot{\iota}_3\}\}$. And

$$Ƭ_{ɧ_i} = \left\{\begin{matrix} \mathfrak{Z}(\mathfrak{U}), \phi, \{\ddot{\iota}_1, \ddot{\iota}_2\}, \{\ddot{\iota}_1, \ddot{\iota}_2, \ddot{\iota}_3, \ddot{\iota}_4\}, \{\ddot{\iota}_1, \ddot{\iota}_2, \ddot{\iota}_3, \ddot{\iota}_5\}, \\ \{\ddot{\iota}_3, \ddot{\iota}_4\}, \{\ddot{\iota}_3, \ddot{\iota}_4, \ddot{\iota}_5\}, \{\ddot{\iota}_2, \ddot{\iota}_3, \ddot{\iota}_4, \ddot{\iota}_5\}, \\ \{\ddot{\iota}_3, \ddot{\iota}_5\}, \{\ddot{\iota}_2, \ddot{\iota}_4, \ddot{\iota}_5\}, \{\ddot{\iota}_1, \ddot{\iota}_2, \ddot{\iota}_4, \ddot{\iota}_5\} \\ . \end{matrix}\right\}.$$

$Ƭ_{ɧ_n} = \{\mathfrak{Z}(\mathfrak{U}), \phi\}$.

$$Ƭ_{ɧ_a} = \left\{\begin{matrix} \mathfrak{Z}(\mathfrak{U}), \phi, \{\ddot{\iota}_1, \ddot{\iota}_2\}, \{\ddot{\iota}_1, \ddot{\iota}_2, \ddot{\iota}_3, \ddot{\iota}_4\}, \{\ddot{\iota}_1, \ddot{\iota}_2, \ddot{\iota}_3, \ddot{\iota}_5\}, \\ \{\ddot{\iota}_3, \ddot{\iota}_4\}, \{\ddot{\iota}_3, \ddot{\iota}_4, \ddot{\iota}_5\}, \{\ddot{\iota}_2, \ddot{\iota}_3, \ddot{\iota}_4, \ddot{\iota}_5\}, \{\ddot{\iota}_1, \ddot{\iota}_3, \ddot{\iota}_4, \ddot{\iota}_5\} \\ , \{\ddot{\iota}_3, \ddot{\iota}_5\}, \{\ddot{\iota}_2, \ddot{\iota}_4, \ddot{\iota}_5\}, \{\ddot{\iota}_1, \ddot{\iota}_2, \ddot{\iota}_4, \ddot{\iota}_5\} \end{matrix}\right\}$$

Table 3.1. $\mathfrak{f}_i(\mathfrak{Z}(\mathfrak{X}))$, $\mathfrak{f}_n(\mathfrak{Z}(\mathfrak{X}))$ and $\mathfrak{f}_a(\mathfrak{Z}(\mathfrak{X}))$ for all $\mathfrak{X} \subseteq \mathfrak{U}$.

**Theorem 3.6.** Let $ß = (\mathbb{V}(\mathfrak{U}), \mathfrak{Z}(\mathfrak{U}))$ be a generalized approximation space and $\mathfrak{X} \subseteq \mathfrak{U}$. Then

(1) $L_i(\mathfrak{Z}(\mathfrak{X})) \cup L_n(\mathfrak{Z}(\mathfrak{X})) \subseteq L_a(\mathfrak{Z}(\mathfrak{X}))$.

(2) $U_a(\mathfrak{Z}(\mathfrak{X})) \subseteq U_i(\mathfrak{Z}(\mathfrak{X})) \cap U_n(\mathfrak{Z}(\mathfrak{X}))$.

(3) $Bd_a(\mathfrak{Z}(\mathfrak{X})) \subseteq Bd_i(\mathfrak{Z}(\mathfrak{X})) \cap Bd_n(\mathfrak{Z}(\mathfrak{X}))$.

(4) $\mathfrak{f}_a(\mathfrak{Z}(\mathfrak{X})) \geq \max\{\mathfrak{f}_i(\mathfrak{Z}(\mathfrak{X})), \mathfrak{f}_n(\mathfrak{Z}(\mathfrak{X}))\}$.

**proof.** (1) and (2) The proof is similar to the proof of proposition (2.20) in [11]

(3) Let $\ddot{t} \in Bd_a(\mathfrak{Z}(\mathfrak{X})) \Rightarrow \ddot{t} \in \left(U_a(\mathfrak{Z}(\mathfrak{X})) - L_a(\mathfrak{Z}(\mathfrak{X}))\right) \Rightarrow \ddot{t} \in U_a(\mathfrak{Z}(\mathfrak{X})) \wedge \ddot{t} \notin L_a(\mathfrak{Z}(\mathfrak{X}))$.
Since $U_a(\mathfrak{Z}(\mathfrak{X})) \subseteq U_i(\mathfrak{Z}(\mathfrak{X})) \cap U_n(\mathfrak{Z}(\mathfrak{X}))$.
And $L_i(\mathfrak{Z}(\mathfrak{X})) \cup L_n(\mathfrak{Z}(\mathfrak{X})) \subseteq L_a(\mathfrak{Z}(\mathfrak{X}))$. Then $\ddot{t} \in \left(U_i(\mathfrak{Z}(\mathfrak{X})) \cap U_n(\mathfrak{Z}(\mathfrak{X}))\right) \wedge \ddot{t} \notin \left(L_i(\mathfrak{Z}(\mathfrak{X})) \cup L_n(\mathfrak{Z}(\mathfrak{X}))\right) \Rightarrow \left(\ddot{t} \in U_i(\mathfrak{Z}(\mathfrak{X})) \wedge \ddot{t} \in U_n(\mathfrak{Z}(\mathfrak{X}))\right) \wedge \left(\ddot{t} \notin L_i(\mathfrak{Z}(\mathfrak{X})) \wedge \ddot{t} \notin L_n(\mathfrak{Z}(\mathfrak{X}))\right) \Rightarrow \left(\ddot{t} \in U_i(\mathfrak{Z}(\mathfrak{X})) \wedge \ddot{t} \notin L_i(\mathfrak{Z}(\mathfrak{X}))\right) \wedge \left(\ddot{t} \in U_n(\mathfrak{Z}(\mathfrak{X})) \wedge \ddot{t} \notin L_n(\mathfrak{Z}(\mathfrak{X}))\right) \Rightarrow \ddot{t} \in \left(U_i(\mathfrak{Z}(\mathfrak{X})) - L_i(\mathfrak{Z}(\mathfrak{X}))\right) \wedge \ddot{t} \in \left(U_n(\mathfrak{Z}(\mathfrak{X})) - L_n(\mathfrak{Z}(\mathfrak{X}))\right) \Rightarrow \ddot{t} \in Bd_i(\mathfrak{Z}(\mathfrak{X})) \wedge \ddot{t} \in Bd_n(\mathfrak{Z}(\mathfrak{X})) \Rightarrow \ddot{t} \in \left(Bd_i(\mathfrak{Z}(\mathfrak{X})) \cap Bd_n(\mathfrak{Z}(\mathfrak{X}))\right)$. Therefore $Bd_a(\mathfrak{Z}(\mathfrak{X})) \subseteq Bd_i(\mathfrak{Z}(\mathfrak{X})) \cap Bd_n(\mathfrak{Z}(\mathfrak{X}))$.

4) By (3) above we get $Bd_a(\mathfrak{Z}(\mathfrak{X})) \subseteq Bd_i(\mathfrak{Z}(\mathfrak{X})) \cap Bd_n(\mathfrak{Z}(\mathfrak{X})) \Rightarrow Bd_a(\mathfrak{Z}(\mathfrak{X})) \subseteq Bd_i(\mathfrak{Z}(\mathfrak{X}))$ and hence $|Bd_a(\mathfrak{Z}(\mathfrak{X}))| \leq |Bd_i(\mathfrak{Z}(\mathfrak{X}))| \Rightarrow \frac{|Bd_a(\mathfrak{Z}(\mathfrak{X}))|}{|\mathfrak{Z}(\mathfrak{U})|} \leq \frac{|Bd_i(\mathfrak{Z}(\mathfrak{X}))|}{|\mathfrak{Z}(\mathfrak{U})|} \Rightarrow 1 - \frac{|Bd_a(\mathfrak{Z}(\mathfrak{X}))|}{|\mathfrak{Z}(\mathfrak{U})|} \geq 1 - \frac{|Bd_i(\mathfrak{Z}(\mathfrak{X}))|}{|\mathfrak{Z}(\mathfrak{U})|} \Rightarrow \mathfrak{f}_a(\mathfrak{Z}(\mathfrak{X})) \geq \mathfrak{f}_i(\mathfrak{Z}(\mathfrak{X}))$.
Using the same way we get $\mathfrak{f}_a(\mathfrak{Z}(\mathfrak{X})) \geq \mathfrak{f}_n(\mathfrak{Z}(\mathfrak{X}))$ thus $\mathfrak{f}_a(\mathfrak{Z}(\mathfrak{X})) \geq \max\{\mathfrak{f}_i(\mathfrak{Z}(\mathfrak{X})), \mathfrak{f}_n(\mathfrak{Z}(\mathfrak{X}))\}$ .

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| $\mathfrak{Z}(\mathfrak{X})$ | $\mathfrak{f}_i(\mathfrak{Z}(\mathfrak{X}))$ | $\mathfrak{f}_n(\mathfrak{Z}(\mathfrak{X}))$ | $\mathfrak{f}_a(\mathfrak{Z}(\mathfrak{X}))$ |
|---|---|---|---|
| $\{\ddot{t}_1\}$ | 4/5 | 0 | 4/5 |
| $\{\ddot{t}_2\}$ | 3/5 | 0 | 4/5 |
| $\{\ddot{t}_3\}$ | 4/5 | 0 | 4/5 |
| $\{\ddot{t}_4\}$ | 4/5 | 0 | 4/5 |
| $\{\ddot{t}_5\}$ | 4/5 | 0 | 4/5 |
| $\{\ddot{t}_1, \ddot{t}_2\}$ | 1 | 0 | 1 |
| $\{\ddot{t}_1, \ddot{t}_3\}$ | 3/5 | 0 | 3/5 |
| $\{\ddot{t}_1, \ddot{t}_4\}$ | 2/5 | 0 | 2/5 |
| $\{\ddot{t}_1, \ddot{t}_5\}$ | 2/5 | 0 | 2/5 |
| $\{\ddot{t}_2, \ddot{t}_3\}$ | 0 | 0 | 0 |
| $\{\ddot{t}_2, \ddot{t}_4\}$ | 2/5 | 0 | 2/5 |
| $\{\ddot{t}_2, \ddot{t}_5\}$ | 2/5 | 0 | 2/5 |
| $\{\ddot{t}_3, \ddot{t}_4\}$ | 1 | 0 | 1 |
| $\{\ddot{t}_3, \ddot{t}_5\}$ | 4/5 | 0 | 4/5 |
| $\{\ddot{t}_4, \ddot{t}_5\}$ | 2/5 | 0 | 2/5 |
| $\{\ddot{t}_1, \ddot{t}_2, \ddot{t}_3\}$ | 2/5 | 0 | 2/5 |
| $\{\ddot{t}_1, \ddot{t}_2, \ddot{t}_4\}$ | 4/5 | 0 | 4/5 |
| $\{\ddot{t}_1, \ddot{t}_2, \ddot{t}_5\}$ | 4/5 | 0 | 4/5 |
| $\{\ddot{t}_2, \ddot{t}_3, \ddot{t}_4\}$ | 2/5 | 0 | 2/5 |
| $\{\ddot{t}_2, \ddot{t}_3, \ddot{t}_5\}$ | 2/5 | 0 | 2/5 |
| $\{\ddot{t}_3, \ddot{t}_4, \ddot{t}_1\}$ | 2/5 | 0 | 2/5 |
| $\{\ddot{t}_3, \ddot{t}_4, \ddot{t}_5\}$ | 1 | 0 | 1 |
| $\{\ddot{t}_4, \ddot{t}_5, \ddot{t}_1\}$ | 0 | 0 | 0 |
| $\{\ddot{t}_4, \ddot{t}_5, \ddot{t}_2\}$ | 3/5 | 0 | 3/5 |
| $\{\ddot{t}_1, \ddot{t}_3, \ddot{t}_5\}$ | 2/5 | 0 | 2/5 |
| $\{\ddot{t}_1, \ddot{t}_2, \ddot{t}_3, \ddot{t}_4\}$ | 4/5 | 0 | 4/5 |
| $\{\ddot{t}_1, \ddot{t}_2, \ddot{t}_3, \ddot{t}_5\}$ | 4/5 | 0 | 4/5 |
| $\{\ddot{t}_2, \ddot{t}_3, \ddot{t}_4, \ddot{t}_5\}$ | 4/5 | 0 | 4/5 |
| $\{\ddot{t}_1, \ddot{t}_3, \ddot{t}_4, \ddot{t}_5\}$ | 3/5 | 0 | 4/5 |
| $\{\ddot{t}_1, \ddot{t}_2, \ddot{t}_4, \ddot{t}_5\}$ | 4/5 | 0 | 4/5 |
| $\mathfrak{Z}(\mathfrak{U})$ | 1 | 1 | 1 |
| $\phi$ | 1 | 1 | 1 |

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