International Journal of Academic and Applied Research (IJAAR)
ISSN: 2643-9603
Vol. 8 Issue 2 February - 2024, Pages: 46-51

On The Problem of Optimal Stability of Nonlinear Stochastic Equation

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Abstract: *In this work , the necessary and at the same time sufficient conditions were found to improve the linear constant regulator. The set task typically involves achieving a specific goal without focusing on optimization, whereas the optimal control problem aims to find the best control strategy to optimize a performance measure, considering system dynamics and constraints. In essence, set tasks aim for achievement, while optimal control problems seek to optimize performance under given conditions is clarified over an unlimited period of time. The essence of the optimal conditions obtained is illustrated by an example*

Keywords: Semi-linear random systems, ideal stability, linear regulator

1- Introduction

Particular importance are the problems of managing , the problem the most developed section of the theory of optimal system of stochastic variables is the optimal solution of linear equations [1]. Formulating this task exclusive of unnecessarily specifying of the current detail point is as follows: it is necessary to reduce the functionality

$$J(Z,v) = E\int_{t_0}^{t_1} Z(u)^T Q(Z(u)) + v(u,z(u))^T Ev(u,z(u))du \Rightarrow \min_v$$

$$dZ(t) = (A_0 Z(h) + B_0 v(h,z(h) + C_0) + \sum_{j=1}^{n} (A_j Z(h) + B_j v(h,Z(h) + C_j)dM_j(h) \qquad (1)$$

It is known [2] that in the case when the equations of the system contain only additive perturbations (Matrix coefficients $A_j, B_j,\ j = 1,n$ equal to zero), for this task, there is a separation principle. If matrix A in standard defined negatively, does not result to problem in this type . The systems in same time of equation (1), the separation principle, in general, does not work , and it shows [4-7] that even with a negatively defined Matrix E in the standard, the problem can be solved . In order to emphasize these and other features in the formula (1) is define : linear the broad sense [8], with position-dependent and control noise [9-11], with multiplicative noise [12].

At the same time, the question of asymptotic stability in the zero-squared average solution of a closed system [13] and the problem optimal stability(OS) over an the interval acquires a meaningful meaning. Note that the control element ensuring the stability of the closed equation , reduces state the equation is reduced zero . The square average, and, other hand, effect the random disturbances on equation . A have been devoted the problem of optimal of semi-linear equations (for example [9-11, 14]), but with more general assumptions about the system and parameters, it is considered in [15], where there are sufficient conditions for the optimal linear constant regulator. In this paper , the necessary and at the same time sufficient conditions are obtained for optimizing a linear constant regulator. This result, with additional about the equation . And a paper contains: Section 2 contains the formulation of the problem , the Section 3 management quality functions, the fourth section , a comparison is made between the studied optimal stability problem and the optimal stability problem in terms of coordinates using a typical example and the fifth section conclusion .

2- Determination of the optimal stability problem

The equation form is considered

$$dZ(h) = (A_0 Z(h) + B_0 v(h,z(h) + C_0) + \sum_{j=1}^{n} (A_j Z(h) + B_j v(h,Z(h) + C_j)dM_j(h) \qquad (2)$$

Where $h \ge h_0$ - time , Z A random operation with values in R^n , M - The standard Wiener process with values in R^k ; $(h,z) \to v(h,z) : [h_0,+\infty) \times R^n \to R^m$ - Boral measurable management strategy ; $A_j \in R^{n\times n}, B_j \in R^{n\times m},\ j = 0,......,k$ - Fixed matrices.

We denote D_{p_0} set control operations $Z(x,v)$, are pairs of stochastic processes Z and control strategies v such as that

1. For the given v , the continued random operation Z is weak solve [16, Section 5.3] of Equation (2) with an initial condition

$$P_{z(h_0)} = P_0 \tag{3}$$

Where the denominator (h_0) of the vector random for the distribution $Z(h_0)$, and P_0 the Measure the probability on R^n the satisfying

$$\int_{R^n} \|z\|^4 P_0(dz) < +\infty$$

It is assumed that $Z(h_0)$ do not rely on $M(h), h \geq h_0$;

2. The conditions are met:

$$E\int_{t_0}^{t_1} \|Z(u)\|^4 du < +\infty \ , \ E\int_{t_0}^{t_1} \|v(u, Z(u)\|)^4 du < +\infty \ , \ h \geq h_0 \tag{4}$$

$$E\int_{t_0}^{+\infty_1} \|Z(u)\|^2 + \|v(u, z(u))\|^2 du < +\infty \ , \ \lim_{t\to+\infty} E\|z(h)\|^2 = 0 \tag{5}$$

(Here and further, the mathematical expectation is taken In the associated probability space z).

Note 1. It is assumed here that with each discrete control operation z there is a complete filtered space $(\Omega \ , F \ , F(h)_{h\geq h_0} \ , P)$ in any continuous random Operation $Z = (z(h), F_h)_{h\geq h_0}$ and the standard operation $M = (M(h), F_h)_{h\geq h_0}$ is given so that prime condition (3) is satisfied and for each $h \geq h_0$, equality (2) is true with the first probability. At the same time, the control process is correctly it is understood

$$z = (\Omega \ , F \ , \ F(t)_{h\geq h_0}, P), (M(h), F_h)_{h\geq h_0}, (Z(h), F_h)_{h\geq h_0}, v).$$

In order to simplify the designations, we will not specify these details further.

3. Auxiliary management quality functions

Similarly, as happened in [15], we will build a quality functional auxiliary control for this purpose . We define some control operations $z = (Z, v) \in D$. It is define [16, theorem 4.1.2] that for each function $(h, z) \to (\phi(h, z) : [h_0, +\infty) \times R^n \to R$, it has continuous derivatives

$\dfrac{\partial \phi}{\partial h}, \dfrac{\partial^2 \phi}{\partial z_i \partial z_j}, i, j = 1,...,n$, is formula correct

$$\phi(h, Z(h) = \phi(h_0, Z(h) + \int_{h_0}^{h} (\frac{\partial \phi}{\partial t}(u, z(u)) + \nabla_z \phi(u, z(u))^T (A_0 Z(u) + B_0 v(u, z(u)) \tag{6}$$

$$+\frac{1}{2}\sum_{j=1}^{k} (A_j Z(u) + B_j v(u, z(u))^T + (A_j Z(u) + B_j v(u, z(u)))du$$

$$\sum_{j=1}^{k} \int_{h_0}^{h} \nabla_u \phi(u, z(u))^T (A_j Z(u) + B_j v(u, z(u)) dM_j(u)$$

Where $\nabla_z(\phi) := (\dfrac{\partial \phi}{\partial z_1}, ..., \dfrac{\partial \phi}{\partial z_n})^T$ - provide more details or specify which function's graduation you're referring $\phi(h,.)$; H_z^{ϕ}

- The function of Hess matrix $\phi(h,.); (H_z^{\phi})_{ij} = \dfrac{\partial^2 \phi}{\partial z_i \partial z_j}, i, j = 1,...,n$.

A function $\phi(h, z) = z^T W_z$, where $W \in R^{n\times n}$ symmetric matrix, we get equal

$$Z(h)WZ(h) = Z(h_0)WZ(h_0) + \int_{h_0}^{h} (2z(u)W(A_0 Z(u) + B_0 v(u, z(u))$$

$$+\sum_{j=1}^{k}(A_j Z(u)+B_j v(u,z(u))^{T} W(A_j Z(u)+B_j v(u,z(u)))du$$

$$\sum_{j=1}^{k}\int_{h_0}^{h}(z(u))^{T}W(A_j Z(u)+B_j v(u,z(u))))dM_j(u)$$

Then, given the properties of [16, theorem 3.2.1] of stochastic integration both are requirements (4) , we will have

$$E(Z(h)^{T}WZ(h))=E(Z(h_0)^{T}WZ(h_0))+E\int_{h_0}^{h}(2z(u)^{T})W(A_0Z(u)+B_0v(u,z(u))$$

$$+\sum_{j=1}^{k}(A_j Z(u)+B_j v(u,z(u))^{T} W(A_j Z(u)+B_j v(u,z(u)))du$$

$$\sum_{j=1}^{k}\int_{h_0}^{h}(z(u))^{T}W(A_j Z(u)+B_j v(u,z(u))))dM_j(u)$$

Aiming h to infinity, taking into account (4) and (5) we get

$$E(Z(h_0)^{T}WZ(h_0))+E\int_{t_0}^{+\infty}(2z(u)^{T})W(A_0Z(u)+B_0v(u,z(u)) \tag{7}$$

$$+\sum_{j=1}^{k}(A_j Z(u)+B_j v(u,z(u))^{T} W(A_j Z(u)+B_j v(u,z(u)))du=0$$

$$\Psi(u):=E(Z(h_0)^{T}WZ(h_0))+E\int_{t_0}^{+\infty}(2z(u)^{T})W(A_0Z(u)+B_0v(u,z(u)) \tag{8}$$

$$+\sum_{j=1}^{k}(A_j Z(u)+B_j v(u,z(u))^{T} W(A_j Z(u)+B_j v(u,z(u)))du+Z(u)^{T}QZ(u)$$

$$+z(u))^{T}Sv(u,z(u))+v(u,z(u))^{T}S^{T}Z(u)+v(u,z(u))^{T}Ev(umz(u)))du$$

From equation (7) and arbitrariness of choice from u , is satisfied

$$\Psi(u)\equiv J\ \ (u)\ \ ,\ u\in D \tag{9}$$

The property that does not depend on the choice of the matrix is considered to be matrix invariance W .

$$\tau(z,v,W):=z^{T}(WA_0+A_0^{T}W+\sum_{j=1}^{k}A_j^{T}WA_j)z+z^{T}(WB_0+\sum_{j=1}^{k}A_j^{T}WB_j+S)v \tag{10}$$

$$+v^{T}(B_0^{T}W+\sum_{j=1}^{k}B_j^{T}WA_j+S^{T})z+v^{T}(\sum_{j=1}^{k}B_j^{T}WB_j+E)v$$

Note that for a constant matrix $W\in R^{n\times n}$, the function $\tau(.,.,W)$ is a linear quadratic function on the set of variables $(z,v)\in R^{n}\times R^{m}$. Using the function τ , the functional Ψ can be rewritten in a more compact form

$$\Psi(z)=tr[WK_0]+E\int_{t_0}^{+\infty}\tau(Z(u),v(u,Z(u)),W)du \tag{11}$$

Where $K_0\in R^{n\times n}$ - is the matrix of the first two moments of the vector $Z(h_0)$, $tr[.]$-the effect of matrices .

4- Optimal stability in terms of coordinates

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In this work it is the allowed D is limited to random operations that tend asymptotically to zero on the square average. However, due to the non-negative certainty of the Matrix Z in equation (8), it is possible to consider the more general problem of optimal stability of the system (2) in terms of coordinates.

Let be given a complete PS $(\Omega, \ F \ ,P)$, the SWP M and the vector random variable n-dimensions $Z_0, E\|Z_0\|^4 < +\infty$ independent of $M(h), h > h_0$. denote $\overline{D}$ the set operations (Z,v) such that:

1-Given v, a continuous random process Z is a strong solution of Equation (2) with the initial condition $Z(t_0) = Z_0$;

2- Equation (8) takes a finite value , and the conditions are met

$$E\int_{h_0}^{h_1}\|Z(v)\|^4 dv < +\infty \ , \ E\int_{h_0}^{h_1}\|v(u,Z(u)\|)^4 dv < +\infty \ , \ h \geq h_0$$

The task of optimal fixation in terms of coordinates will be called the task of searching for a control strategy $\overline{v}$ such that there is an acceptable control operation $\overline{z} = (\overline{Z}, \overline{v})$ in the equation (8)

$$J(\ \overline{z}) = \min_z J(z)$$

Using a typical example, let's compare this task with the task of optimal installation.

Example1. The following equation and a control quality criterion

$$dZ_1(h) = -\frac{1}{2}Z_1(h)dh + v(h,Z(h)))dM(h) \qquad (12)$$

$$dZ_2(h) = (\frac{1}{2}Z_2(h) + v(h,Z(h))dh) \qquad (13)$$

$$J(\ z) = \ E\int_0^{+\infty}(Z_1^2(u) + v^2(t,Z(u)))du \qquad (14)$$

Where $h \geq 0$ - is the time $Z = Z_1 \ \ Z_2)^T$ - RO in R^2 ; M - Standard one-dimensiona; $(h,z) \to v(h,z) : R \times R^2 \to R$ - Management strategy; $z = (Z,v)$ -The management process.

Using the theorem, we obtain a solution to the problem of optimal stability. Matrices

$$W = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \quad , \ L = \begin{pmatrix} 0 & 1 \end{pmatrix}$$

In this case, The Matrix W is defined positively. We prove that $v(h,z) = -L_z$ is stability. The equation of the matrix of the second moments $K(h)$ of the state vector $Z(h)$ of a closed system (12), (13) it has the form [16] .

$$\frac{d}{dh}K(h) \ \ = -K(h) + \begin{pmatrix} K_{22}(h) & 0 \\ 0 & 0 \end{pmatrix}$$

$$K(h) = \begin{pmatrix} K_{11}(h) & K_{21}(h) \\ K_{12}(h) & K_{22}(h) \end{pmatrix}$$

The linear equation can be rewritten as a vector linear differential equation

$$\frac{d}{dh}K(h) = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \ [K(h)]$$

Therefore the control strategy $v(h,z) = -L_z$ stabilizes. Given the result, then

$$J \quad (z) = 2$$

We solve problem of optimal stability in terms of system coordinates (12), (13) with equation (14). The difference between of OCP and the optimal stability problem is that a random process controlled by Z may not meet the requirements (5). Let's take advantage of the fact that Z_2 is not explicitly included either in the standard or in the equation of Z_1. The problem of optimal stability of the equation (12)

with the standard (14):

$$dZ_1(h) \quad = -\frac{1}{2} Z_1 \quad (h)dh + v(h, Z(h)))dM(h) \quad , Z_1(0) = 1 \qquad (15)$$

$$J(z) = E \int_0^{+\infty} (Z_1^2(u) + v^2(t, Z(u)))du$$

We find the optimal stability regulator $\tilde{v}$ and the corresponding value $\tilde{W}$

$$\tilde{v}(t,z) \equiv 0 \;,\; \tilde{W} = 1$$

The existing management strategy ensures the stability of equation (15), the value of the criterion of the management process $\tilde{z} = (\tilde{Z}_1, \tilde{v})$ is equal to $J(\tilde{z}) = 1$.

Note 2: The fact was found that the control strategy $\tilde{v}$ that provides a value for the parameter than optimal stability v of a constant prime state does not contradict the conditions of the obtained ideality. In fact, let P0 a probability point $Z(0) = \begin{pmatrix}1 & 1\end{pmatrix}$ is the solve of equations (12), (13) and control strategy $\tilde{v}$. It's easy to see that the component $\tilde{Z}_2(h)$ increases indefinitely at Infinity. Therefore, the control operation $\tilde{z} = (\tilde{Z}_1, \tilde{v})$ does not belong to group DP_0 and condition (7) is not violated

5-Conclusion

Problem of OS of a pseudo-(linear- random -system)over an unlimited period of time is considered. With sufficiently broad assumptions about the control system and quality parameters, the necessary and at the same time sufficient conditions are obtained for optimizing a linear stationary controller in a wide category of permissible controls. A typical example shows the teams between the studied optimal stability is the terms in coordinates.

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