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On Neutrosophic Study Of Pseudo BH-SubAlgebra And Ideals With There Structure In Graph Theory

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Abstract— *In this work, it presents a new kind of BH-subalgebra for the neutrosophic triple set, which is named the neutrosophic BH-subalgebra. It investigates this neutrosophic BH-algebra through some significant properties of BH-algebra. It also uses upper bounds, lower bounds, and some important characteristics to study the behavior of neutrosophic BH-subalgebra [NBHS] and ideals [NBHI] and study neutrosophic BH-pseudo subalgebra [NPBHS] and ideals [NPBHI] on BH-algebra, additional we present there in graph theory structure.*

Keywords— BH-algebra, Pseudo BH-algebra, Neutrosophic BH-subalgebra, Neutrosophic Ideals, Neutrosophic Pseudo BH-subalgebra and Neutrosophic Pseudo Ideals.

1. INTRODUCTION

K. ISEKI and Y. IMAI defined and investigated the BCK-algebra and BCI-algebra. A generality of BCK-algebra was introduced in 1966 [2]. An idea popped up in BH-algebra in 1998 by Y. B. Jun, in addition, Y.B. Jun et al. introduced the pseudo-BH algebra in 2015 [6]. In 2017, A.H. Nouri and H.H. Abbass thoughtfully considered some kinds of ideals of pseudo-BH algebra [7]. Fuzzy sets were presented by Zadeh in such a way that most writers deem the year 1965 to be the start of fuzzy logic as a subset of fuzzy sets [1]. The Smarandache-proposed neutrosophic sets (NSs) are a potent mathematical tool for dealing with partial, ambiguous, and inconsistent information in the real world. They are a generalization of intuitionistic fuzzy sets [8], interval valued intuitionistic fuzzy sets [9], and the theory of fuzzy sets [10]. The truth-membership function (t), indeterminacy-membership function (i), and falsity-membership function (f) separately define the neutrosophic sets and are located within the real standard or nonstandard unit interval]0, 1+[. Wang et al. [11] presented the idea of single-valued neutrosophic sets (SVNS), a subclass of the neutrosophic sets, to make it easier to utilize NS in practical applications. A generalization of intuitionistic fuzzy sets, the SVNS has three independent membership functions with values that fall within the unit interval [0,1]. The graph is an ordered pair G = (V, E), where V is a non-empty set called vertices and E is an ordered pair of V called edges [12].Graph theory is currently a significant area of applied mathematics and is typically thought of as a subfield of combinatorics. In many disciplines, including geometry, algebra, number theory, topology, optimization, and computer science, graph is a common tool for addressing combinatorial issues [13]. The most crucial point to keep in mind is that the model turns into a fuzzy graph when there is ambiguity about the set of vertices, the set of edges, or both. There have been numerous studies on fuzzy graphs and intuitionistic fuzzy graphs [14]; in each of these studies, the vertex sets and edge sets were regarded as fuzzy and/or intuitionistic fuzzy sets.

However, fuzzy graphs and intuitionistic fuzzy graphs fail when the relationships between nodes (or vertices) in problems are uncertain. Samarandache [15] defined four main categories of neutrosophic graphs for this purpose, two of which were based on literal indeterminacy (I) and were known as I-edge neutrosophic graph and I-vertex neutrosophic graph, respectively. These concepts are thoroughly studied and have grown in popularity among researchers as a result of their applications to real-world issues [16]. The (T, I, F)-Edge neutrosophic graph and the (T, I, F)-vertex neutrosophic graph are the two additional graphs that are built on (T, I, F) components; nevertheless, these concepts are not at all developed. Since there is a gap in the literature on the study of single valued neutrosophic graphs (SVN-graph), we will concentrate on it in this paper.

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2. PRELIMINARIES

We present here the basics that we relied on in this paper.

Definition (2.1) [3]: - Assume that X is a non-empty set with a binary operation $*$ and a constant 0 is named a BH-algebra if: $\forall\, x, y \in$ X

$i.\ \ x * x = 0.$

$ii.\ \ x * y = 0\ \&\ y * x = 0 \implies x = y.$

$iii.\ x * 0 = x.$

Definition (2.2) [3]: - A BH-subalgebra or subalgebra of a BH-algebra (X,$*$, 0) is a non-empty subset S of X, for every $x, y \in$ X such that:

$x * y \in$ S.

Definition (2.3) [6]: - A pseudo BH-algebra(simply P.BH) is non-empty set X with a constant 0 and binary operations $*$, $\#$ satisfies the next conditions:

$i.\ \ x * x = x \# x = 0, \forall x \in$ X.

$ii.\ \ x * y = 0\ \&\ y \# x = 0 \implies x = y, \forall x, y \in$ X.

$iii.\ x * 0 = x \# 0 = x, \forall x \in$ X.

Remark (2.4) [7]: - Let X be a P.BH-algebra, we define the relation "$\leq$" on X by: $x \leq y \Leftrightarrow x * y = 0\ \&\ x \# y = 0$.

Definition (2.5) [6]: - A non-empty set S of a P.BH-algebra $(X, *, \#, 0)$is named a Pseudo BH-subalgebra of X, if achieved:

$x * y\ \&\ x \# y \in$ S, $\forall x, y \in$ X.

Definition (2.6) [6]: - Assume I is a non-empty subset of a P. BH of X, then I is named a Pseudo ideal of X, denoted by P. I if achieved: $\forall x, y \in$ X

$i.\ \ 0 \in$ I.

$ii.$ if $x * y \in$ I, $x \# y \in$ I $\&\ y \in$ I $\Longrightarrow x \in$ I.

Definition (2.7) [4]: - If X be a space of points and let $x \in$ X. A neutrosophic set A in X is characterized by T_A, I_A, F_A:X$\longrightarrow]^-0, 1^+[$. Where $T_A(x), I_A(x), F_A(x)$ are the (truth , indeterminacy and falsity) membership functions respectively T_A, the neutrosophic set can be represented as $A = \{(x, T_A(x), I_A(x), F_A(x)) \mid x \in$ X $\}$. There is no restriction on the sum of $T_A(x), I_A(x)$ and $F_A(x)$, so $^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$.

Definition (2.8) [5]: - The complement of Neutrosophic value set

A = $(T_A(x), I_A(x), F_A(x))$ is denoted by A^c and it is defined by

$\overline{T_A}(x) = [1 - T^+(x), 1 - T^-(x)]$.

$\overline{I_A}(x) = [1 - I^+(x), 1 - I^-(x)]$.

$\overline{F_A}(x) = [1 - F^+(x), 1 - F^-(x)]$.

Remark(2.9): When the period is reduced to to the interval [0,1], the set A is called a single-valued neurosophic set.

3. NEUTROSOPHIC PSEUDO BH-SUBALGEBRAS AND IDEALS.

Remark(3.1):- A ={$(x, T_A(x), I_A(x), F_A(x)) \mid x \in$ X} =

$(T_A(x), I_A(x), F_A(x))$ or (T_A, I_A, F_A). Where T means membership value, I mean indeterminacy membership value and F means non-membership value with $T_A(x), I_A(x)$ and $F_A(x)$:$\longrightarrow]^-0, 1^+[$, such that $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Definition(3.2):- Let X be a BH-algebra, then A is a **Neutrosophic BH-Subalgebra (simply NBHS)** if satisfies the following conditions : $\forall\, x, y \in$ X

$i.\ \ T_A(x) \geq min\{T_A(x * y),\ T_A(y)\}.$

$ii.\ I_A(x) \geq min\{\ I_A(x * y),\ I_A(y)\}.$

$iii.\ F_A(x) \leq max\{F_A(x * y),\ F_A(y)\}.$

And A is a **Neutrosophic ideal** of a BH-algebra X **(simply NBHI)** if satisfies the following : $\forall\, x, y \in$ X

$i.$ (1) $T_A(0) \geq T_A(x)$ & (2) $F_A(0) \leq F_A(x)$ & (3) $I_A(0) \geq I_A(x)$.

$ii.\ T_A(x) \geq min\{T_A(x * y),\ T_A(y)\}.$

$iii.\ I_A(x) \geq min\{\ I_A(x * y),\ I_A(y)\}.$

$iv.\ F_A(x) \leq max\{F_A(x * y),\ F_A(y)\}.$

Definition(3.3): [7] A SVNG **G** = (M, N) with underlying set of V is defined to be a pair of G = (V, E) which is defined as

(i) $T_M : V \rightarrow [0, 1]$, $F_M : V\ [0, 1]$ and $I_M : V \rightarrow [0, 1]$ represents the degree of true membership function, degree of false membership function, and degree of indeterminacy membership function of the element m ∈ V, respectively, where

$0 \leqslant T_M(\mathrm{m}) + I_M(\mathrm{m}) + F_M(\mathrm{m}) \leqslant 3$, ∀ m ∈ V.

(ii) The function $T_N : E \rightarrow [0, 1]$, $I_N : E \rightarrow [0, 1]$ and $F_N : E \rightarrow [0, 1]$ are defined by

$$T_F(mn) \leq T_M(m) \wedge T_M(n)$$

$$I_F(mn) \leq I_M(m) \wedge I_M(n)$$

$$F_F(mn) \geq F_M(m) \wedge F_M(n)$$

It is free of any restriction so $0 \leqslant T_N(\mathrm{mn}) + I_N(\mathrm{mn}) + F_N(\mathrm{mn}) \leqslant 3$.

Definition(3.4): [7] (Complement). The complement of a single valued neutrosophic graph $\overline{G}$ of the graph **G** = (M, N) with underlying graph $G^* = (V, E)$ of V is defined by

1) $\bar{V} = V$

2) $\overline{T_A}(v_i) = T_A(v_i)$, $\overline{I_A}(v_i) = I_A(v_i), and\ \overline{F_A}(v_i) = F_A(v_i)$

3) $\overline{T_B}(v_i, v_j) = \min[T_A(v_i), T_A(v_j)] - T_B(v_i, v_j)$,

$\overline{I_B}(v_i, v_j) = \min[I_A(v_i), I_A(v_i)] - I_B(v_i, v_j)$, and

$\overline{F_B}(v_i, v_j) = |max[F_A(v_i), F_A(v_i)] - F_B(v_i, v_j)|, \forall\ v_i, v_j \in V$

Remark(3.5):

1) any edge e=uv in a SVNG G is called strong edge if the following satisfied:

$T_B(v_i, v_j) = \min[T_A(v_i), T_A(v_j)]$,

$I_B(v_i, v_j) = \min[I_A(v_i), I_A(v_i)]$, and

$F_B(v_i, v_j) = max[F_A(v_i), F_A(v_i)], \forall\ v_i, v_j \in V$

2) any graph G with all strong edge $uv \in E$ for every pair of vertices $u, v \in V$a is called complete graph.

3) every strong edge e in G is omitted in $\bar{G}$

Proposition(3.6): for every complete SVNG G has complete underline graph G^*

Remark(3.7): The inverse of proposition is not always true.

Example(3.8):- Assume that X = {0 , x_1 , x_2 , x_3} is a BH-algebra with the following Cayley table :

*	0	x_1	x_2	x_3

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0	0	0	0	0
x_1	x_1	0	0	x_1
x_2	x_2	x_2	0	x_2
x_3	x_3	x_3	x_3	0

Define the neutrosophic A = $(T_A(x), I_A(x), F_A(x))$ by:

$$T_A(x) = \begin{cases} 0.6 & if \quad x = 0, x_1, x_3 \\ 0.5 & if \quad x = x_2 \end{cases},$$

$$F_A(x) = \begin{cases} 0.1 & if \quad x = 0, x_1, x_3 \\ 0.4 & if \quad x = x_2 \end{cases},$$

$$I_A(x) = \begin{cases} 0.5 & if \quad x = 0, x_1, x_3 \\ 0.4 & if \quad x = x_2 \end{cases}$$

Then

$T_A(0 * 0) = T_A(0) = 0.6$

$F_A(0 * 0) = F_A(0) = 0.1$

$I_A(0 * 0) = I_A(0) = 0.5$

$T_A(0 * x_1) = T_A(x_1) = 0.6$

$F_A(0 * x_1) = F_A(x_1) = 0.1$

$I_A(0 * x_1) = I_A(x_1) = 0.5$

$T_A(0 * x_2) = T_A(x_2) = 0.5$

$F_A(0 * x_2) = F_A(x_2) = 0.4$

$I_A(0 * x_2) = I_A(x_2) = 0.4$

$T_A(0 * x_3) = T_A(x_3) = 0.6$

$F_A(0 * x_3) = F_A(x_3) = 0.1$

$I_A(0 * x_3) = I_A(x_3) = 0.5$

$T_A(x_1 * 0) = T_A(0) = 0.6$

$F_A(x_1 * 0) = F_A(0) = 0.1$

$I_A(x_1 * 0) = I_A(0) = 0.5$

$i.\ T_A(0) = T_A(x_1) = T_A(x_3) = 0.6 \geq T_A$

$F_A(0) = F_A(x_1) = F_A(x_3) = 0.1 \leq F_A(x$

$I_A(0) = I_A(x_1) = I_A(x_3) = 0.5 \geq I_A($

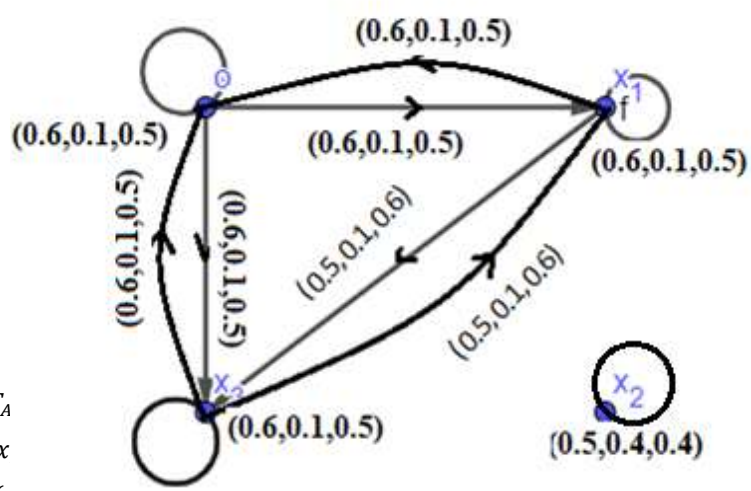

Figure 1: Graph G

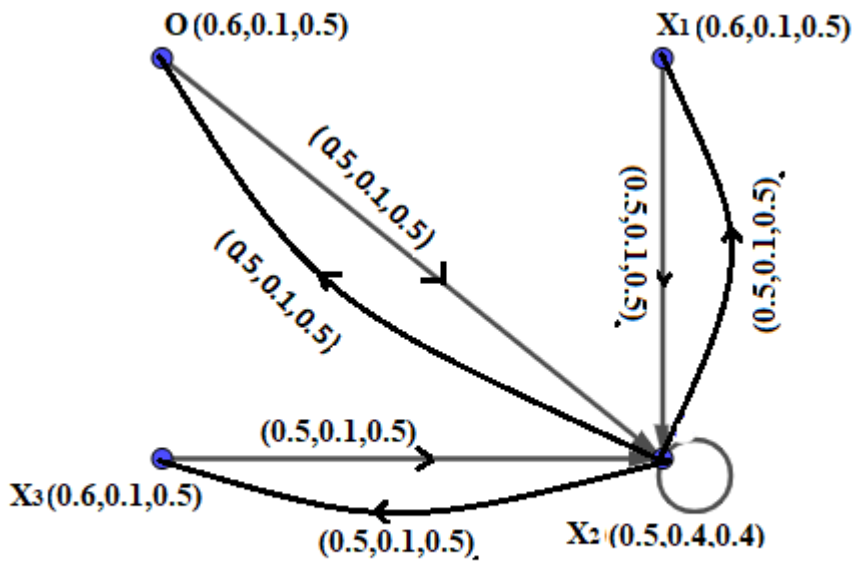

Figure 2 : Complement $\overline{G}$ of the graph G

$ii.$ $T_A(x) \geq min\{T_A(x * y),\ T_A(y)\}$ is verified.

$iii.$ $I_A(x) \geq min\{I_A(x * y),\ I_A(y)\}$ is verified.

$iv.$ $F_A(x) \leq max\{F_A(x * y),\ F_A(y)\}$ is verified.

Thus A = $(T_A(x), I_A(x), F_A(x))$ is a NBHI of X.

Definition(3.9):- Let X be a BH-algebra, then A is **Neutrosophic Pseudo BH-Subalgebra (simply NPBHS)** it satisfies the following conditions : $\forall\ x, y \in$ X

$i.$ $T_A(x) \geq inf_y\{T_A(x * y), T_A(x \# y),\ T_A(y)\}$.

$ii.$ $I_A(x) \geq inf_y\{I_A(x * y), I_A(x \# y),\ I_A(y)\}$.

$iii.$ $F_A(x) \leq sup_y\{F_A(x * y), F_A(x \# y),\ F_A(y)\}$.

And A is a **Neutrosophic Pseudo ideal** of a BH-algebra X **(simply NPBHI)** if satisfies the following : $\forall\ x, y \in$ X

$i.$ (1) $T_A(0) \geq T_A(x)$ & (2) $F_A(0) \leq F_A(x)$ & (3) $I_A(0) \geq I_A(x)$.

$ii.$ $T_A(x) \geq inf_y\{T_A(x * y), T_A(x \# y),\ T_A(y)\}$.

$iii.$ $I_A(x) \geq inf_y\{I_A(x * y), I_A(x \# y),\ I_A(y)\}$.

$iv.$ $F_A(x) \leq sup_y\{F_A(x * y), F_A(x \# y),\ F_A(y)\}$.

Example(3.10):- Assume that X = $\{0, x_1, x_2, x_3\}$ is a P.BH-algebra with the following Cayley tables :

$*$	0	x_1	x_2	x_3
0	0	0	0	0
x_1	x_1	0	0	x_1
x_2	x_2	x_2	0	x_2
x_3	x_3	x_3	x_3	0

$\#$	0	x_1	x_2	x_3
0	0	0	0	0
x_1	x_1	0	0	x_3
x_2	x_2	x_2	0	x_2
x_3	x_3	x_3	x_3	0

Define the neutrosophic A = $(T_A(x), I_A(x), F_A(x))$ by:

$$T_A(x) = \begin{cases} 0.7 & if \quad x = 0, x_1, x_3 \\ 0.6 & if \quad x = x_2 \end{cases},$$

$$F_A(x) = \begin{cases} 0.3 & if \quad x = 0, x_1, x_3 \\ 0.4 & if \quad x = x_2 \end{cases},$$

$$I_A(x) = \begin{cases} 0.6 & if \quad x = 0, x_1, x_3 \\ 0.4 & if \quad x = x_2 \end{cases}$$

Then

i. $T_A(0) = T_A(x_1) = T_A(x_3) = 0.7 \geq T_A(x_2) = 0.6$ &

$F_A(0) = F_A(x_1) = F_A(x_3) = 0.3 \leq F_A(x_2) = 0.4$ &

$I_A(0) = I_A(x_1) = I_A(x_3) = 0.6 \geq I_A(x_2) = 0.4.$

$ii.$ $T_A(x) \geq inf_y\{T_A(x * y), T_A(x \# y), T_A(y)\}$ is verified.

$iii.$ $F_A(x) \leq sup_y\{F_A(x * y), F_A(x \# y), F_A(y)\}$ is verified.

$iv.$ $I_A(x) \geq inf_y\{I_A(x * y), I_A(x \# y), I_A(y)\}$ is verified.

Thus A = $(T_A(x), I_A(x), F_A(x))$ is a NPBHI of X.

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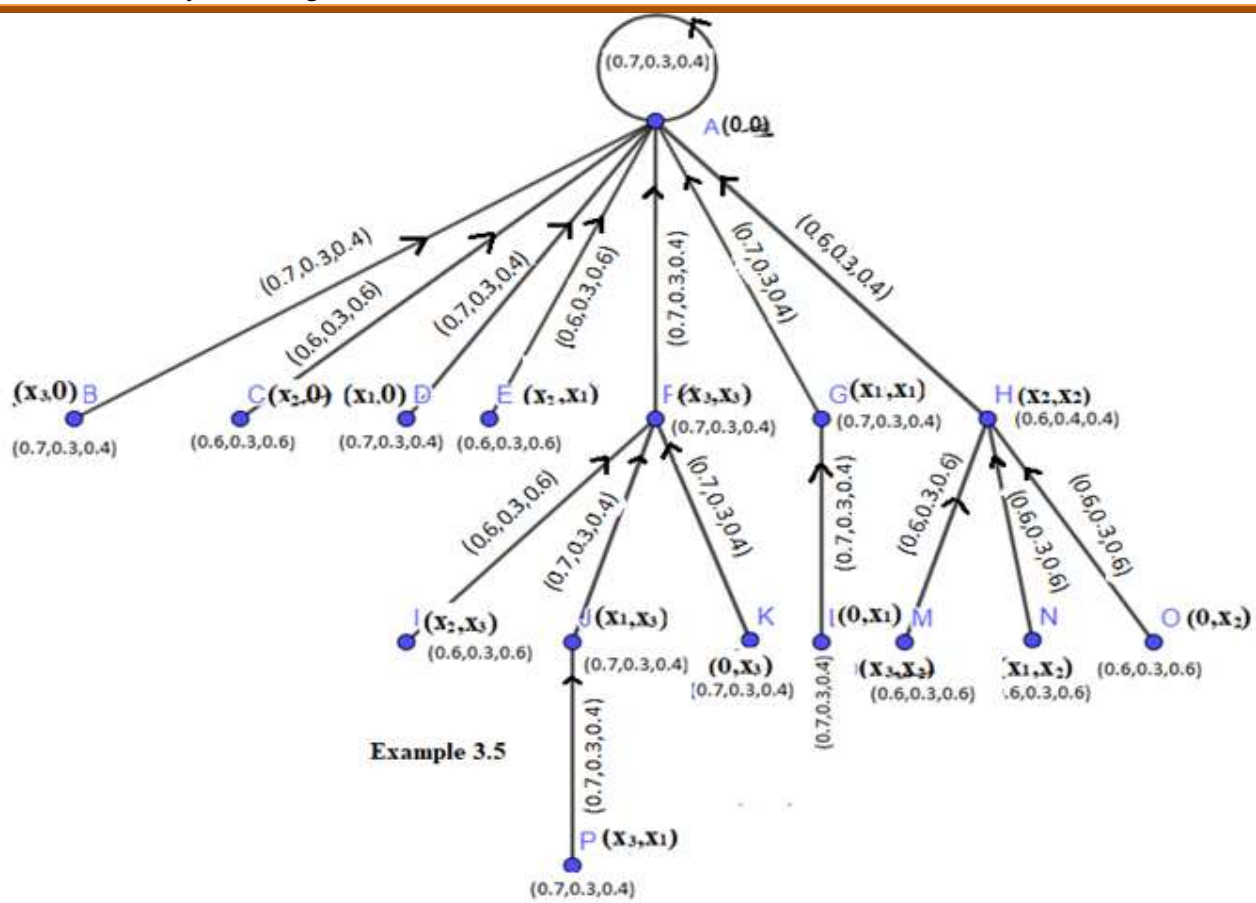

figure 3 neutrosophic P.BH-algebra Graph

We note that the graph is union of loop graph at (0,0) and a tree digraph with a root at (0,0)

Remark(3.11): the complement of the neutrosophic P.BH-algebra graph is not neutrosophic P.BH-algebra graph since it is not satisfied Cayley table

Example(3.12):- Let X = Z be the set of all integer numbers defined

A = $(T_A(x), I_A(x), F_A(x))$ is a NBHI by

$T_A(x) = \begin{cases} \vartheta_1 & if \quad x \in Z^+ \cup \{0\} \\ \vartheta_\circ & if \quad x \in Z^- \end{cases}$ so that $\vartheta_1 > \vartheta_\circ$

$I_A(x) = \begin{cases} \alpha_1 & if \quad x \in Z^+ \cup \{0\} \\ \alpha_\circ & if \quad x \in Z^- \end{cases}$ so that $\alpha_1 > \alpha_\circ$

$F_A(x) = \begin{cases} \lambda_1 & if \quad x \in Z^+ \\ \lambda_\circ & if \quad x \in Z^- \cup \{0\} \end{cases}$ so that $\lambda_1 < \lambda_\circ$

Such that ϑ_1, $\vartheta_\circ$, λ_1, $\lambda_\circ$, α_1 & $\alpha_\circ \in [0,1]$, so that $\vartheta_1 + \lambda_1 + \alpha_1 \leq 3$ & $0 \leq \vartheta_\circ + \lambda_\circ + \alpha_\circ \leq 3$, then A = $(T_A(x), I_A(x), F_A(x))$ is a NPBHI of X.

Lemma (3.13): - Let A = $(T_A(x), I_A(x), F_A(x))$ be a NPBHI of X and

$x \leq y$. Then $T_A(x) \geq T_A(y)$, $I_A(x) \geq I_A(y)$. and $F_A(x) \leq F_A(y)$

Proof: - Suppose that $x \leq y$, then $x * y = 0$ & $x \# y = 0$. Now,

$T_A(x * 0) = T_A(x) \geq inf\ \{T_A(x \square y), T_A(x \# y), T_A(y)\} =$

$inf\ \{T_A(0), T_A(0), T_A(y)\} = T_A(y)$, thus $T_A(x) \geq T_A(y)$.

Now, $F_A(x * 0) = F_A(x) \leq sup\ \{F_A(x \square\square y), F_A(x \# y), F_A(y)\} =$

$sup\ \{F_A(0), F_A(0), F_A(y)\} = F_A(y)$, then $F_A(x) \leq F_A(y)$. In the same way we can prove the third solution.

Proposition(3.14):- If $\{A_i, i \in \Omega\}$, is a family of NBHI of a BH-algebra X, such that $\cap_{i\in\Omega} A_i = (inf_y T_{A_i}(x), sup_y F_{A_i}(x), inf_y I_A(x))$, then $\cap_{i\in\Omega} A_i$ is a NBHI of X.

Proof: - $\cap_{i\in\Omega} A_i = (inf_y T_{A_i}(x), sup_y F_{A_i}(x), inf_y I_{A_i}(x))$, let $x, y \in$ X. Now,

$i.\ T_{A_i}(0) \geq T_{A_i}(x), \forall x \in$ X, $\forall i \in \Omega \Rightarrow inf_{y_{i\in\Omega}} T_{A_i}(0) \geq inf_{y_{i\in\Omega}} T_{A_i}(x) \Rightarrow T_{\underset{i\in\Omega}{\cap} A_i}(0) \geq T_{\underset{i\in\Omega}{\cap} A_i}(x)$ and $F_{A_i}(0) \leq F_{A_i}(x), \forall x \in$ X, $\forall$i $\in \Omega$ $\Rightarrow sup_{y_{i\in\Omega}} F_{A_i}(0) \leq sup_{y_{i\in\Omega}} F_{A_i}(x) \quad \Rightarrow F_{\underset{i\in\Omega}{\cup} A_i}(0) \leq F_{\underset{i\in\Omega}{\cup} A_i}(x)$, and

$I_{A_i}(0) \geq I_{A_i}(x), \forall x \in$ X, $\forall i \in \Omega \Rightarrow inf_{y_{i\in\Omega}} I_{A_i}(0) \geq inf_{y_{i\in\Omega}} I_{A_i}(x) \Rightarrow$

$I_{\underset{i\in\Omega}{\cap} A_i}(0) \geq I_{\underset{i\in\Omega}{\cap} A_i}(x)$.

$ii.$ Let $x, y \in$ X, $T_{\underset{i\in\Omega}{\cap} A_i}(x) = inf_y\ \{T_{A_i}(x)\}$ by (3.2)

$\geq inf_y\{\ min\{T_{A_i}(x * y),\ T_{A_i}(y)\}\} = inf_y\{T_{\underset{i\in\Omega}{\cap} A_i}(x * y), T_{\underset{i\in\Omega}{\cap} A_i}(y)\}$.

$iii.$ Let $x, y \in$ X, $F_{\underset{i\in\Omega}{\cup} A_i}(x) = sup_y\{F_{A_i}(x)\}$ by (3.2)

$\leq sup_y\{max\{\ F_{A_i}(x * y),\ F_{A_i}(y)\}\} = sup_y\{F_{\underset{i\in\Omega}{\cup} A_i}(x * y), F_{\underset{i\in\Omega}{\cup} A_i}(y)\}$.

$iv.$ Let $x, y \in$ X, $I_{\underset{i\in\Omega}{\cap} A_i}(x) = inf_y\{I_{A_i}(x)\}$ by (3.2)

$\geq inf_y\{min\{I_{A_i}(x * y),\ I_{A_i}(y)\}\} = inf_y\{I_{\underset{i\in\Omega}{\cap} A_i}(x * y), I_{\underset{i\in\Omega}{\cap} A_i}(y)\}$. Therefore $\cap_{i\in\Omega} A_i$ is a **NBHI** of X.

Proposition(3.15):- If $\{\ A_i,\ i \in \Omega\}$, is a family of NPBHI of a BH-algebra X, such that $\cap_{i\in\Omega} A_i = (inf_y T_{A_i}(x), sup_y F_{A_i}(x), inf_y I_A(x))$, then $\cap_{i\in\Omega} A_i$ is a NPBHI of X.

Proof: - The prove is in the same way as above proposition.

Theorem(3.16):- Let X be a P.BH-algebra, then a subset

A = $(T_A(x), I_A(x), F_A(x))$ is a NPBHI of X if and only if the fuzzy sets $\overline{T_A}$, $\overline{I_A}$ & $\overline{F_A}$ are Fuzzy Pseudo Ideal of X.

Proof:- Let A = $(T_A(x), I_A(x), F_A(x))$ be an NPBHI of X. for every

$x, y \in$ X, we have $\overline{T_A}(0) = 1 - T_A(0) \leq 1 - \overline{T_A}(x) = \overline{T_A}(x)$ &

$\overline{F_A}(0) = 1 - F_A(0) \geq 1 - F_A(x) = \overline{F_A}(x)$ & $\overline{I_A}(0) = 1 - I_A(0) \leq 1 - \overline{I_A}(x) = \overline{I_A}(x)$. Now, $\overline{T_A}(x) = 1 - T_A(x) \leq 1 - inf_y\{T_A(x * y), T_A(x \# y), T_A(y)\}$

$= sup_y\{1 - T_A(x * y), 1 - T_A(x \# y), 1 - T_A(y)\}$

$= sup_y\{\ \overline{T_A}(x * y), \overline{T_A}(x \# y), \overline{T_A}(y)\}$, then $\overline{T_A}$ is a F. P. I of X. And

$\overline{F_A}(x) = 1 - F_A(x) \geq 1 - sup_y\{F_A(x * y), F_A(x \# y), F_A(y)\}$

$= inf_y\{1 - F_A(x * y), 1 - F_A(x \# y), 1 - F_A(y)\}$

$= inf_y\ \{\ \overline{F_A}(x * y), \overline{F_A}(x \# y), \overline{F_A}(y)\}$, then $\overline{F_A}$ is a F. P. I of X. And $\overline{I_A}(x) = 1 - I_A(x) \leq 1 - inf_y\{I_A(x * y), I_A(x \# y), I_A(y)\}$

$= sup_y\{1 - I_A(x * y), 1 - I_A(x \# y), 1 - I_A(y)\}$

$= sup_y\{\ \overline{I_A}(x * y), \overline{I_A}(x \# y), \overline{I_A}(y)\}$, then $\overline{I_A}$ is a F. P. I of X.

Conversely, assume that $\overline{T_A}$, $\overline{I_A}$ & $\overline{F_A}$ are F. P. I of X, $\forall\ x, y \in$ X, we get

$i.\ \overline{T_A}(0) \leq \overline{T_A}(x) \Longrightarrow 1 - T_A(0) \leq 1 - T_A(x) \Longrightarrow T_A(0) \geq T_A(x)$ &

$\overline{F_A}(0) \geq \overline{F_A}(x) \Longrightarrow$ 1- $F_A(0) \geq$ 1- $F_A(x) \quad \Longrightarrow F_A(0) \leq F_A(x)$ &

$\overline{I_A}(0) \leq \overline{I_A}(x) \Longrightarrow$ 1- $I_A(0) \leq$ 1- $I_A(x) \Longrightarrow I_A(0) \geq I_A(x)$.

$ii.$ 1- $T_A(x) = \overline{T_A}(x) \leq sup_y\{\overline{T_A}(x * y), \overline{T_A}(x \# y), \overline{T_A}(y)\} =$

$sup_y\{$1- $T_A(x * y)$, 1- $T_A(x \# y)$, 1- $T_A(y)\} =$

1- $sup_y\{T_A(x * y), T_A(x \# y), T_A(y)\}$, therefore

$T_A(x) \geq inf_y\{T_A(x * y), T_A(x \# y), T_A(y)\}$.

$iii.$ 1- $F_A(x) = \overline{F_A}(x) \geq inf_y \{\overline{F_A}(x * y), \overline{F_A}(x \# y), \overline{F_A}(y)\} =$

$inf_y\{$1- $F_A(x * y)$, 1- $F_A(x \# y)$, 1- $F_A(y)\} =$

1- $inf_y\{F_A(x * y), F_A(x \# y), F_A(y)\}$, that is

$F_A(x) \leq sup_y\{F_A(x * y), F_A(x \# y), F_A(y)\}$.

$iv.$ 1- $I_A(x) = \overline{I_A}(x) \leq sup_y\{\overline{I_A}(x * y), \overline{I_A}(x \# y), \overline{I_A}(y)\} =$

$sup_y\{$1- $I_A(x * y)$, 1- $I_A(x \# y)$, 1- $I_A(y)\} =$

1- $sup_y\{I_A(x * y), I_A(x \# y), I_A(y)\}$, therefore

$I_A(x) \geq inf_y\{I_A(x * y), I_A(x \# y), I_A(y)\}$.

Hence A = $(T_A(x), I_A(x), F_A(x))$ is a NPBHI of X.

Definition (3.17): - Let X be a P.BH-algebra, then A is a **Neutrosophic Pseudo *n*-fold Closed ideal** of a BH-algebra X **(simply NPn-FCBHI)** if satisfies the following: $\forall\, x \in$ X

$i.\ \ min\ \{T_A(0 * x^n), T_A(0 \# x^n)\} \geq T_A(x)$.

$ii.\ min\ \{I_A(0 * x^n), I_A(0 \# x^n)\} \geq I_A(x)$.

$iii.\ max\ \{F_A(0 * x^n), F_A(0 \# x^n)\} \leq F_A(x)$.

Example(3.18):- Assume that X = $\{0, x_1, x_2, x_3\}$ is a P.BH-algebra with the following Cayley tables :

$*$	0	x_1	x_2	x_3
0	0	x_1	x_2	x_2
x_1	x_1	0	x_1	x_1
x_2	x_2	x_2	0	x_3
x_3	x_3	x_3	x_1	0

$\#$	0	x_1	x_2	x_3
0	0	x_2	x_2	x_2
x_1	x_1	0	x_3	x_1
x_2	x_2	x_2	0	x_2
x_3	x_3	x_3	x_3	0

Define the neutrosophic A = $(T_A(x), I_A(x), F_A(x))$ by:

$$T_A(x) = \begin{cases} 0.6 & if \quad x = 0 \\ 0.5 & if \quad x = x_1, x_2, x_3 \end{cases}$$

$$F_A(x) = \begin{cases} 0.3 & if \quad x = 0 \\ 0.4 & if \quad x = x_1, x_2, x_3 \end{cases}$$

$$I_A(x) = \begin{cases} 0.7 & if \quad x = 0 \\ 0.6 & if \quad x = x_1, x_2, x_3 \end{cases}$$

Then

$min\{T_A(0 * 0^n), T_A(0 \# 0^n)\} \geq T_A(0)$

$min\{T_A(0 * {x_1}^n), T_A(0 \# {x_1}^n)\} \geq T_A(x_1)$

$min\{T_A(0 * {x_2}^n), T_A(0 \# {x_2}^n)\} \geq T_A(x_2)$

$min\{T_A(0 * {x_3}^n), T_A(0 \# {x_3}^n)\} \geq T_A(x_3)$ &

$max\ \{F_A(0 * 0^n), F_A(0 \# 0^n)\} \leq F_A(0)$

$max\ \{F_A(0 * {x_1}^n), F_A(0 \# {x_1}^n)\} \leq F_A(x_1)$

$max\ \{F_A(0 * {x_2}^n), F_A(0 \# {x_2}^n)\} \leq F_A(x_2)$

$max\ \{F_A(0 * {x_3}^n), F_A(0 \# {x_3}^n)\} \leq F_A(x_3)$ &

$min\{I_A(0 * 0^n), I_A(0 \# 0^n)\} \geq I_A(0)$

$min\{I_A(0 * {x_1}^n), I_A(0 \# {x_1}^n)\} \geq I_A(x_1)$

$min\{I_A(0 * {x_2}^n), I_A(0 \# {x_2}^n)\} \geq I_A(x_2)$

$min\{I_A(0 * {x_3}^n), I_A(0 \# {x_3}^n)\} \geq I_A(x_3)$.

Hence A = $(T_A(x), I_A(x), F_A(x))$ is a NPn-FCBHI of X.

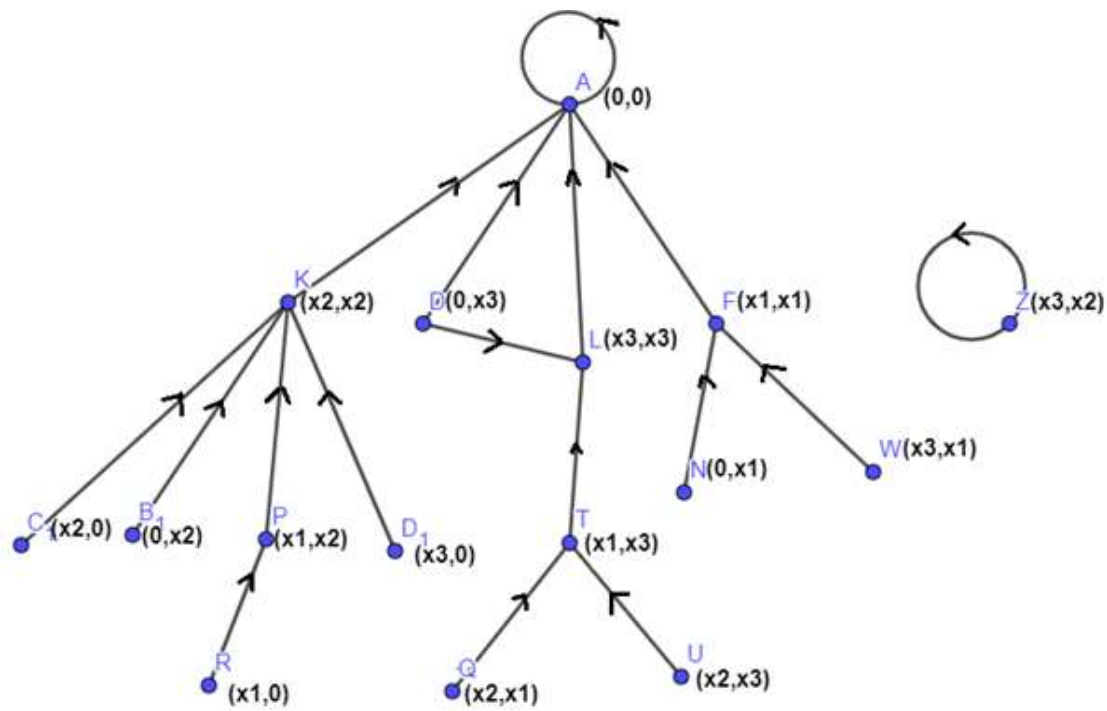

Figure 4: Neutrosophic P.BH algebra graph

We note that the digraph has two loops and single cycle

Theorem(3.19):- Let X be a P.BH-algebra and A = ($T_A(x)$, $I_A(x)$, $F_A(x)$) is a NPBHI of X, then A is NPn-FCBHI if and only if the set upper α_1-level U(T_A,α_1) is P. n-F. C. I of X, $\forall\alpha_1 \in]^-0, 1^+[$ & the set lower α_2-level L (F_A,α_2) is P. n-F. C. I of X, $\forall\alpha_2 \in]^-0, 1^+[$.

Proof:- Let A= ($T_A(x)$, $I_A(x)$, $F_A(x)$) be a NPn-FCBHI of X, and U(T_A,α_1) $\neq \emptyset \neq$ L(F_A,α_2) for every α_1, $\alpha_2 \in]^-0, 1^+[$. Obviously, 0 ∈ U (T_A,α_1) ∩ L (F_A,α_2), since $T_A(0) \geq \alpha_1$ & $F_A(0) \leq \alpha_2$. Assume that $x \in$ X such that, $(0 * x^n)$ & $(0 \# x^n) \in$ U (T_A,α_1) then, $T_A(0 * x^n) \geq \alpha_1$ and $T_A(0 \# x^n) \geq \alpha_1$. It follows that $T_A(x) \geq min\ \{T_A(0 * x^n), T_A(0 \# x^n)\} \geq \alpha_1$ so $x \in$ U(T_A,α_1), therefore U(T_A,α_1) is a P. n-F. C. I. Now, assume that $x \in$ X such that $(0 * x^n)$ & $(0 \# x^n) \in$ L(F_A,α_2) then, $F_A(0 * x^n) \leq \alpha_2$ & $F_A(0 \# x^n) \leq \alpha_2$. It follows that $F_A(x) \leq max\{F_A(0 * x^n), F_A(0 \# x^n)\} \leq \alpha_2$ so $x \in$ L (F_A,α_2), therefore, L (F_A,α_2) is a P. n-F. C. I.

Conversely, assume that α_1, $\alpha_2 \in]^-0, 1^+[$ and U(T_A,α_1) & L(F_A,α_2) are P. n-F. C. I, $\forall\ x \in$ X, let $T_A(x) = \alpha_1$ & $F_A(x) = \alpha_2$ then, $x \in$ U(T_A,α_1) ∩ L (F_A,α_2) & U(T_A,α_1) $\neq \emptyset \neq$ L(F_A,α_2) since U(T_A,α_1) & L(F_A,α_2) are P. n-F. C. I of X then,

0 ∈ U (T_A,α_1) ∩ L (F_A,α_2). Hence $T_A(0) \geq \alpha_1 = T_A(x)$ & $F_A(0) \leq \alpha_2 = F_A(x)$, $\forall\ x \in$ X. Now we take the opposite, let $v \in$ X such that

$T_A(v) < min\ \{T_A(0 * v^n), T_A(0 \# v^n)\}$. Now let $\alpha_3 = (0.5)(T_A(v) + min\ \{T_A(0 * v^n), T_A(0 \# v^n)\})$ then,

$T_A(v) < \alpha_3 < min\ \{T_A(0 * v^n), T_A(0 \# v^n)\}$. Hence $v \notin$ U (T_A,α_3), but $(0 * v^n)$ and $(0 \# v^n) \in$ U (T_A,α_3). Thus,

U (T_A,α_3) is not P. n-F. C. I of X. And let $k \in$ X such that $F_A(k) > max\ \{F_A(0 * k^n), F_A(0 \# k^n)\}$, now, let $\alpha_4 = (0.5)\ (F_A(k) + max\ \{F_A(0 * k^n), F_A(0 \# k^n)\})$ then,

$max\ \{F_A(0 * k^n), F_A(0 \# k^n)\} < \alpha_4 < F_A(k)$. Hence

$(0 * k^n) \in$ L(F_A,α_4) and $(0 \# k^n) \in$ L(F_A,α_4), but $k \notin$ L(F_A,α_4), therefore L(F_A,α_4) is not P. n-F. C. I of X, This is impossible from the assumption, therefore, A = ($T_A(x)$, $I_A(x)$, $F_A(x)$) is NPn-FCBHI of X.

Definition (3.20): -Let X be a BH-algebra, a subsets A=($T_A(x)$, $I_A(x)$, $F_A(x)$) and B = ($T_B(x)$, $I_B(x)$, $F_B(x)$) are two NPBHI of X. Define the intersection by form:

$(A_{TIF} \cap B_{TIF})(x)$={$max(T_A(x), T_B(x))$, $min(F_A(x), F_B(x))$, $max(I_A(x), I_B(x))$.

Proposition (3.21): - Let A=($T_A(x)$, $I_A(x)$, $F_A(x)$), B=($T_B(x)$, $I_B(x)$, $F_B(x)$) are two NPBHI of X. The intersection $(A_{TIF} \cap B_{TIF})(x)$ also NPBHI of X.

Proof:- Let A=($T_A(x)$, $I_A(x)$, $F_A(x)$), B = ($T_B(x)$, $I_B(x)$, $F_B(x)$), then $(A_{TIF} \cap B_{TIF})(x)$ = {$max(T_A(x), T_B(x))$, $min(F_A(x), F_B(x))$, $max(I_A(x), I_B(x))$. If $max(T_A(x), T_B(x)) = T_A(x)$ or $T_B(x)$ & if $min(F_A(x), F_B(x)) = F_A(x)$ or $F_B(x)$ & if $max(I_A(x), I_B(x)) = I_A(x)$ or $I_B(x)$. Thus $(A_{TIF} \cap B_{TIF})(x)$={($T_A(x)$ or $T_B(x)$), ($F_A(x)$ or $F_B(x)$), ($I_A(x)$ or $I_B(x)$). Therefore, the results are as follows

$(A_{TIF} \cap B_{TIF})(x)$={($T_A(x)$, ($F_A(x)$, ($I_A(x)$) ∨ $(A_{TIF} \cap B_{TIF})(x)$={($T_A(x)$, ($F_B(x)$, ($I_A(x)$) ∨

$(A_{TIF} \cap B_{TIF})(x)$={($T_A(x)$, ($F_B(x)$, ($I_B(x)$) ∨

$(A_{TIF} \cap B_{TIF})(x)$={($T_A(x)$, ($F_A(x)$, ($I_B(x)$) ∨

$(A_{TIF} \cap B_{TIF})(x)$={($T_B(x)$, ($F_A(x)$, ($I_A(x)$) ∨

$(A_{TIF} \cap B_{TIF})(x)$={($T_B(x)$, ($F_A(x)$, ($I_B(x)$) ∨

$(A_{TIF} \cap B_{TIF})(x)$={($T_B(x)$, ($F_B(x)$, ($I_A(x)$) ∨

$(A_{TIF} \cap B_{TIF})(x)$={($T_B(x)$, ($F_B(x)$, ($I_B(x)$). Thus $(A_{TIF} \cap B_{TIF})(x)$ is NPBHI of X.

Conclusion

In this work, we presented BH-algebra and pseudo BH-algebra on neutrosophic groups with neutrosophic graph stracture in graph theory, and presented many new characteristics and examples. And studied the relationship between the concept of neutrosophic BH-algebra and neutrosophic pseudo BH-algebra. We recommend studying BH-algebra with union and multiplication in neutrosophic groups.

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