Certain properties of b-closed ideal in a BCI-algebra

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Abstract*—In this paper has been devoted for defining the notion of b-closed BCI-algebra by adopting notions of BH-algebra. Also, we have studied some types of ideal, that could be proved by BCI-algebra. In addition, we have introduced some theorems and propositions related to BCI-algebra detailed with proofs.*

Keywords—BCI-algebra, Closed ideals, P-ideals, Positive implicative ideals, Quasi-associative ideals

INTRODUCTION

In 1966,the notion of BCK-algebras was found firstly by (Y.Imai) and (K.Iseki) [17]. On the other hand, (K.Iseki) had defined the BCI–algebra [8] at the same year. Also, (C. S. Hoo) introduced the notion of an ideal and a closed ideal in 1991,for BCI-algebra[4]. A generalization of BCH-algebras that is called the BH-algebra used defined by (Jun et al) in 1998 [16]. In 2000, (Y. L. Liu, J. Meng, X. H Zhang and Z. Caiyue) gave us the notions of q-Ideals and a-Ideals in BCI-Algebras[18], and (A. Namdar and A. B. Saeid) found the concept of n-fold implicative ideal [2]. A fantastic ideal in a BCI-algebra was introduced by (A. B. Saeid) in 2010[1]. In 2011, "H.M.A.saeed introduced the notion of some type of ideals in a BH-algebra" [5]. In this paper, we have introduced the notion of some type of ideals in a BCI-algebra.

1. BASIC CONCEPTS AND NOTATIONS ABOUT BCI,BCK,BCH, AND BH- ALGEBRA:

Some basic concepts are recalled about BCI-algebra , BCK-algebra , BCH-algebra, BH-algebra in this section .

Definition(1.1) : [8,9,19]

Assume a set $Y \neq \emptyset$, BCI − algebra Y is said to be an algebra (Y,∘ ,0) of type (2,0) , ∘ be binary operation and 0 is a constant , satisfies the coditions: $\forall$ w, v, k$\in$ Y:

$$((w \circ v) \circ (w \circ k)) \circ (k \circ v) = 0.$$

$$(w \circ (w \circ v)) \circ v = 0.$$

$$w \circ w = 0.$$

$$w \circ v = 0 \text{ and } v \circ w = 0 \Rightarrow w = v.$$

Example (1.2): [1]

Assume Y = {0, 1, i, j, k}and defined the binary operation ∘ as follows:

∘	0	1	i	j	k
0	0	0	i	i	i
1	1	0	i	i	i
i	i	i	0	0	0
j	j	i	1	0	0
k	k	i	1	1	0

Thus, $(Y, \circ, 0)$ is a BCI − algebra.

Definition(1.3) : [6,9,17]

Each BCK − algebra Y be a BCI − algebra Y if satisfies the condition:

$0 \circ w = 0 \; \forall \, w \in Y$.

Remark (1.4):[8,9]

Each BCK − algebra Y be BCI − algebra Y, but the vice is not true, in example (1.2), Y be a BCI − algebra , but not a BCK − algebra as $i = 0 \circ i \neq 0$.

Proposition(1.5): [7,11,12]

If BCH − algebra Y, then the following conditions satisfies $\forall\, w, v, k \in Y$.

$$w \circ 0 = w.$$

$$(w \circ (w \circ v)) \circ v = 0.$$

$$0 \circ (w \circ v) = (0 \circ w) \circ (0 \circ v).$$

$$0 \circ (0 \circ (0 \circ w)) = 0 \circ w.$$

$\omega \leq v \Rightarrow 0 \circ w = 0 \circ v$.

Remark (1.6):[11,14]

A BCH-algebra Y is known a proper if it is not a BCI-algebra, where each BCI − algebra Y be a BCH − algebra Y, but the vice is not true as in the following example.

Example (1.7): [13]

Let Y={0, 1, 2, 3} and ∘ be a binary operation is define by:

∘	0	1	2	3
0	0	0	0	0
1	1	0	3	3
2	2	0	0	2
3	3	0	0	0

Thus $(Y, \circ, 0)$ be a BCH − algebra Y but is not a BCI − algebra Y since

$$((2 \circ 3) \circ (2 \circ 1)) \circ (1 \circ 3) = (2 \circ 0) \circ 3 = 2 \circ 3 = 2 \neq 0$$

Definition(1.8) : [3,15,16]

Suppose $Y \neq \emptyset$,BH − algebra Y with a binary operation ∘ and a constant 0 satisfies the conditions:

$$w \circ w = 0, \forall\, w \in Y.$$

$$w \circ v = 0 \text{ and } v \circ w = 0 \Rightarrow w = v, \forall\, w, v \in Y.$$

$$w \circ 0 = w, \forall\, w \in Y.$$

Example (1.9): [16]

Assume $Y = \{0, 1, 2, 3\}$ and defined the binary operation $\circ$ by:

$\circ$	0	1	2	3
0	0	3	0	2
1	1	0	0	0
2	2	2	0	3
3	3	3	1	0

Hence $(Y, \circ, 0)$ be a BH-algebra Y .

Remark (1.10):[16]

Each BCH − algebra Y be a BH − algebra Y, but the vice is not true . In example(1.9), the BH − algebra Y is not a BCH − algebra.

Since $(0 \circ 1) \circ 2 = 3 \circ 2 = 1 \neq (0 \circ 2) \circ 1 = 0 \circ 1 = 3$.

Definition(1.11) : [16]

Suppose $\emptyset \neq S \subseteq Y$, of a BH-algebra Y is denoted as a subalgebra (BH − algebra)of Y if $w \circ v \in S$ $\forall w, v \in S$.

Remark (1.12):[5]

According to remark(1.4) , remark(1.6) and remark(1.10) we obtain:

Each BCK − algebra is a BCH − algebra.

Each BCK − algebra is a BH − algebra.

Each BCI − algebra is a BH − algebra.

Definition(1.13) : [5]

The set $Y+ = \{ w \in Y : 0 \circ w = 0 \}$ is dnoted by the BCA-part of Y where Y be a BH − algebra .

2. Basic Concepts and Notations about ideal of BCI and BH − algebra :

We introduce the basic concepts about ideals, P-ideals, closed ideals, implicative ideals, , positive implicative ideals, quasi-associative ideals , in this section .

Definition(2.1) : [10]

Assume $\emptyset \neq R \subseteq Y$, $(Y, \circ, 0)$ is a BCI − algebra, R be denoted an ideal of Y if R satisfying the conditions:

$$0 \in R.$$

$w \circ v \in R$ and $v \in R \Rightarrow w \in R$.

Definition(2.2) : [3,16]

Suppose that R is a subset of a BH-algebra Y, that is nonempty, then R is said to be an ideal of Y if the following conditions are satisfied:

$0 \in R$.

$w \circ v \in R$ and $v \in R$ imply $w \in R$

Definition(2.3) : [5]

Suppose $\emptyset \neq R \subseteq Y$, R is an ideal of a $BH-$ algebra Y, R is said to be a closed ideal of Y if $\forall$ w∈R, we obtaind $0 \circ w \in R$.

Definition(2.4) : [5]

Assume $\emptyset \neq R \subseteq Y$ and R is an ideal of a $BH-$ algebra Y, hence R is said to be a $P-$ ideal of Y if the following conditions are held:

$$0 \in R.$$

$(w \circ k) \circ (v \circ k) \in R$ and $v \in R \Rightarrow w \in R$, for all $w, v, k \in Y$.

Definition(2.5) : [5]

Assume $\emptyset \neq R \subseteq Y$, and R is an ideal of a $BH-$ algebra Y, thus R is said to be an implicative ideal of Y the following conditions are held:

$$0 \in R.$$

$(w \circ (v \circ w)) \circ k \in R$ and $k \in R \Rightarrow w \in R$, for all $w, v, k \in Y$.

Definition(2.6) : [5]

Suppose $\emptyset \neq R \subseteq Y$ and R is an ideal of a $BH-$ algebra Y, thus R is said to be a positive implicative ideal if the following conditions are satisfied:

$0 \in R$.

$$(w \circ v) \circ k \in R \text{ and } v \circ k \in R \Rightarrow w \circ k \in R \text{ for all } w, v, k \in Y.$$

Definition(2.7) : [5]

Suppose R is an ideal of a $BH-$ algebra Y, thus R denoted by a quasi-associative ideal if for each $w \in Y$, we obtaind $0 \circ (0 \circ w) = 0 \circ w$.

Proposition(2.8):[1] If Y is an associative BCI-algebra, then each ideal is a fantastic ideal of Y.

Definition(2.9) : [5]

Suppose that Y is a BH-algebra and R is an ideal of Y, therefore R is said to be a closed ideal w.r.t. an element b∈Y if $b \circ (0 \circ w) \in R$, $\forall w \in R$.

Remark (2.10):[5]

Assume R be an ideal of a $BH-$ algebra Y such that $R = \{0\}$, then R is said to be $0-$ closed ideal, also if $R = Y$ then R called a b-closed ideal, for all b∈Y.

Definition(2.11) : [5]

Let Y is a closed BCI-algebra w.r.t. b, iff each proper ideal is a b-closed ideal such that Y is a $BCI-$ algebra with b∈Y.

3-The Main Results:

The devoted of study the concept for a closed ideal will be presented In this section with a closed BCI-algebra, for that we involve these notions with some types of ideals in BCI-algebra which we mentioned in the paper.

Theorem(3.1):

Suppose that Y= Y+ where Y a BH-algebra. If R is an ideal of Y, then R is a b-closed ideal, ∀b∈R.

Proof:

Assume b∈R.

To prove R is a b-closed ideal, assume Y BCI- algebra:

⇒ Y BH-algebra [By Remark(1.12) , each BCI-algebra is a BH- algebra]

Suppose $v \in R$,then we have

$b \circ (0 \circ v) = b \circ 0$ [By definition(1.13), Since Y=Y+ and 0∘ Y = 0, ∀v∈Y+]

= b [By definition(1.8) ,Since v∘ 0 = v, ∀v∈Y]

$$\Rightarrow\ b \circ (0 \circ v) = b \in R$$

Hence, R is said a b − closed ideal of Y, ∀ b∈R.

Theorem(3.2):

If Y is a BCI-algebra and R be an ideal of Y, then R is a b-closed ideal, ∀b∈R.

Proof:

Suppose Y BCI- algebra

⇒ Y BH-algebra [By Remark(1.12),(3)]

Suppose b∈R and Since R be an ideal of a BCI − algebra Y.

⇒ R is an ideal of a BH-algebra Y [By Remark(1.12),(3)]

But, $0 \circ v = 0,\ \forall v \in Y$ [By definition(1.3)]

⇒ $Y = Y_+$ [By definition(1.13)]

⇒ By Theorem (3.1) we obtaind R is a b − closed ideal of Y .

We obtain R is a b − closed ideal of Y, ∀b ∈ R.

Theorem(3.3):

If { Ri , i∈ρ} is a collection of b-closed ideals of a BCI-algebra Y, hence

$(\bigcap_{i\in\rho} R_i)$ is a b-closed ideal of Y.

Proof:

We must show $(\bigcap_{i\epsilon\rho} R_i)$ is an ideal .

(1) $0 \in R_i$, ∀ i∈ρ [By definition(2.1)]

⇒$0 \in (\bigcap_{i\in\rho} R_i)$

(2) Suppose w∘ v∈$(\bigcap_{i\in\rho} R_i)$ and y ∈$(\bigcap_{i\in\rho} R_i)$

⇒ w∘ v∈Ri and y ∈ Ri ,∀ i∈ρ

⇒ w∈ Ri , ∀ i∈ρ [By definition(2.1),Since each Ri is ideal, ∀ i∈ρ]

⇒ w ∈ $(\bigcap_{i\in\rho} R_i)$

Thus , $(\bigcap_{i\in\rho} R_i)$ is an ideal of Y.

To prove $(\bigcap_{i\epsilon\rho} R_i)$ is a b-closed ideal

Suppose x∈$(\bigcap_{i\in\rho} R_i)$,then w ∈Ri ,∀ i∈ρ

⇒ b∘ (0∘ w) ∈ Ri, ∀ i∈ρ [By definition(2.9), Since Ri is b-closed ideal,∀ i∈ρ]

⇒ b∘ (0∘ w) ∈$(\bigcap_{i\in\rho} R_i)$

$(\bigcap_{i\in\rho} R_i)$ is a b-closed ideal of Y.

Theorem(3.4):

If { Ri , i∈ρ } is a collection of b − closed ideals of a BCI − algebra Y,

therefore $(\bigcup_{i\in\rho} R_i)$ is a b-closed ideal of Y.

Proof:

In order to show that $(\bigcup_{i\in\rho} R_i)$ is an ideal:

$1 -\ 0 \in Ri\ ,\ \forall\, i \in \rho \qquad [By\ definition (2.1)]$

$\Rightarrow\ 0 \in (\bigcup_{i\in\rho} R_i\ ).$

$2 -\ Suppose\ w \circ v\ \in (\bigcup_{i\in\rho} R_i\ )\ and\ \ v\ \in (\bigcup_{i\in\rho} R_i\ )$

$\exists Rj\ , Rk\ \in \{\ Ri\ \}\ i \in \rho\ , where\ w \circ v \in Rj\ and\ v\ \in Rk\ ,$

$\Rightarrow\ either\ Rj \subseteq Rk\ \ or\ Rk \subseteq Ri \qquad [\ Since\ \ \{Ri\}\ i \in \rho\ is\ a\ chain\ ]$

$\Rightarrow\ either\ w \circ v \in Rj\ \ and\ \ v \in Rj \qquad or \qquad w \circ v \in Rk\ \ and\ \ v \in Rk$

$\Rightarrow\ either\ w\ \in Rj\ \ \ or\ \ v\ \in Rk \qquad [By\ definition (2.1), Since\ Rj\ and\ Rk\ are\ ideals]$

$\Rightarrow\ w\ \in (\bigcup_{i\in\rho} R_i\ )$

$\Rightarrow\ (\bigcup_{i\in\rho} R_i\ )\ \ is\ an\ ideal$

$To\ show\ \ (\bigcup_{i\in\rho} R_i\ )\ \ is\ b - closed\ ideal\ of\ Y.$

$let\ w\ \in (\bigcup_{i\in\rho} R_i\ )$

$\exists Rj\ \in \{\ Ri\ \}\ i \in \rho\ \ \ where\ \ \ w\ \in Rj$

$\Rightarrow\ b \circ (0 \circ w)\ \in Rj \qquad [By\ definition (2.9)]$

$\Rightarrow\ b \circ (0 \circ w)\ \in (\bigcup_{i\in\rho} R_i\ )$

Hence $(\bigcup_{i\in\rho} R_i)$,is said to be a b − closed ideal of Y.

Proposition(3.5):

If Y be a BCI-algebra, then each p − ideal of Y be an ideal of Y.

Proof:

Suppose R is a p − ideal .

Suppose Y BCI − algebra .

$\Rightarrow\ Y\ \ BH - algebra\ [By\ Remark (1.12), Since\ each\ BCI - algebra\ is\ a\ BH -\ algebra]$

$To\ prove\ \ R\ \ is\ an\ ideal$

$1 -\ 0 \in R \qquad [By\ definition (2.4)\ of\ a\ p - ideal\ , Since\ R\ is\ a\ p - ideal]$

$2 -\ Suppose\ w, v \in Y\ such\ that\ w \circ v \in R\ \ and\ v \in R$

$$Choose\ k = 0, then:$$

$$(w \circ k) \circ (v \circ k) = (w \circ 0) \circ (v \circ 0)$$

$$= w \circ v \quad [By\ definition(1.8)\ of\ a\ BH - algebra\ , Since\ w \circ 0 = w,\ \forall w \in Y]$$

$$\Rightarrow \ (w \circ k) \circ (v \circ k) \in R \ \ and \ \ v \in R$$

$\Rightarrow \ x \in R$ [By definition(2.4) of a p-ideal]

Hence, R be an ideal of Y.

Theorem(3.6):

If Y be a BCI-algebra ,where Y = Y+ and R be a p − ideal of Y, then R is a b − closed ideal of Y, ∀b∈R .

Proof:

Suppose Y BCI − algebra

⇒ Y BH − algebra [By Remark(1.12)Since each BCI-algebra is a BH- algebra]

Assume b∈R, since R is a p-ideal ⇒ By proposition (3.5) we obtained

R is an ideal of Y. But b∈R ⇒ By Theorem (3.1) we have to

R be a b − closed ideal of Y implies R is a b − closed ideal of Y, ∀b∈R .

Theorem(3.7):

If R is a p − ideal of Yand Y be a BCI − algebra , then R is a b − closed ideal of Y, ∀b∈R .

Proof:

Suppose Y BCI − algebra

⇒ Y BH − algebra [By Remark(1.12),Since each BCI-algebra is a BH- algebra]

Suppose b∈R, Since R is a p − ideal of a BCI − algebra Y.

⇒ R be a p − ideal of a BCI − algebra R [By Remark(1.12)Since each BCI − algebra is a BH − algebra.]

But, 0∘ w=0, ∀w∈Y [By definition(1.3)]

⇒ $Y = Y_+$ [By definition(1.13)]

⇒ By Theorem (3.6) we obtained that R represents a b-closed ideal of the BCI-algebra Y.

Thus, R be a b − closed ideal of Y, ∀b ∈ R .

Proposition(3.8):

If Y be a BCI − algebra, then each implicative ideal is an ideal of Y.

Proof:

Suppose Y BCI- algebra

⇒ Y BH-algebra [By Remark(1.12),Since each BCI-algebra is a BH- algebra]

Now, by supposing that R representing an implicative ideal , so in order to have R presenting an ideal:

1- 0∈R [By definition(2.5) of an implicative ideal ,Since R is an implicative ideal]

2- Let w, k∈Y such that w∘ k ∈R and k∈R

$$Choose\ v = w, then$$

$$(w \circ (v \circ w)) \circ k = (w \circ (w \circ w)) \circ k$$

$$= (w \circ 0) \circ k \qquad [By\ definition(1.8), Since\ w \circ w = 0,\ \forall w \in Y]$$

$$\Rightarrow\ (w \circ (v \circ w)) \circ k \in R \quad and \quad k \in R$$

$\Rightarrow\ w \in R$ [By definition(2.9),Since R is an implicative ideal]

Thus, R is an ideal of Y.

Theorem(3.9):

If R be an implicative ideal of BCI − algebra Y, where Y = Y+ and , then R be a b − closed ideal of Y, ∀b∈R .

Suppose b∈R and since R be an implicative ideal .

we obtained by Proposition (3.8), R is an ideal of Y.

Now but b∈R ⇒ By Theorem (3.1) we have that R representing a b-closed ideal of the BCI − algebra. .

Thus , R be a b − closed ideal of Y, ∀b∈R

Theorem(3.10):

If R be an implicative ideal of BCI − algebra Y,thus R is called b − closed ideal of Y, ∀b∈R .

Proof:

Suppose b∈R, Since R by imposition be an implicative ideal of a BCI − algebra Y.

⇒ R be an implicative ideal of a BH − algebra Y. [By remark(1.12) ,(3)]

But, 0∘ w=0, ∀w∈Y [By definition(1.3)]

⇒ Y = Y+ [By definition(1.13)]

⇒ we obtained R be a b − closed ideal of Y . [By Theorem (3.9)]

Thus, we have R be a b − closed ideal of Y, ∀b ∈ R .

Proposition(3.11):

Each positive implicative ideal in BCI − algebra Y is an ideal of Y.

Proof:

Suppose Y BCI − algebra .

⇒ Y BH − algebra . [By Remark(1.12),Since each BCI-algebra is a BH- algebra]

Suppose that R represents a positive implicative ideal, so to have R be an ideal:

1- 0∈R [By definition(2.6),Since R is a positive implicative ideal]

2- Suppose w, v∈Y where w ∘ v∈R and v∈R

$$Choose\ k = 0. Then$$

$$(w \circ v) \circ k = (w \circ v) \circ 0$$

$$= (w \circ v) \quad [By\ definition(1.8)\ of\ a\ BH-algebra, Since\ w \circ 0 = w,\ \forall w \in Y\ ]$$

$$And$$

$$v \circ k = v \circ 0 = v \quad [By\ definition(1.8)\ of\ a\ BH-algebra\ , Since\ w \circ 0 = w,\ \forall w \in Y]$$

$$\Rightarrow\ (w \circ v) \circ k \in R\ \ and\ \ v \circ k \in R$$

$$\Rightarrow\ w \circ k \in R \quad [By\ definition(2.6), Since\ R\ is\ an\ implicative\ ideal\ ]$$

$$But\ w \circ k = w \circ 0 = w \quad [By\ definition(1.8)\ of\ a\ BH-algebra\ , Since\ w \circ 0 = w,\ \forall w \in Y]$$

$$\Rightarrow\ w \in R$$

Thus, R is an ideal of Y.

Theorem(3.12):

If R be a positive implicative ideal of Y such that Y be a BCI − algebra ,Y = Y+. Thus, R is a b − closed ideal of Y for all b belongs to R.

Proof:

Assume b∈R, since R is a positive implicative ideal

⇒ we obtaind R is an ideal of Y . [By Proposition(3.11)]

But b∈R implies we found R is a b − closed ideal of Y. [By Theorem(3.1)]

Thus, R be a b − closed ideal of Y, ∀b∈R.

Theorem(3.13):

If R be a positive implicative ideal of Y where Ybe a BCI − algebra, then R is a b − closed ideal of Y, ∀b∈R .

Proof:

Suppose b∈R, Since R is a positive implicative ideal from a BCI − algebra Y.

$\Rightarrow\ R\ is\ said\ to\ be\ \ a\ positive\ implicative\ ideal\ of\ aBH - algebra\ Y.$ $[By\ remark(1.12), Since\ each\ BCI -\ algebra\ is\ a\ BH -\ algebra)]$

$But, 0 \circ w = 0,\ \forall w \in Y.$ $[By\ definition(1.3)]$

$\Rightarrow\ Y\ =\ Y+\ \ .$ $[By\ definition(1.13)]$

$\Rightarrow\ we\ obtained\ \ R\ is\ a\ b - closed\ ideal\ of\ Y.$ $[By\ Theorem\ (3.12)]$

$Hence, R\ be\ a\ b - closed\ ideal\ of\ Y,\ \forall b\ \in R.$

Theorem(3.14):

If Y is a BCI − algebra such that = Y_+ , then Y is 0 − closed BCI − algebra .

Proof:

Suppose Y BCI- algebra

⇒ Y BH-algebra [By Remark(1.12),Since each BCI-algebra is a BH- algebra]

Suppose R be an ideal of Y

To prove R is 0-closed ideal of Y.

Suppose w∈R

0∘ (0∘ w) = 0∘ 0 [by definition(1.13),Since 0∘ w = 0, ∀w∈Y+]

= 0 [By definition(1.8)Since w ∘ w = 0, ∀w∈Y]

But 0∈R [By definition(2.2),Since R is an ideal]

$\Rightarrow\ 0 \circ (0 \circ w)\ =\ 0 \in R$

$\Rightarrow\ R\ is\ 0 - closed\ ideal\ of\ Y.$

$Thus,\ \ Y\ is\ 0 - closed\ BCI - algebra.$

$Theorem(3.15\ ):$

$Each\ BCI - algebra\ is\ 0 - closed\ BH - algebra$

$Proof:$

$Suppose\ \ Y\ be\ a\ BCI - algebra$

$\Rightarrow\ Y\ is\ BH - algebra$ $[By\ remark(1.12), each\ BCI - algebra\ is\ a\ BH - algebra\ \ ]$

$But\ 0 \circ w\ =\ 0,\ \forall w \in Y$ $[By\ definition(2.2)]$

$\Rightarrow\ Y = Y+$ $\quad [By\ definition(1.13)\ of\ Y+]$

$\Rightarrow\ By\ Theorem(3.14)\ we\ obtained$

$Y\ is\ a\ 0-closed\ BH-algebra.$

Theorem(3. 16):

If Y be 0-closed BCI-algebra ,then each quasi-associative ideal is closed ideal.

Proof:

$Suppose\ \ Y\ \ BCI-\ algebra$

$\Rightarrow\ Y\ \ BH-algebra\ [By\ Remark(1.12), Since\ each\ BCI-algebra\ is\ a\ BH-\ algebra\ ]$

$Suppose\ \ R\ be\ a\ quasi-associative\ ideal\ of\ Y\ .$

$To\ prove\ \ R\ \ be\ a\ closed\ ideal\ of\ \ Y.$

$Suppose\ \ w\in R$

$\Rightarrow\ 0\circ(0\circ w)\ \ =\ 0\circ w$ $\quad [By\ definition(2.7)\ of\ a\ quasi-associative\ ideal]$

$But\ R\ be\ a\ 0-closed\ ideal\ of\ Y\ .[Since\ Y\ is\ a\ 0-closed\ BH-algebra$
$\Rightarrow each\ ideal\ is\ \ 0-closed\ ideal. by\ definition(2.11)\ of\ a\ b-closed\ BH-algebra]$

$\Rightarrow\ 0\circ(0\circ w)\in R$

$\Rightarrow\ 0\circ w\in R$ $\quad [Since\ 0\circ(0\circ w)\ \ =\ 0\circ w]$

$\Rightarrow\ R\ is\ a\ closed\ ideal.$

Theorem(3.17):

If Y be a BCI-algebra ,then each quasi-associative ideal is subalgebra.

Proof:

$Assume\ \ Y\ \ BCI-\ algebra$

$\Rightarrow\ Y\ \ BH-algebra\ [By\ Remark(1.12), Since\ each\ BCI-algebra\ is\ a\ BH-\ algebra]$

$Assume\ \ R\ \ be\ a\ quasi-associative\ ideal$

$Assume\ \ w, v\in R$

$To\ prove\ \ w\circ v\in R$

$0\circ(\ 0\circ(w\circ v))\ =\ 0\circ((0\circ w)\circ(0\circ v))$ $\quad [By\ proposition(1.5)]$

$=\ (0\ \circ\ (0\ \circ\ w)\ )\ \circ\ (\ 0\ \circ\ (0\ \circ\ v)\ )$ $\quad [By\ proposition(1.5)]$

$=\ (0\ \circ\ w)\ \circ\ (0\ \circ\ v)$ $\quad [By\ definition(2.7)]$

$=\ 0\circ(w\circ v)$ $\quad [By\ proposition(1.5)]$

$\Rightarrow\ 0\circ(0\circ(w\circ v))\ =\ 0\circ(w\circ v)$

$\Rightarrow\ w\circ v\in R$ [By definition(2.7)]

Thus, R is a subalgebra.

Conclusions :

By this paper , we have proved that some theorems and propositions which are related to b- closed ideal of a BCI-algebra, also we introduced a study in some ideal types of a BCI-algebra ,with detaild discssions, addition to that we have bind this notion with other kinds of ideal that involved in this paper .

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