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Functions in Discrete Mathematics

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Abstract: *In discrete mathematics, functions are mathematical relations between sets that map each element in one set to exactly one element in another set. They're often represented by input-output pairs, where each input has a unique output. Functions play a crucial role in various areas like computer science, cryptography, and graph theory, offering a formal way to describe relationships between elements. They're fundamental for understanding and solving problems in discrete mathematics.*

Keywords: Function, collection, injective, codomain, element, number,sets

Introduction:

Functions are an important part of discrete mathematics. This article is all about functions, their types, and other details of functions. A function assigns exactly one element of a set to each element of the other set. Functions are the rules that assign one input to one output. The function can be represented as f: A ⇢ B. A is called the domain of the function and B is called the codomain function.

Functions:

- A function assigns exactly one element of one set to each element of other sets.
- A function is a rule that assigns each input exactly one output.
- A function f from A to B is an assignment of exactly one element of B to each element of A (where A and B are non-empty sets).
- A function f from set A to set B is represented as f: A ⇢ B where A is called the domain of f and B is called as codomain of f.
- If b is a unique element of B to element a of A assigned by function F then, it is written as f(a) = b.
- Function f maps A to B means f is a function from A to B i.e. f: A ⇢ B

Domain of a function:

- If f is a function from set A to set B then, A is called the domain of function f.
- The set of all inputs for a function is called its domain.

Codomain of a function:

- If f is a function from set A to set B then, B is called the codomain of function f.
- The set of all allowable outputs for a function is called its codomain.

Pre-image and Image of a function:

A function f: A ⇢ B such that for each a ∈ A, there exists a unique b ∈ B such that (a, b) ∈ R then, a is called the pre-image of f and b is called the image of f.

Types of function:

One-One function (or Injective Function):

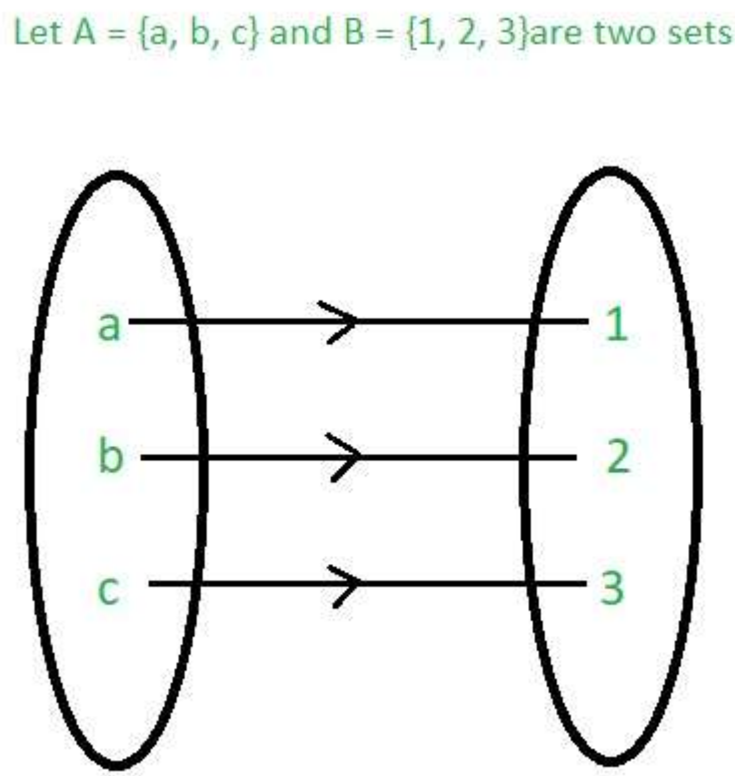
A function in which one element of the domain is connected to one element of the codomain.

A function f: A ⇢ B is said to be a one-one (injective) function if different elements of A have different images in B.

f: A ⇢ B is one-one

⇒ a ≠ b ⇒ f(a) ≠ f(b)   for all a, b ∈ A

⇒ f(a) = f(b) ⇒ a = b   for all a, b ∈ A



ONE-ONE FUNCTION

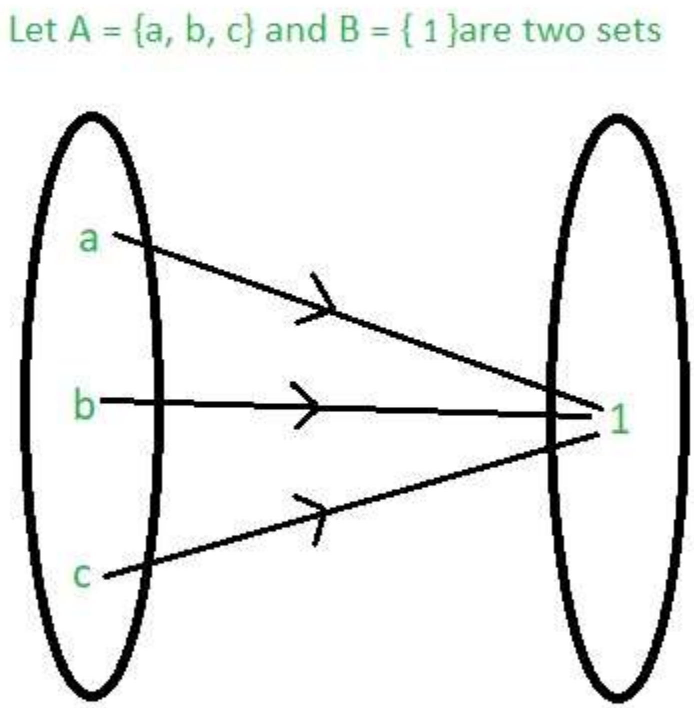
Many-One function:

A function f: A ⇢ B is said to be a many-one function if two or more elements of set A have the same image in B.

A function f: A ⇢ B is a many-one function if it is not a one-one function.

f: A ⇢ B is many-one

⇒ a ≠ b but f(a) = f(b)   for all a, b ∈ A



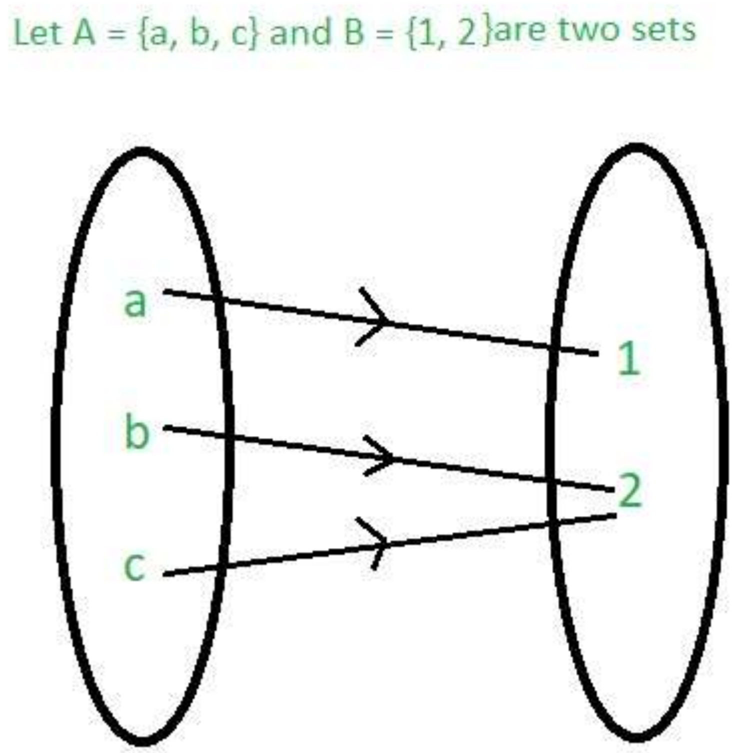
MANY-ONE FUNCTION

Onto function(or Surjective Function):

A function f: A -> B is said to be onto (surjective) function if every element of B is an image of some element of A i.e. f(A) = B or range of f is the codomain of f.

A function in which every element of the codomain has one pre-image.

f: A ⇢ B is onto if for each b∈ B, there exists a∈ A such that f(a) = b.



ONTO FUNCTION

Into Function:

A function f: A ⇢ B is said to be an into a function if there exists an element in B with no pre-image in A.

A function f: A⇢ B is into function when it is not onto.

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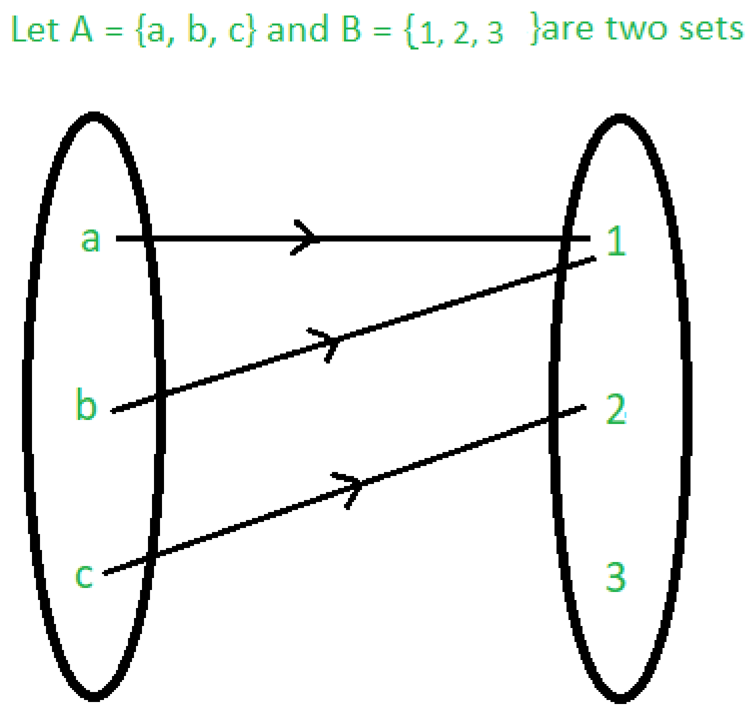


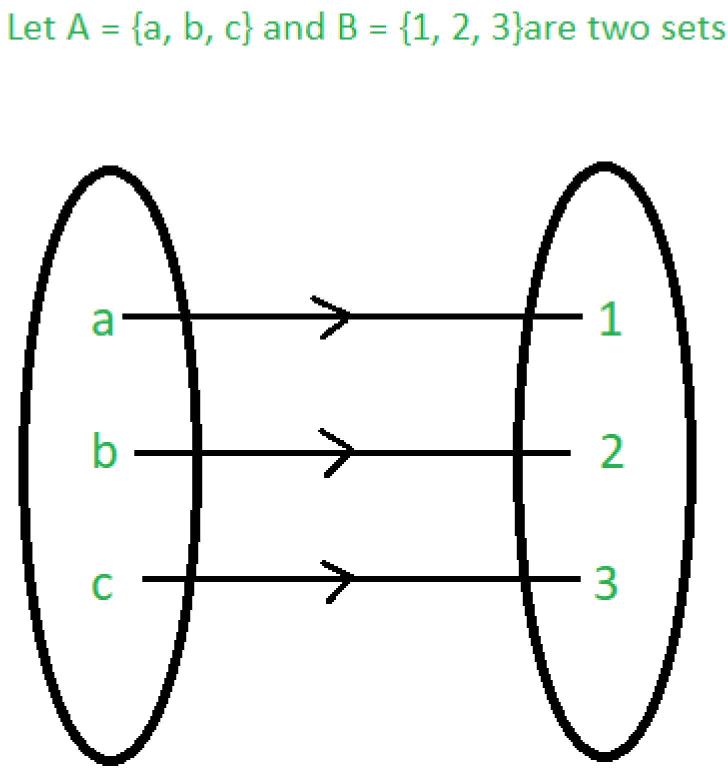
INTO FUNCTION

One-One Correspondent function(or Bijective Function or One-One Onto Function):

A function which is both one-one and onto (both injective and surjective) is called one-one correspondent(bijective) function.

f : A ⇢ B is one-one correspondent (bijective) if:

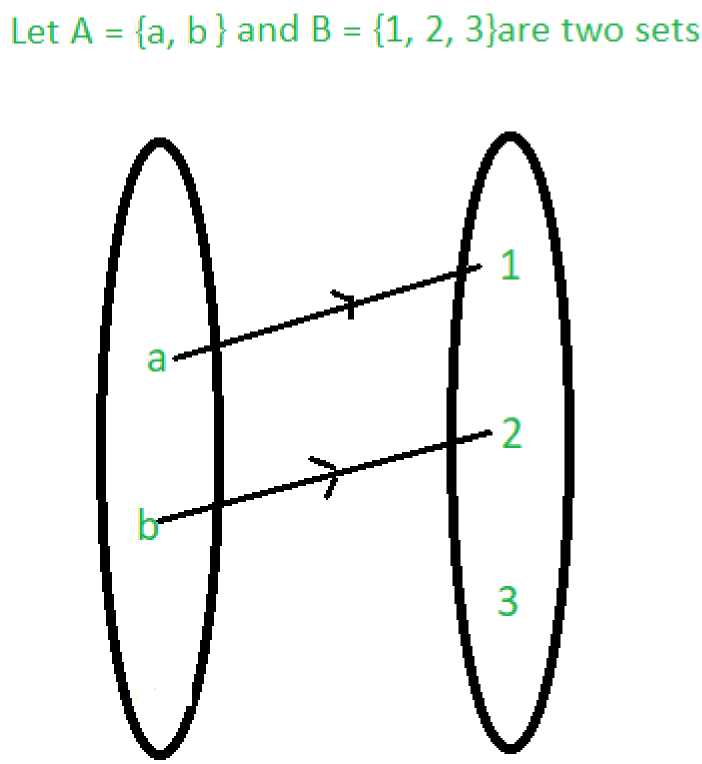
- one-one i.e. $f(a) = f(b) \Rightarrow a = b$ for all $a, b \in A$
- onto i.e. for each $b \in B$, there exists $a \in A$ such that $f(a) = b$.



ONE-ONE CORRESPONDENT FUNCTION

One-One Into function:

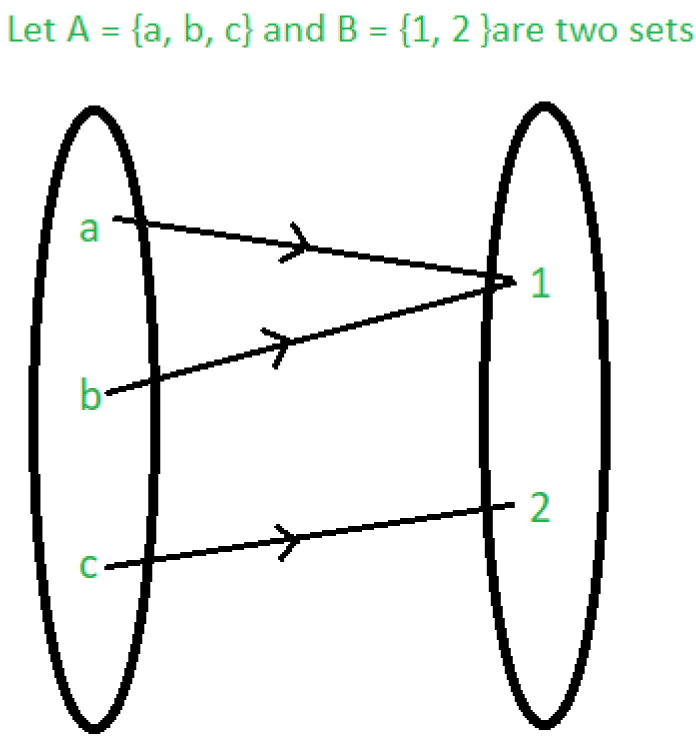
A function that is both one-one and into is called one-one into function.



ONE-ONE INTO FUNCTION

Many-one onto function:

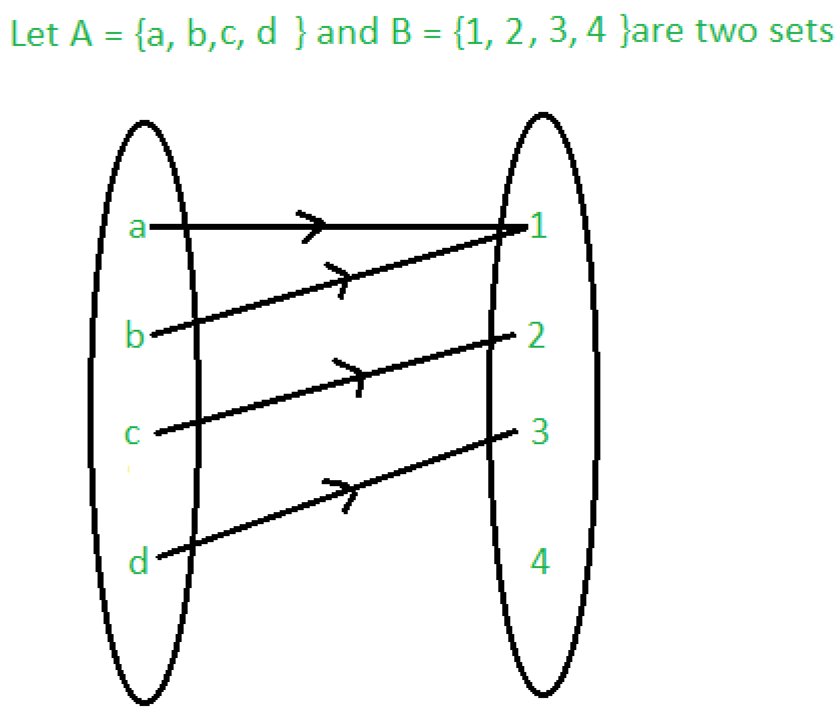
A function that is both many-one and onto is called many-one onto function.



MANY-ONE ONTO FUNCTION

Many-one into a function:

A function that is both many-one and into is called many-one into function.



MANY-ONE INTO FUNCTION

Inverse of a function:

Let $f: A \dashrightarrow B$ be a bijection then, a function $g: B \dashrightarrow A$ which associates each element $b \in B$ to a different element $a \in A$ such that $f(a) = b$ is called the inverse of f.

$f(a) = b \leftrightarrow$ □ $g(b) = a$

Composition of functions :-

Let $f: A \dashrightarrow B$ and $g: B \dashrightarrow C$ be two functions then, a function $gof: A \dashrightarrow C$ is defined by

$(gof)(x) = g(f(x))$, for all $x \in A$

is called the composition of f and g.

Note:

Let X and Y be two sets with m and n elements and a function is defined as f : X->Y then,

- Total number of functions = n^m
- Total number of one-one function = ${}^nP_m$
- Total number of onto functions = $n^m - {}^nC_1(n-1)^m + {}^nC_2(n-2)^m - \ldots\ldots\ldots\ldots.. + (-1)^{n-1}\,{}^nC_{n-1}1^m$ if $m \geq n$.

For the composition of functions f and g be two functions :

- $fog \neq gof$

- If f and g both are one-one function then fog is also one-one.
- If f and g both are onto function then fog is also onto.
- If f and fog both are one-one function then g is also one-one.
- If f and fog both are onto function then it is not necessary that g is also onto.
- $(fog)^{-1} = g^{-1} o\ f^{-1}$
- $f^{-1} o\ f = f^{-1}(f(a)) = f^{-1}(b) = a$
- $fof^{-1} = f(f^{-1}(b)) = f(a) = b$

Conclusion:

In conclusion, functions in discrete mathematics are foundational tools for describing relationships between elements of sets. They provide a formal way to represent mappings between sets, ensuring that each input corresponds to exactly one output. Understanding functions is essential for tackling a wide range of problems across various fields, making them a fundamental concept in discrete mathematics.

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