

Transformational Learning Experiences on Productive-Failure Approach in Mathematics

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Abstract: *A learning failure is not always a denial of knowledge but an opportunity to be productive in the academic pursuit in the field of mathematics. This qualitative case study explored the students' experiences with the utilization of the Productive-Failure Approach in teaching Mathematics. This employed thematic analysis to analyze the qualitative data from the ten (10) BSEd Mathematics students via Individual Interviews and Focus-Group Discussion. Findings revealed four (4) emergent themes on the utilization of the productive-failure approach in mathematics: (1) prior knowledge activation in mathematics; (2) engaging mathematics learning processes; (3) exerting effort in learning mathematics; and (4) involvement and interaction in learning mathematics. The findings of this study signify that the use of the Productive-Failure Approach helped the students thrive in the learning process; thus, it is recommended that mathematics teachers strengthen their utilization of the Productive-Failure Approach in teaching Mathematics.*

Keywords— Learning experiences, Productive-failure approach, Mathematics

1. INTRODUCTION

Mathematics teachers shoulder a multifaceted responsibility that extends beyond mere instruction, cultivating foundational skills like number sense and operational fluency. They integrate problem-solving, inspiring resilience and perseverance. Their mandate includes instilling a passion for mathematics and equipping students with essential tools for lifelong learning (Zhi-song, 2009).

In mathematics, encountering failure is common, especially when students face unfamiliar problems. Traditionally, teachers introduce concepts through direct instruction, followed by problem-solving. In the 21st century, teaching methods must evolve to enhance learning retention. Productive failure flips this sequence, engaging students in problem-solving first, and then teaching concepts and procedures afterward (Kapur, 2010).

The failure-based approach, as described by (Kapur & Bielaczyc, 2012), involves students naturally failing to discover correct solutions, activating their prior knowledge to facilitate learning. This intentional failure, framed as part of the learning process, primes students' brains to grasp new concepts after initial setbacks. By developing tasks that challenge students and necessitate the use of prior knowledge, this approach encourages learning rooted in students' experiences and insights, fostering a deeper understanding of the topics in mathematics.

Productive-failure Approach is not just an adage about persevering through challenges; it is an active and practical teaching approach that empowers students not only to do well on short-term measures of knowledge but also provides enhanced creative thinking, conceptual understanding, and helps students to transfer or utilize learning to real-life situations (Kapur, 2016).

In a productive failure learning approach, the instruction of the new concepts of the lesson is preceded by unguided exploration and inquiry. As learners scarcely know how to decipher the given problems during this exploration phase successfully, they are anticipated to make mistakes. Although failure and errors seem persuasive and are commonly considered undesirable outcomes of the learning process, advocates of the productive failure approach strongly believe that this is not primarily the case (Darabi et al., 2018).

Kapur, Dickson, & Yhing (2009) emphasized that students should receive instruction that is connected to their learning experiences during the lesson. With this proposition, they theorized that failure can be useful and productive when students are facilitated immediately after making mistakes. When the instruction is connected to the experiences of the students, it becomes relevant and meaningful. Consequentially, students learn to recognize the cause of the failure. Additionally, VanLehn, Siler, Murray, Yamauchi, & Baggett (2003) pointed out that successful learning is associated with failure. This failure permits the students to reach an impasse and are not able to solve the given problem by delaying the instruction.

This approach might seem like frustration to students because of their inability to solve the problem, that is why the teacher must motivate the students in order not to feel bad and instead spike the learners' interest in solving the problem. And that's the right place to get students before teaching them something. After students commit mistakes and encounter failure in their own discovery and

problem-solving process, the teacher facilitates a discussion that highlights various student attempts and teaches the new concept, consolidating students' understanding of the processes required to complete the task (Perry et al., 2005).

In the Philippine context, Mathematics presents itself as a challenging academic discipline. The researcher has observed a notable deficiency in students' capacity to comprehend and retain mathematical concepts effectively. This trend, if left unaddressed, poses significant obstacles to the cultivation of globally competitive individuals, as evidenced by the outcomes of assessments like the Programme for International Student Assessment (PISA). Motivated by this concern, the researcher utilized the Productive-Failure approach, aiming to stimulate student interest, foster critical thinking skills, and promote long-term retention of mathematical knowledge. Consequently, the researcher seeks to investigate the experiences of students regarding the implementation of the Productive-Failure approach in Mathematics.

2. METHODS

This study utilized a case study qualitative research method to explore the experiences of students regarding the implementation of the Productive-Failure approach in Mathematics.

Thematic analysis was utilized in analyzing the qualitative data of this study. Thematic analysis is one of the most commonly used forms of inductive analysis in qualitative research. It is the technique of finding patterns or themes within qualitative data (Braun & Clarke, 2006). The method above aimed to interpret and identify themes that describe the students' experiences on the implementation of the productive-failure approach in teaching mathematics. A structured interview questionnaire with open-ended questions was utilized to gather accurate and reliable data on how they perceive the implementation of the productive-failure approach. The gathered data were examined, categorized, and analyzed to uncover patterns, determine meanings, and construct conclusions.

3. RESULTS AND DISCUSSION

The transformational learning experiences of the students on the implementation of the productive-failure approach in teaching mathematics were: (1) *Prior Knowledge Activation in Mathematics* includes using prior knowledge and mental effort and engagement; (2) *Engaging Mathematics Learning Processes* integrates learning by doing activities, elaborating ideas and building self-confidence; (3) *Exerting Effort in Learning Mathematics* incorporates eager to do the activity, trying to generate many solutions, and defending answer; and (4) *Involvement and Interaction in Learning Mathematics* considers engaging in group work (collaboration), asking help, asking questions, and sharing of ideas.

Eight (8) out of ten (10) students responded that prior knowledge activation was one of the scenarios on the implementation of the productive-failure approach because it allows them to use their prior knowledge to solve novel problems. Eight (8) out of ten (10) students responded that engaging mathematics learning processes was one of the things that the productive-failure approach can do during the learning process. Six (6) out of ten (10) students responded that exerting effort was another situation that happened during the implementation of the productive-failure approach. Additionally, there are four (4) out of ten (10) students also responded that involvement and interaction were also one of the circumstances that occurred in the implementation of the productive-failure approach.

Prior Knowledge Activation in Mathematics

The manifestation of the productive-failure approach is evident in how students leverage their prior knowledge to solve novel problems, as affirmed by the following statements.

"The part of the delivery of the lesson that challenged me the most was when we were given a problem and we were tasked to solve it, even though we didn't know yet that topic. I was very challenged at that time because I tried my best to solve it, and using my prior knowledge, I was forcing myself to calculate properly and look for an answer." (P1, P3, P5, P6)

"The part of the delivery of the lesson that helped me most was when Sir gave us activities without discussing them. In that way, it helped me to find an answer on my own though it was hard, it helped me to remember and use my stock knowledge. In that part, I was eager to answer even if I was doubtful to answer it." (P2, P7, P8, P10)

DeCaro & Rittle-Johnson (2012) and Schwartz et al. (2011) supported the above statements on the Productive-Failure Approach, which prioritizes problem-solving before explanation of concepts and procedures. This method, though cognitively demanding, enhances encoding and schema assembly, leading to improved learning outcomes (Kapur, 2010). Furthermore, initiating problem-solving with prior knowledge allows students to discern critical features of new concepts more effectively (Romorosa et al, 2023).

Engaging Mathematics Learning Processes

This theme is evident in the students' experiences emphasizing that the productive-failure approach is indeed an engaging and effective way for students to explore and generate several solutions to solve novel problems. This is supported by the statements below.

"During the presentation of our answers, I enjoyed it so much because even though we didn't know the correct answer to the said activities, we confidently presented it in front of our classmates. With the use of the new approach (productive-failure), our math lessons and learning activities became engaging because we were given the chance to elaborate our ideas and apply our prior knowledge to solve the given problem." (P3, P4, P8, P9)

"Productive-failure approach helped me to engage myself during the learning process because it allowed me to explore and formulate many solutions just to solve the given problems. I enjoyed the collaboration of answers of my classmates and our groupings. Without knowing the correct answer to the given problem, we had cooperation to answer even we were doubtful if it was wrong or right." (P2, P5, P7, P10)

Throughout the productive failure cycle, students receive frequent feedback precisely when they need it most—while actively refining their problem-solving abilities (Luzano, 2020). The ability of students to persist in problem-solving, despite heightened cognitive demands, may grant them a sense of empowerment, thereby enhancing their engagement with the task (diSessa et al., 1991).

Exerting Effort in Learning Mathematics

This is one of the grounds that students give maximum effort just to find the exact solution for the given novel problems during their mathematics class and was supported by the quotes below.

"I enjoyed my math class especially when we answered the problem (equation) and presented it in our class because we were trying our best to deliver it properly and correctly in front of my classmates and my teacher. Though it was hard to solve the given problems there was a part of the lesson that I enjoyed the most during the discussion or presenting our solutions because we did our best and exerted more effort to defend our answer no matter if it was right or wrong." (P1, P5, P10)

"When our teacher immediately gave us the problem to be solved, I did my best and exerted more effort to solve it since we were doing it individually. When we presented our solutions though we were doubtful, we still gave our best exerted effort, and defended our answer. The part of the lesson that I enjoy the most is when we answer the problem because I don't know how to solve it, but we tried our best to generate as many solutions as we can." (P2, P7, P9)

During the initial phase of the productive-failure approach, known as the generation phase, students are provided with the opportunity to merge and organize pertinent ideas and solution methods (Kapur & Bielaczyc, 2012). In this phase, students engaged in a productive failure lesson delve into available solutions based on their existing knowledge. As they explore various solutions, they may encounter the realization that their comprehension of the topic is inadequate. Consequently, students must attempt different solutions built upon their prerequisite knowledge, with the likelihood of encountering obstacles during the problem-solving process (Loibl, Roll & Rummel, 2016; Luzano & Ubalde, 2023).

Involvement and Interaction in Learning Mathematics

This is one of the themes that emerged as to why students are provided with the opportunity to engage and interact with their classmates, as supported by the quotes below.

"The delivery of lesson that helped me most was when we answered it by the group because we shared our ideas, particularly our formulated solutions and we are helping each other and trying our best to choose the best answer." (P6, P7)

"I enjoy the collaboration of answers of my classmates or our groupings. Without knowing the correct answer to the given problem, we asked help and questions from each member of the group to derive the answer to the given problem. When we work as a group, I enjoy it so much because as a member, I can share some of my ideas, learn something, and give a little contribution to my group." (P4, P9)

The statements emphasize how collaborative learning improves understanding and problem-solving skills among students. Group engagement encourages active participation, idea exchange, and collective solution formulation (Francisco, 2013; Luzano, 2023). These foster shared responsibility and mutual support in comprehending complex concepts and finding optimal solutions (Luzano, 2024). Seeking help and questioning peers within the group promotes critical thinking and deepens conceptual understanding. Ultimately, collaborative learning enhances knowledge acquisition and cultivates fulfillment among group members, enriching the learning experience (Cáceres et al., 2018).

4. CONCLUSION AND RECOMMENDATION

This qualitative exploration has concluded that the adoption of the Productive-Failure Approach in teaching Mathematics significantly enhances students' academic performance compared to traditional methods. The research findings affirm that students who engage in problem-solving before formal instruction exhibit notably greater achievement than those who receive instruction first. Analysis of the process reveals that when students activate and draw upon their prior knowledge to generate sub-optimal or incorrect solutions, the learning process becomes meaningful and productive, effectively preparing them for subsequent instruction.

Consequently, implementing the Productive-Failure Approach can enhance both the teaching and learning of Mathematics, fostering improved performance and facilitating the transfer of knowledge over the short and long term. This underscores the importance of educators considering methods that involve students in failed problem-solving attempts, whether their own or those of their peers, as viable alternatives for promoting enhanced mathematical learning outcomes.

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