International Journal of Engineering and Information Systems (IJEAIS)
ISSN: 2643-640X
Vol. 8 Issue 5 May - 2024, Pages: 53-57

Solving linear Boundary Value Problems to Find an Approximate Value of (∞) the Second Boundary Condition

Ebaa Abdulsattar Jaber[1],Hayder Salih Dakhil[2] ,Aseel Sami Hamd[3],Noor Razak Jafar[4],Ali Hasan Abdoul Khaled[5]

1 Mathematics Department, College of Education for Pure Sciences, University of Babylon. Babylon, Iraq.
edu792.a.abadalsataar@uobabylon.edu.iq
2 Mathematics Department, College of Education for Pure Sciences,General Directorate of Education ,Babylon. Babylon, Iraq.
hydrsalhalwady@gmail.com
3 Mathematics Department, College of Education for Pure Sciences, University of Babylon. Babylon, Iraq.
Pure.aseel.najm@uobabylon.edu.iq
4 Mathematics Department, College of Education for Pure Sciences, University of Babylon. Babylon, Iraq.
noorrazak1993@gmail.com
5 Mathematics Department, College of Education for Pure Sciences, University of Babylon. Babylon, Iraq.
edu716.ali.hasan@uobabylon.edu.iq

Abstract: *The search aims to solve the problems of the second-order linear boundary value and the second boundary condition of infinity (∞). The research uses the finite difference method to determine approximate values.*

Keywords: Ordinary differential equations, Numerical methods, Approximate Value, Boundary Condition, Runge-Kutta.

1. Introduction

L. Fox (in 1947) tried to add conditions to solve ordinary and partial differential equations using numerical methods, such as the finite difference method, to obtain the most accurate results[1].In (1975), V. Pereyra and E.G. Sewell supported the development of grid selection for using the convergence of finite differences to irregular grids in boundary value problems [2].In 1981, R.D. Skeel reported on general techniques for obtaining accurate results for solving differential equations that can be used computationally [3]. In 1985, S. Gupta explained finite difference methods using the Runge-Kutta method and its laws for solving differential equations with two-point boundary conditions. S. Gupta described a diversified system and a specific differential solution using implicit methods[4]. In 1996, Jeffr. Cash solved initial value problems using the Runge-Kutta method [5]. In 2001, Jeffr. Cash discovered a new algorithm to solve nonlinear boundary value problems , and the best results were reached[6].

2. Two-point boundary value problems[8]

The article discusses the finite difference method for solving D.E where

f(x) = y″ 0 < x < 1 (1)

We need initial conditions y_0 (x) =a and boundary conditions y(∞)=B(x) so this problem is called 2-point BVP. With the expectation of a value at two different points, such that y_{n+1} is an approximation of the solution y_n,
we obtain a set of algebraic equations

y″=y_{n+1} -2y_n+y_{n-1})/h^2 (2)

(h represents the distance between the points from the algebraic conditions).
To know the extent to which y is able to approximate the function y(x), we know that the difference approximation The central y_n, provides an accurate approximation of degree 2 fory_{n-1}, knowing that the values of y_n are at each point and calculating a complete set of discrete values, and this is more complicated, as we find that applying the y function to the discrete values gives the required values for y_n, hoping to give it a small error rate.

3. Deferred correction

Over the following years, some scientists have applied new techniques to a variety of questions in Differential and integral equations. This method is used for numerical solutions to first-order two-point nonlinear boundary value problems Lindberg. The basic idea of deferred correction was introduced by the scientist L. Fox, who assumed different forms of error improvement[2]. The scientist Lindberg developed in particular. We will provide a brief description of this method.

y'= F (x, y) , d ≤ x ≤ w ,
(y(d),y(w)) = 0 (3)

International Journal of Engineering and Information Systems (IJEAIS)
ISSN: 2643-640X
Vol. 8 Issue 5 May - 2024, Pages: 53-57

Let to be Θ_p formula of Runge-Kutta the rank P It is used for division (3)and so the result is a non-linear equation

Θ_p =0 (4)

Lindberg's idea of reducing error is based on assumptions Θ_{p+r} the formula Runge – Kutta of order p+r.

Θ_p (η̄) = - Θ_{p+r}(η) (5)

- Θ_{p+r} is the local truncation error formula for the lowest-order formula Θ_p

In this strategy, we want the answer to be more exact in terms of convergence.

4. Algorithm

4 .1. An algorithm for solving linear boundary value problems with (ꝏ) boundary conditions

$y'' = f_n(x)\, y_n(x) + g_n(x)$ of second order.

The Boundary conditions is

y (d)=D

y (ꝏ)= W

1. change the second Boundary conditions

y (ꝏ)= W

y(ꝏ) $\longrightarrow$ W, w $\longrightarrow$ ꝏ

y(w^N)=W

$w^{(N)}$= a+(N+1)h

chose ϵ is the value very small.

2. Now we will use the finite difference method

$(y_{n+1}$ -2y_n+y_{n-1}) / $h^2 =$ f_n(x) y_n(x)+ g_n(x)

h is The distances between the two places.

y_{n+1} -2y_n+y_{n-1} = $h^2(f_n\, y_n + g_n)$

y_{n+1} -2y_n+y_{n-1} = $h^2 f_n\, y_n + h^2\, g_n$

y_{n+1} =2y_n-y_{n-1}+$h^2 f_n\, y_n + h^2\, g_n$

y_{n+1} = (2 +$h^2 f_n$)y_n -y_{n-1}+$h^2\, g_n$ (3) when n =1,2,3,…………

We enter values n into equation (3) when n=1 we get.

y_2 = (2 +$h^2 f_1$)y_1 −y_0+$h^2\, g_1$

y_{n+1} = (2 +$h^2 f_1$)y_1 −y_0+$h^2\, g_1$

y_1= (y_{n+1}+y_0- $h^2 g_1$)/ (2 +$h^2 f_1$)

When n=2 we get

y_3 = (2 +$h^2 f_2$)y_2- y_1+$h^2\, g_2$

y_2=($y_3 + y_1$ -$h^2\, g_2$)/ (2 +$h^2 f_2$)

y_2 =($y_{n+1} + y_n$ -$h^2\, g_2$)/ (2 +$h^2 f_2$)

3. We continue to solve until we get results less than ϵ

$$\left| y_{(n)}^{(N+1)} - y_{(n)}^{(N)} \right| < \epsilon$$

4. Break

5. Applications

Example:

y''= 2y- sin(x)

Boundary conditions y (0)=1 ,y(∞)=0, ϵ =10^{-10} , h=0.5

y (x)= sin(x)

$\frac{-y_{n+1} + 2y_n - y_{n-1}}{h^2}$ +2y_n = sin (x)

$-y_{n+1} + 2y_n - y_{n-1}$ + 0.5 y_n = 0.25 sin ($\frac{n}{2}$)

y_{n+1}= 2.5 $y_n - y_{n-1}$ − 0.25 sin ($\frac{n}{2}$) (4)

when n = 1,2,3 ……… we substitute into the equation (4)

y_2= 2.5 $y_1 - y_0$ − 0.25 sin ($\frac{1}{2}$)

Put y_0 = 1 , y_2= 0 (given in the question)

0= 2.5 $y_1 - 1$ − 0.25 sin ($\frac{1}{2}$)

$y_1 = \frac{1+0.25\sin(\frac{1}{2})}{2.5}$ = 0.40087

y_3= 2.5 $y_2 - y_1$ − 0.25 sin ($\frac{2}{2}$)

y_3= 2.5 (2.5 $y_1 - y_0$ − 0.25 sin ($\frac{1}{2}$)) $- y_1$ − 0.25 sin (1)

y_3 = 6.25 y_1 $-2.5y_0$ − 0.625 sin ($\frac{1}{2}$) $-y_1$ − 0.25 sin (1)

y_3 = 5.25 y_1 −2.5 − 0.625 sin ($\frac{1}{2}$) − 0.25 sin (1)

0 = 5.25 y_1 −2.5 − 0.625 sin ($\frac{1}{2}$) − 0.25 sin (1)

$y_1 = \frac{2.5+0.25\sin(1)+0.625\sin(\frac{1}{2})}{5.25}$ = 0.47805

$|0.47805 - 0.40087|$ < 10^{-10}

$|0.07718|$ > 10^{-10}

We continue solving until we reach n = 9

b^{N+1}= a+(n+1)/h

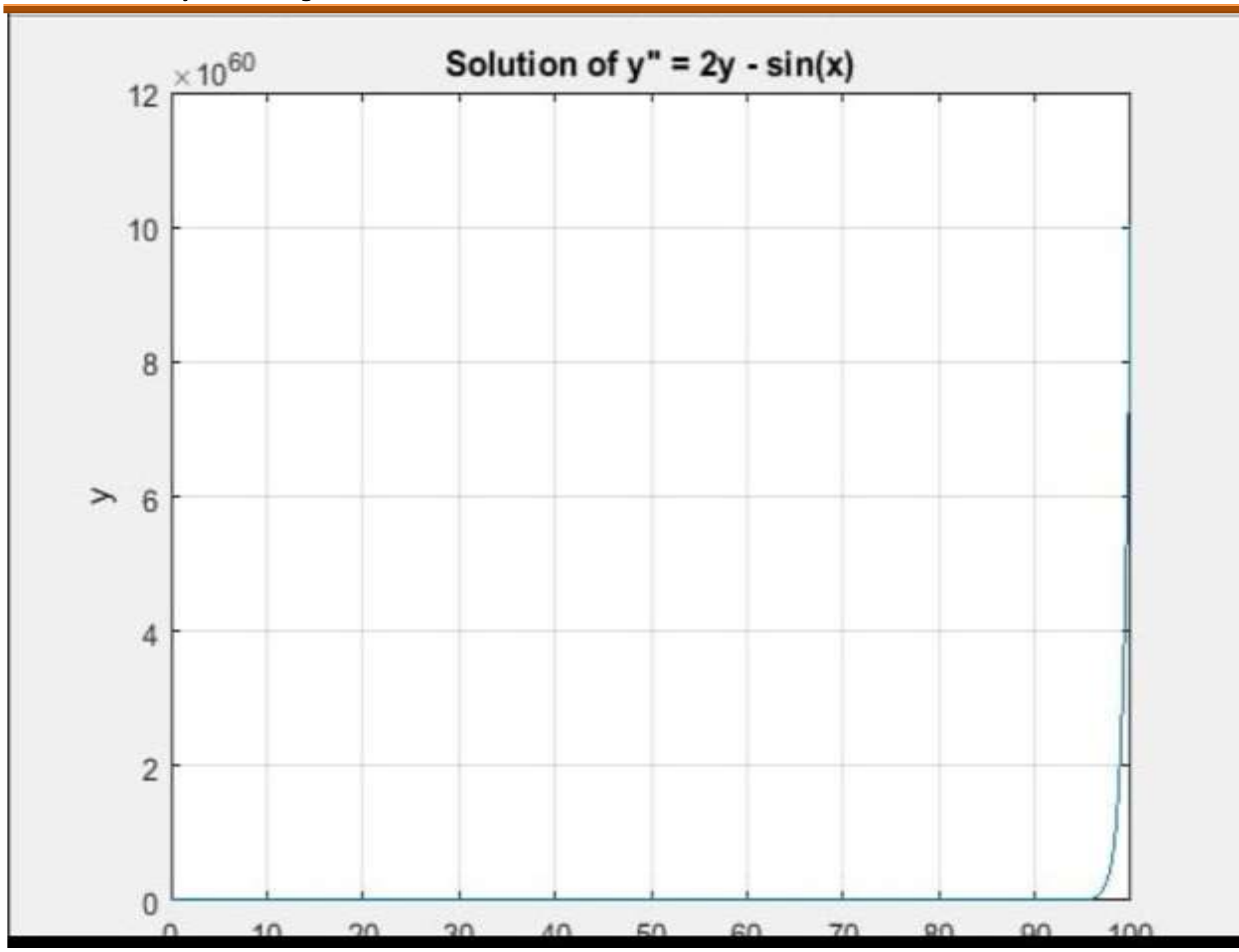
$b^{(10)}$ =0 + (10)(($\frac{1}{2}$) = 5

We get Boundary Condition y(0)=1, y(5) ≅ 0

When we advance the solution, we obtain the best approximation of the solution.

Now we plot the result using MATLAB

International Journal of Engineering and Information Systems (IJEAIS)
ISSN: 2643-640X
Vol. 8 Issue 5 May - 2024, Pages: 53-57



Figur-1-

6.conclusion

The finite difference method solved second-order linear boundary value problems with a second boundary condition. The MATLAB application was utilized.

References.

[1] Fox, L., Some improvement in the use of relaxation methods of ordinary and partial differential equations, Proc. Roy. Soc.,190, 31-59,(1947).

[2] Pereyra, V. and E. G. Sewell, Mesh selection for discrete solution of boundary problems in ordinary differential equations, Numer, Math.,23, 621-268,(1975).

[3] Skeel, R. D., A theoretical framework for proving accuracy results for deferred corrections, SIAM J. Numer, Anal,. 19, 171-196,(1981).

[4] Gupta, S., An adaptive boundary value Runge-Kutta solver for first order boundary value problems, SIAM J. Numer. Anal.25, 114-115., (1985).

[5] J. C. Butcher, JeffR. Cash, DESI Methods for Stiff Initial-Value Problems, ACM Trans. Math. Softw. 22(4): 401-422(1996).

[6] JeffR. Cash, An automatic continuation strategy for the solution of singularly perturbed nonlinear boundary value problems, ACM Trans. Math. Softw. 27(2): 245-266(2001).

[7] Cartwright, J. H., & Piro, O. (1992). The dynamics of Runge–Kutta methods. International Journal of Bifurcation and Chaos, 2(03), 427-449.Forslind, B., & Lindberg, M. (Eds.). (2003). *Skin, hair, and nails: structure and function*. CRC Press.

[8] Cash, J. R., Moore, G., & Wright, R. W. (2001). An automatic continuation strategy for the solution of singularly perturbed nonlinear boundary value problems. ACM Transactions on Mathematical Software (TOMS), 27(2), 245-266.

[9] P. G. CIARLET, M. H. SCHULTZ, AND R. S. VARGA, Numerical methods of high-order accuracy for nonlinear boundary value problems I. One dimensionalproblem, Numer. Math., 9 (1967), pp. 394-430.

[10] Drábek, P., & Fonda, A. (Eds.). (2006). Handbook of differential equations: ordinary differential equations. Elsevier.

[11] LeVeque, R. J. (2007). Finite difference methods for ordinary and partial differential equations: steady-state and time-dependent problems. Society for Industrial and Applied Mathematics.