International Journal of Engineering and Information Systems (IJEAIS)
ISSN: 2643-640X
Vol. 8 Issue 6 June - 2024, Pages: 1-4

On Some Properties of A BCC-algebra

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***ABSTRACT*—** In this paper, we are studying the properties of the BCC-algebra , closed ideal and completely closed ideal.

KEYWORDS— Bcc-Algebra, completely closed ideal.

INTRODUCTION :

In 1966, Imai and Iski [8, 9] dened two classes of algebras are called BCK-algebras and BCIalgebras as algebras connected with some logics. In 1984, Komori [9] and [7] introduced so-called the BCC-algebras, to solve some problems on BCK-algebras. In [2] redened the notion of BCCalgebras by using a dual form of the ordinary algebra. Further study of BCC-algebras was continued in [1, 3, 5, 6, 7]. In this paper, we are studying the properties of the BCC-algebra , closed ideal and completely closed ideal..

1.PRELIMINARIES:

Definition (1.1)[12]:

A *BCC-algebra X* is an abstract algebra $(X, *, 0)$ of type $(2; 0)$ satisfying the following axioms:

(i) $((x * y) * (z * y)) * (x * z) = 0$,

(ii) $x * x = 0$,

(iii) $x * 0 = x$,

(iv) $x * y = y * x = 0 \Rightarrow x = y$.

(v) $0 * x = 0$

is called a *BCC-algebra*. A BCC-algebra with the condition

is called a *BCK-algebra*.

Definition(1.2)[12]:

A non-empty subset A of a BCC-algebra X is called a *BCK-ideal* if

(vii) $0 \in A$,

(viii) $x * y \in A$ and $y \in A$ imply $x \in A$,

and a *BCC-ideal* if it satisfies (vii) and

(ix) $(x * y) * z \in A$ and $y \in A$ imply $x * z \in A$.

Putting $z = 0$, we can see that a BCC-ideal is a BCK-ideal. The converse is not true (cf. [6]). This means that a BCC-ideal is a BCK-ideal with some additional property.

***Example*(1.3):**

Consider the set $G = \{0, a, b, c, d\}$ with the operation * defined by the following table:

*	0	a	b	c	d
0	0	0	0	0	0
a	a	0	a	0	0
b	b	b	0	0	0
c	c	c	a	0	0
d	d	c	d	c	0

Then $(G, *, 0)$ is a BCC-algebra, The subset $A = \{0, a\}$ is a BCK-ideal of this BCC-ideal since $(d * a) * c \in A$ and $d * c \notin A$ **.**

Definition (1.4)[6]:

A nonempty subset S of a BCC-algebra X is called a **BCC-Subalgebra** or **Subalgebra** of X if x*y∈S for all x, y∈S.

Example(1.5):

Let X be the BCC-algebra **Example(1.3)** Then the set S={0,a} is a Subalgebra of a BCC-algebra X. Since

0*0 =0∈S, 0*a=0∈S, a*0=a∈S and a*a=0∈S.

Definition (1.6)[6]:

Let X be a BCC-algebra and I be a subset of X. Then I is called a BCC-ideal of X if it satisfies following conditions:

1) $0 \in I$,

2) $x*y \in I$ and $y \in I \Rightarrow x \in I$,

3) $x \in I$ and $y \in X \Rightarrow x*y \in I$, $I*X \subseteq I$.

2.Main results

In this section, we review some a new definitions and Proposition of Bcc-algebras ,that we results in our work.

Definition (2.1):

A BCC-algebra (X,*,0) is said to be positive implicative if it satisfies for all x ,y and $z \in X$, $(x*z)*(y*z)=(x*y)*z$.

Example(2.2):

Let X = {0,a,b,c,d} be a set with the following table:

*	0	a	b	c	d
0	0	0	0	0	0
a	a	0	a	0	0
b	b	b	0	0	0
c	c	c	a	0	0
d	d	c	d	c	0

Then (X,*,0) is a positive implicative BCC-algebra.

Definition (2.3):

1. A BCC-algebra (X,*,0) is said to be 0-commutative if:
$x*(0*y)=y*(0*x)$ for all x , y $\in X$.
2. A non-empty subset N of Bcc-algebra X is said to be normal of X if :

$(x*a)*(y*b) \in N$ for any $x*y$, $a*b \in N$, $\forall x, y, a, b \in X$.

Theorem (2.4):

Every normal subset N of a BCC-algebra X is a subalgebra of X.

Proof:

If x ,$y \in N$, then $x*0$, $y*0 \in N$. Since N is normal,
$x*y=(x*y)*(0*0) \in N$. Thus N is a subalgebra of X.

Remark: The converse of above theorem does not hold. Indeed, N={0,c} is a subalgebra of X, but it is not normal, since $c*0, b*b \in N$, but $(c*b)*(0*b)=a \notin N$.

Definition (2.5):

A BCC-algebra X satisfying in condition $0*x=0 \Rightarrow x=0$ is called a

P-semisimple BCC-algebra.

Example(2.6):

Consider the BCC-algebra **Example(1.5)** X is a **p-semisimple** BCC-algebra since $0*x = 0 \Rightarrow x = 0$.

Definition (2.7):

1. Let X be a BCC-algebra. Then the set $X+=\{x \in X:0*x=0\}$ is called the **BCA-part of X** .
2. Let X be a BCC-algebra. Then the set $med(X)=\{x \in X:0*(0*x)=x\}$ is called the **medial part** of X.

Example(2.8):

Consider the BCC-algebra **Example(1.5)** The set med (X)={0} is a medial part of a BCC-algebra X, since $0*(0*0)=0*0=0$

Remark:

Let X and Y be Bcc-algebras. A mapping $f:X \to Y$ is called

a homomorphism if $f(x*y)=f(x)*f(y)$ for any x, $y \in X$.

a homomorphism f is called a monomorphism (resp., epimorphism) if it

is injective (resp., surjective). A bijective homomorphism is called an

isomorphism. Two BH-algebras X and Y are said to be isomorphic, written $X \cong Y$, if there exists an isomorphism $f:X\to Y$. For any homomorphism $f:X \to Y$, the set $\{x\in X:f(x)=0\}$ is called the kernel of f, denoted by Ker(f), and the set $\{f(x):x\in X\}$ is called the image of f, denoted by Im(f). Notice that f(0)=0 for any homomorphism f.

Definition (2.9):

A mapping $f:X\to X$ on a Bcc-algebra (X,*,0) is called a Bcc-**endomorphism** if it is a homomorphism.

Definition (2.10):

Let X be a BCC-algebra. For a fixed $a\in X$, we define a map Ra $:X\to X$ such that Ra(x)=x*a for all $x\in X$, and call Ra a right map on X. The set of all right maps on X is denoted by R(X). A left map La is defined by a similar way, and the set of all left maps on X is denoted by L(X).

Definition (2.11):

Let I be a nonempty subset of a Bcc-algebra X. Then I is called an ideal of X if it satisfies:

1) $0\in I$.

2) $x*y\in I$ and $y\in I$ imply $x\in I$.

Definition (2.12):

Let X be a BCC-algebra and I be a subset of X. Then I is called a BCC-ideal of X if it satisfies following conditions:

1) $0\in I$,

2) $x*y\in I$ and $y\in I \Rightarrow x\in I$,

3) $x\in I$ and $y\in X\Rightarrow x*y\in I$, $I*X\subseteq I$.

Definition (2.13):

An Bcc-ideal I in Bcc-algebra X is said to be closed Bcc-ideal if it also is sub-algebra.

Proposition (2.14):

Let f be isomorphism from a Bcc-algebra X into a Bcc-algebra Y. If I is closed Bcc-ideal in X, then f(I) is closed Bcc-ideal in

Proof :

Let a, b $\in$ f(I) such that a = f(x), b = f(y), when x, y $\in$ I
Since $a * b = f(x) * f(y) = f(x * y)$
Since I is closed Bcc-ideal, then $x * y \in I$, thus $f(x * y) \in f(I)$
Then f(I) is closed Bcc-ideal**.**

Proposition (2.15):

Let f be epimorphism from a Bcc-algebra X into a Bcc-algebra Y. If J is closed Bcc-ideal in Y, then $f^{-1}(J)$ is closed Bcc-ideal in.

Proof :

Let x, y $\in f^{-1}(J)$, since f(x), f(y) $\in$ J & J is closed Bcc-ideal Then
$f(x) * f(y) \in J$, thus $f(x * y) \in J$
Then $x * y \in f^{-1}(J)$, thus $f^{-1}(J)$ is closed Bcc-ideal .

Definition (2.16):

Let I and J be two subset of X such that $I \subseteq J$. Then I is said to be closed with respect to J if $x * y \in J$, $\forall$ $y \in I$, $y \neq 0$, then $x * y \in I$.

Proposition (2.17):

The union of family of closed with respect to J is closed with respect to J.

Proof :

Let $\{ Ii : i \in \Delta \}$ be a family of closed with respect to J and
Let $x * y \in J$, $\forall$ $y \in \bigcup i\in\Delta$ Ii , $y \neq 0$, since $\forall$ $i \in \Delta$, Ii is closed with respect to J then $\exists$ $j \in \Delta$ such that $x * y \in J$, $\forall$ $y \in$ Ij, $y \neq 0$, then $x * y \in$ Ij
Thus $x * y \in \bigcup i\in\Delta$ Ii, then $\bigcup i\in\Delta$ Ii is closed with respect to.

Proposition (2.18):

Let I be Bcc-ideal and $\emptyset \neq I \subseteq J$, if I is closed with respect to J, then J is Bcc-ideal .

Proof:

Let $x * y \in J$, $\forall$ $y \in J$, $Y \neq 0$
Since $I \subseteq J$, then $x * y \in J$, $\forall$ $y \in I$, $y \neq 0$.

Since I is closed with respect to J, then $x * y \in I$, $\forall y \in I$, $y \neq 0$, since I is Bcc-ideal, thus
$x \in I$, consequently $x \in J$, then J is Bcc-ideal .

Proposition (2.19):

Let X be a Bcc-algebra and I is Bcc-ideal if X is implicative with respect to I, then I is ideal .

Proof:

Let $x * y \in I$, $y \in I$ such that $y \neq 0$
Since $(x * (x * y)) * y = 0 \in I$, $\forall y \in I$, $y \neq 0$ Since I is Bcc-ideal,
then $x * (x * y) \in I$ Since $x * (x * y) = x$, thus $\in I$, then I is ideal .

Proposition (2.20):

Let X be a commutative BCK-algebra has at lest two element. If X is implicative with respect to I .Then X = I has only two elements .

Proof:

Let $x, y \in X$ such that $x \neq 0$, $y \neq 0$, since X is implicative with respect to I , then $x * (x * y) = x$ & $y * (y * x) = y$ But X is commutative, then $x = y$.

Definition (2.21):

An ideal I of a BCC-algebra X is called a **normal ideal** if $x*(x*y)\in I$ implies $y*(y*x)\in I$, for all $x, y\in X$.

Example(2.22):

Let X={0,a,b,c}. The following table shows the BCC-algebra structure on X.

*	0	a	b	c
0	0	0	0	0
a	a	0	0	a
b	b	b	0	0
c	c	c	b	0

The set I={0,a} is a normal ideal.

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