International Journal of Engineering and Information Systems (IJEAIS)
ISSN: 2643-640X
Vol. 8 Issue 6 June - 2024, Pages: 43-46

Using Itkin's Method to Improve the Results of Double Integrals Numerically

Rana Hasan hilal

Department of Mathematics / Faculty of education for Girls /University of Kufa, Najaf , Iraq

***Abstract**: In this research, we discuss the method of the compound midpoint rule on two dimensions to calculate the double integrals numerically, then use the Itkin delta square method to improve the results. We obtained good results in terms of approaching the true value.*

Keyword: *Numerical integration ; double integrals ;* Itkin *Accelerating*

Introduction

We present a method for calculating double integrals by applying the Itkin method to the values resulting from applying the midpoint rule to both dimensions (inner and outer x, y) when n (the number of subintervals into which it is divided) is equal to m (the number of subintervals into which it is divided) and . We chose the midpoint because it can be used on integrals whose integrals are known. We symbolize this method with the symbol where the Wrightkin acceleration method is the midpoint rule applied to both dimensions, and its general formula is:

$$MM\left(h\right)=h^2\sum_{j=1}^{m}\sum_{i=1}^{n}f\left(x_i,y_j\right)$$

Akkar [2]

$$x_i=a+\frac{2i-1}{2}h \ , \ i=1,2,...,n \qquad y_j=a+\frac{2j-1}{2}h \ ,$$

$$j=1,2,...,m \ .$$

When we start by putting in the formula above, we calculate the approximate value of the double integral where it is calculated and it is equal to and we prove this approximate value in our tables when we calculate where

$$MM\left(h\right)=h^2\sum_{j=1}^{m}\sum_{i=1}^{n}f\left(x_i,y_j\right).$$

Then we put any

We also prove this value in our tables as the approximate value of the double integral. We can improve the two approximate values that we obtained by applying the Itkin acceleration method to them, and thus we obtain a value for the double integral by the Itkin method with the midpoint and the two dimensions and . Thus, we continue to apply the rule MMfor the rest of the values, then n=m apply Itkin acceleration to them until the values approach the analytical value of the integral until we obtain the value with the desired accuracy that we choose

1.2 <u>Aitken's delta-squared process</u>

Assume the sequence $\{x\}$ as $\{x\}=\{x_1,x_2,...,x_k,...\}$ converging linearly to a certain final value α

$$\alpha - x_{i+1} = C_i\left(\alpha - x_i\right) \quad , \quad \left|C_i\right| < 1$$

$$C_i \to C$$

We notice that C_i It will be almost constant and we can write $\alpha - x_{i+1} \cong \bar{C}\left(\alpha - x_i\right)$ We also note $\alpha - x_{i+2} \cong \bar{C}\left(\alpha - x_{i+1}\right)$

Therefore: $\dfrac{\alpha - x_{i+2}}{\alpha - x_{i+1}} \cong \dfrac{\alpha - x_{i+1}}{\alpha - x_i}$

$$\boxed{\alpha \cong \frac{x_i . x_{i+2} - x_{i+1}^2}{x_{i+2} - 2x_{i+1} + x_i}}$$

This is the Aitken delta quadratic method (Ralston [1]).

When using the values n of the sequence $\{x\}$ We get another sequence $n-2$ of values that approaches faster to α .

1.3 example

1- $I = \int_1^2\int_1^2 ln(x+y)dxdy$ Its analytical value is 1.08913865206603, rounded to fourteen decimal places.

2- $I = \int_0^1\int_0^1 xe^{-(x+y)}dxdy$ Its analytical value is 0.167032242958898, rounded to fifteen decimal places.

3- $I = \int_0^1\int_0^1 \sin(\frac{\pi}{2}(x+y))dxdy$ Its analytical value is 0.810569469138702 rounded to fifteen decimal places.

1.4 Results

1-When calculating $I = \int_1^2\int_1^2 ln(x+y)dxdy$ the integral numerically using Aitken acceleration with the MM rule, we obtained

From Table (1) we obtained a correct value for nine decimal places when (sub-interval) when setting. Nasser[3], Akkar [2].

2-From Table (2), when calculating $I = \int_0^1\int_0^1 xe^{-(x+y)}dxdy$ the integral numerically using Aitken acceleration with the MM rule, we obtained a correct value for nine decimal places when (sub-period) when we put. Nasser [3], Akkar [2].

International Journal of Engineering and Information Systems (IJEAIS)
ISSN: 2643-640X
Vol. 8 Issue 6 June - 2024, Pages: 43-46

3- From Table (3) when calculating $I = \int_0^1 \int_0^1 \sin(\frac{\pi}{2}(x + y))dxdy$ the integral numerically using Aitken acceleration with the MM rule, we obtained a correct value close to eleven decimal places when (subperiod). When we put using Aitken acceleration with the MM rule, the result in Table (3) was correct Nasser[3], Akkar [2].

m=n	قيم قاعدةMM	AMM	AMM
1	1.09861228866811		
2	1.09156956942644		
4	1.08975064597941	1.08911729761810	
8	1.08929192371573	1.08913722151573	
16	1.08917698716071	1.08913856086425	1.08913865738199
32	1.08914823691532	1.08913864633522	1.08913865228321

Tabla1

Double integral calculation $I = \int_1^2 \int_1^2 ln(x + y)dxdy$

m=n	MM	AMM	AMM
1	0.183939720585721		
2	0.171714931552731		
4	0.168233163548213	0.166846605240987	
8	0.167334392455161	0.167021659027521	
16	0.167107900747999	0.167031595428381	0.167032193379105
32	0.167051164939255	0.167032202699818	0.167032242229764

Table2

Double integral calculation $I = \int_0^1 \int_0^1 xe^{-(x+y)}dxdy$

m=n	MM	AMM	AMM
1	0.183939720585721		
2	0.171714931552731		
4	0.168233163548213	0.166846605240987	
8	0.167334392455161	0.167021659027521	
16	0.167107900747999	0.167031595428381	0.167032193379105
32	0.167051164939255	0.167032202699818	0.167032242229764

Table3

Double integral calculation $I = \int_0^1 \int_0^1 \sin(\frac{\pi}{2}(x + y))dxdy$

1.5Conclusion

It is clear from the tables of this chapter that when calculating double integrals with continuous integrals using the Aitken acceleration with the AMM rule (resulting from applying the midpoint rule to the dimensions Y and x) it gave better results in terms of speed of convergence and with a small number of subperiods. In the first example, we obtained an integer value to nine decimal places when (subinterval) after using Aitken acceleration with the AMM rule when setting

In the second example, we obtained an integer value to nine decimal places when (subinterval) when we set

In the third example, using Aitken acceleration with the MM rule, we obtained an integer value close to eleven decimal places when (subinterval). When we set

Thus, we can rely on the AMM method in calculating double integrals with continuous integrals.

[1] Anthony Ralston ,"A First Course in Numerical Analysis " McGraw –Hil Book Company ,1965.

[2] Akkar, Batoul Hatem, “Some numerical methods for calculating double and triple integrals,” Master’s thesis submitted to the University of Kufa, 2010.

[3]Nasser, Rasul Hassan, “Comparison between the two acceleration methods, Itkin and Robbery in calculating [3]

integrals numerically”, Master’s thesis submitted to the University of Kufa, 2011.