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Using Rombric acceleration to numerically improve the results of values of double integrals

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Abstract: *The main goal of this research is to numerically calculate double integrals with continuous integrals.Using the MT method ((the combined method of using the R rumbrack acceleration method with the trapezoidal rule on the outer dimension Y and Mid – point rule on the inner dimensionX,when the number of partial periods into which the external integration period is divided is equal to the number of partial periods into which the internal integration period is divided, it gives good results in terms of accuracy and speed of approaching the real value, and thus the method of calculating double integrals numerically can be relied upon.*

Keyword: **Rombric** acceleration, trapezoidal rule,function

1. Introduction

The subject of numerical analysis is characterized by devising various methods to find approximate solutions to specific mathematical problems in an effective manner. The efficiency of these methods depends on both the accuracy and ease with which they can be implemented. Modern numerical analysis is the numerical interface to the broad field of applied analysis

In this research, we discuss a numerical method composed of the trapezoidal base and the Simpson rule, then improving the results using the R **Rombric** acceleration. We will symbolize this method with the **Rombric** acceleration with the symbol RMT : S represents the Mid-point rule, T represents the trapezoidal base, and R the **Rombric** acceleration represents the following. The derivation of the method follows.

1.1 Trapezoidal 's rule on the outer dimension and the Mid-point rule on the inner dimension (MT)

Applying Trapezoidal rule to the one-way integral $\int_c^d f(x,y)dy$ we get:

$$T = \int_a^b f(x,y)dx = \frac{h}{2}\left[f(a,y) + f(b,y) + 2\sum_{i=1}^{n-1} f(x_i,y)]\right] + \frac{(b-a)}{-12}h^2\frac{\partial^2 f(\mu_1,y)}{\partial x^2} + \frac{(b-a)}{720}h^4\frac{\partial^4 f(\mu_2,y)}{\partial x^4} - \frac{(b-a)}{30240}h^6\frac{\partial^6 f(\mu_3,y)}{\partial x^6} + \cdots \qquad \cdots(1)$$

Wears $\mu_i \in (a,b)$ and $i = 1,2,3,\cdots$, i=1,2,3,$\cdots$n-1, $x_i = a + ih$, $h = \frac{b-a}{n}$

To calculate the integration on the external dimension by applying the midpoint rule to each term of equation (1), we obtain:

$$i)\ \int_c^d \frac{h}{2} f(a,y)dy = \frac{h^2}{2}\sum_{j=0}^{n-1} f(a, y_j + \tfrac{h}{2}) + \frac{h}{2}\left[\frac{(d-c)}{6}h^2\frac{\partial^2 f(a,\lambda_{11})}{\partial y^2} - \frac{7(d-c)}{360}h^4\frac{\partial^4 f(a,\lambda_{12})}{\partial y^4} + \frac{31(d-c)}{15120}h^6\frac{\partial^6 f(a,\lambda_{13})}{\partial y^6} - \cdots\right] + \qquad \cdots(2)$$

$$ii)\ \int_c^d \frac{h}{2} f(b,y)dy = \frac{h^2}{2}\sum_{j=0}^{n-1} f(b, y_j + \tfrac{h}{2}) +$$
$$\frac{h}{2}\left[\frac{(d-c)}{6}h^2\frac{\partial^2 f(b,\lambda_{21})}{\partial y^2} - \frac{7(d-c)}{360}h^4\frac{\partial^4 f(b,\lambda_{22})}{\partial y^4} + \frac{31(d-c)}{15120}h^6\frac{\partial^6 f(b,\lambda_{23})}{\partial y^6} - \cdots\right] + \qquad \cdots(3)$$

$$iii)\ \int_c^d h\sum_{i=1}^{n-1} f(x_i,y)dy = h^2\sum_{j=0}^{n-1}\sum_{i=1}^{n-1} f(x_i, y_j + \tfrac{h}{2}) +$$
$$h\sum_{i=1}^{n-1}\left[\frac{(d-c)}{6}h^2\frac{\partial^2 f(x_i,\lambda_{2+i1})}{\partial y^2} - \frac{7(d-c)}{360}h^4\frac{\partial^4 f(x_i,\lambda_{2+i2})}{\partial y^4} + \frac{31(d-c)}{15120}h^6\frac{\partial^6 f(x_i,\lambda_{2+i3})}{\partial y^6} + \cdots\right] \quad (4)$$

whears $y_j = c + jh \qquad j = 0,1,2,3,\cdots,n-1 \qquad \mathrm{x_i} = a + ih \qquad \mathrm{i=1,2,3,\cdots n\text{-}1}$

By combining equations (4), (3), (2) and integrating the error formula in equation (1), we get

$$MT = \int_c^d\int_a^b f(x,y)dxdy = \frac{h^2}{2}\sum_{j=0}^{n-1}\left(f(a,y_j+\tfrac{h}{2}) + f(b,y_j+\tfrac{h}{2}) + 2\sum_{i=1}^{n-1} f(x_i,y_j+\tfrac{h}{2})\right) +$$
$$\int_c^d\left[\frac{(b-a)}{-12}h^2\frac{\partial^2 f(\mu_1,y)}{\partial x^2} + \frac{(b-a)}{720}h^4\frac{\partial^4 f(\mu_2,y)}{\partial x^4} - \frac{(b-a)}{30240}h^6\frac{\partial^6 f(\mu_3,y)}{\partial x^6} + \cdots\right]dy +$$
$$\frac{h}{2}\left[\frac{(d-c)}{6}h^2\frac{\partial^2 f(a,\lambda_{11})}{\partial y^2} - \frac{7(d-c)}{360}h^4\frac{\partial^4 f(a,\lambda_{12})}{\partial y^4} + \frac{31(d-c)}{15120}h^6\frac{\partial^6 f(a,\lambda_{13})}{\partial y^6} - \cdots\right] +$$
$$\frac{h}{2}\left[\frac{(d-c)}{6}h^2\frac{\partial^2 f(b,\lambda_{21})}{\partial y^2} - \frac{7(d-c)}{360}h^4\frac{\partial^4 f(b,\lambda_{22})}{\partial y^4} + \frac{31(d-c)}{15120}h^6\frac{\partial^6 f(b,\lambda_{23})}{\partial y^6} - \cdots\right] +$$
$$h\sum_{i=1}^{n-1}\left[\frac{(d-c)}{6}h^2\frac{\partial^2 f(x_i,\lambda_{2+i1})}{\partial y^2} - \frac{7(d-c)}{360}h^4\frac{\partial^4 f(x_i,\lambda_{2+i2})}{\partial y^4} + \frac{31(d-c)}{15120}h^6\frac{\partial^6 f(x_i,\lambda_{2+i3})}{\partial y^6} - \right] \qquad \cdots(5)$$

whears $\mu_1,\mu_2,\mu_3,\cdots \in (a,b)$ and $\lambda_{kl} \in (c,d)$ and $k = 1,2,3,\cdots,n+1$, $l = 1,2,3,\cdots$ Since the partial derivatives of the function f with respect to the variable x are continuous in its integration region, using the mean value of integration theorem] we get

$$MT = \frac{h^2}{2}\sum_{j=0}^{n-1}\left(f(a,y_j+\tfrac{h}{2}) + f(b,y_j+\tfrac{h}{2}) + 2\sum_{i=1}^{n-1} f(x_i,y_j+\tfrac{h}{2})\right) +$$
$$(b-a)(d-c)\left[\frac{h^2}{-12}\frac{\partial^2 f(\mu_1,\theta_1)}{\partial x^2} + \frac{h^4}{720}\frac{\partial^4 f(\mu_2,\theta_2)}{\partial x^4} - \frac{h^6}{30240}\frac{\partial^6 f(\mu_3,\theta_3)}{\partial x^3} + \cdots\right] +$$
$$h^2\left[\frac{h}{2}\frac{(d-c)}{6}\frac{\partial^2 f(a,\lambda_{11})}{\partial y^2} + \frac{h}{2}\frac{(d-c)}{6}\frac{\partial^2 f(b,\lambda_{21})}{\partial y^2} + h\sum_{i=1}^{n-1}\frac{(d-c)}{6}\frac{\partial^2 f(x_i,\lambda_{2+i1})}{\partial y^2}\right] +$$
$$h^4\left[\frac{h}{2}\frac{7(d-c)}{-360}\frac{\partial^4 f(a,\lambda_{12})}{\partial y^4} + \frac{h}{2}\frac{7(d-c)}{-360}\frac{\partial^4 f(b,\lambda_{22})}{\partial y^4} + h\sum_{i=1}^{n-1}\frac{7(d-c)}{-360}\frac{\partial^4 f(x_i,\lambda_{2+i2})}{\partial y^4}\right] + \cdots \qquad \cdots(2.13)$$

Where θ_i values belong to the period (c, d) and $i = 1,2,3,\cdots$ from here we notice that the value of the integration in (2) using the MT rule with correction limits becomes

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$$MT = \frac{h^2}{2}\sum_{j=0}^{n-1}\left(f(a, y_j + \tfrac{h}{2}) + f(b, y_j + \tfrac{h}{2}) + 2\sum_{i=1}^{n-1} f(x_i, y_j + \tfrac{h}{2})\right) + A_{MT}h^2 + B_{MT}h^4 + C_{MT}h^6 + \cdots$$

Where $A_{MT}, B_{MT}, C_{MT}, \cdots$ are constants whose values depend on the partial derivatives of the function f with respect to the variables y and x and that $,(j = 0,1,2,\cdots,n-1\ ,\ y_j = c + jh)$, $(\mathrm{i}=1,2,3,\cdots \mathrm{n}\text{-}1\ ,\ \mathrm{x_i} = a + ih)$

So the correction limits for the MT rule are:

$$E_{MT}(h) = A_{MT}h^2 + B_{MT}h^4 + C_{MT}h^6 + \cdots$$

2.example

1- $\int_3^4\int_2^3 xy\ \ln(x+y)dxdy$ Its analytical value is (15. 74097730678463) rounded to fourteen decimal places.

2- $\int_1^2\int_0^1 xe^{(-x-y)}dxdy$ Its analytical value is (0. 06144772819733) rounded to fourteen decimal places.

3- $\int_1^2\int_1^2 \ln(x+y)dxdy$ Its analytical value is (1. 08913865206603) rounded to fourteen decimal places.

Note:In all the integrations above, the integrator is defined at each point in its integration region. Therefore, when using the rule (RM) to improve the integration results, we use the following correction limits: $E = Ah^2 + Bh^4 + Ch^6 + \cdots$

3.Results

1- For integration $\int_3^4\int_2^3 xy\ \ln(x+y)dxdy$ When applying the rule (MT), the value is correct to three decimal places when n = 32, n = 32, and n = 16, respectively, but when using Romberg acceleration with the method (RMT), the value is correct to at least fourteen decimal places when n = 32. (partial period), 2^{10} as in Table (1)

2- integration $\int_1^2\int_0^1 xe^{(-x-y)}dxdy$ When applying the rule (MT), the value is correct to three decimal places when n = 32, n = 32, and n = 16, respectively, but when using Romberg acceleration with the method (RMT), the value is correct to at least fourteen decimal places when n = 32

as in Table (2)

3- integration $\int_1^2\int_1^2 \ln(x+y)dxdy$ When applying the rule (MT), the value is correct to four decimal places when n = 32, n = 32, and n = 16, respectively, but when using Romberg acceleration with the method (RMT), the value is correct to at least fourteen decimal places when n = 32

as in Table (3)

=m	MT	k=2	k=4	k=6	k=8	k=10
1	15.79357975156784					
2	15.75411299330246	15.74095740721399				
4	15.74426029419750	15.74097606116252	15.74097730475909			
8	15.74179799523047	15.74097722890812	15.74097730675783	15.74097730678956		
16	15.74118247524535	15.74097730191698	15.74097730678424	15.74097730678466	15.74097730678464	
32	15.74102859867164	15.74097730648040	15.74097730678463	15.74097730678463	15.74097730678463	15.74097730678463
tabel(1)						15.74097730678463

n	TM	k=2	k=4	k=6	k=8	k=10
1	0.07630378969308					
2	0.06515477092705	0.06143843133837				
4	0.06237366398552	0.06144662833834	0.06144717480501			
8	0.06167915412129	0.06144765083321	0.06144771899953	0.06144772763754		
16	0.06150558094924	0.06144772322523	0.06144772805137	0.06144772819505	0.06144772819723	
32	0.06146219115063	0.06144772788443	0.06144772819504	0.06144772819732	0.06144772819733	0.06144772819733
table(2)						0.06144772819733

n	MT	k=2	k=4	k=6	k=8	k=10
1	1.08452685018476					
2	1.08793172253238	1.08906667998158				
4	1.08883320145205	1.08913369442527	1.08913816205485			
8	1.08906205060568	1.08913833365690	1.08913864293900	1.08913865057209		
16	1.08911948666992	1.08913863202467	1.08913865191586	1.08913865205835	1.08913865206417	
32	1.08913385977589	1.08913865081121	1.08913865206365	1.08913865206600	1.08913865206603	1.08913865206603
table(3)						1.08913865206603

4.Results

In the first integration, when applying the TM rule, we obtained a correct value to three decimal places when n = 32, but after using Rombrock acceleration, the value became correct to twelve decimal places when n = 32.

As for the second integration, when applying the TM rule, we obtained an integer value to four decimal places when n = 32, and by using Rombrock acceleration, we obtained an integer value to fourteen decimal places when n = 32.

In the third and fourth integrals, when applying the TM rule, we obtained integer values for four decimal places when n = 32, and using Rombrock acceleration, we obtained integer values for fourteen decimal places when n = 16 and when n = 32.

We conclude from the above that the RMT method is good in terms of accuracy and speed of approaching the true value, and therefore it can be relied upon in calculating binary integrals numerically.

Sources

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