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Design of Discrete-time LQR for Optimal Positioning Control of Deep Space Satellite Antenna

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Abstract: *In optimally positioning of a satellite, a cost effective process can be achieved regarding the anticipated link margin necessary to receive most of the of the satellite transmission data from certain angle of elevation (or azimuth position). In this work, an optimal control system based on discrete-time linear quadratic regulator (LQR) has been designed and implemented with a DC servomotor-based antenna positioning control system in MATLAB/Simulink environment. The system response was initially analyzed considering the uncompensated state. The results obtained were presented in terms of steady-state transient response characteristics, which were rise time, peak time, settling time, overshoot, and steady-state error. In the uncompensated state, the rise time was 0.52s, peak time was 2.28s, settling time was 5.34s, overshoot was 34.6%, and steady state error was 0.181 rad. Considering the high value of overshoot, a LQR was designed and incorporated into the system to form a closed loop control system, and simulation conducted revealed that the oscillation or cycling (or instability) associated with the uncompensated because of high overshoot (34.6%) was largely reduced to 6.43% and steady state error of 0 was achieved. The performance of LQR was compared with that of PID controller. The comparison indicated that the LQR provided better stable system than PID controller by offering overshoot of 6.43% against 8.09%. Hence, for optimal control of deep space antenna, the optimal control scheme (LQR) will provide improved stability than PID controller.*

Keywords—Position control; Satellite antenna; Optimal control; LQR; PID controller

1. Introduction

Communication is facilitated between observers on the earth and satellites by the means of antennas. However, the earth station antenna usually suffers from the problem of pointing accuracy or satellite tracking inadequacy due to improper alignment or positioning of the antenna's dish structure for efficient satellite communication. Thus, this has become a major problem since effective communication between the antenna and the satellite requires that the antenna is aligned so as to correctly aim at satellite location. It is not easy to point accurately since every antenna is dedicated to a specific satellite [1]. Hence, it is important to have a control system integrated with the antenna that provides optimum positioning for quality signal transmission and reception.

In optimally positioning of a satellite, a cost effective process can be achieved regarding the anticipated link margin necessary to receive most of the of the satellite transmission data from certain angle of elevation (or azimuth position). In order to achieve this, a controller is included as a subsystem and it is linked to the existing antenna system in order to improve its performance. A controller offers required command to the entire system to guarantee that the desired performance response is met. This means, the control system makes sure that stability, steady state and enhanced transient responses, cost and robustness are realized. These requirements are applicable to designing antenna servo control system for satellite ground station since the objective is to have a system with a robust tracking and reduced steady state error and improved transient response. Antenna servo control system for satellite ground station is the regulation of speed and position of a motorized antenna based on a feedback control mechanism.

One of the control strategies that can provide cost effectiveness and timeliness is the optimal controller implemented in modern control system. In this paper, an optimal control of deep space antenna positioning system is considered.

2. Literature Review

This section presents some of the most recent studies on satellite antenna positioning. Tracking of satellite by deep space antenna positioning control system was achieved using weighted cultural artificial fish swarm algorithm by Salawudeen et al. [2]. Series arranged antenna loop that included a proportional integral and derivative (PID) controller was used to implement a novel smart antenna system in which automatic control commenced at any time the transceiver receive low signal strength and feed it to process comparator unit [3]. PID tuned compensator was used to provide robust control for DC servo motor based antenna positioning control system by Eze et al. [4]. Fuzzy-PID controller has been used to solve the problem of pointing accuracy or tracking in satellite antenna positioning control system in [5] and [6]. Satellite antenna in distributed mobile telemedicine nodes has been control using back propagation neural network, based PID controller, full state feedback control scheme, and PID plus pre-filter transfer function in [7-9]. Model following control based PID (MFC-PID) controller was used to achieve effective positioning control for DC servo motor based antenna system [10]. Antenna azimuth positioning system subject to external disturbance was improved using nonlinear PID controller whose parameters were tuned by particle swarm optimization (PSO) [11]. PID tuned digital and cascade compensators have been separately used to implement positioning control of ground station satellite antenna in MATALB/Simulink [12,13]. Linear quadratic regulator has been separately applied in the control of DC servomotor based antenna by Chishti et al. [14] and Aloo et al. [15].

Regarding the related surveyed papers much study about the use of optimal control for deep space antenna has not been carried out. Therefore, this paper attempts to fill this gap by implementing a discrete time (digital) optimal controller based on LQR to replace the continuous time LQR implemented in [14] and [15].

3. SYSTEM DESIGN

State-space analysis is used to evaluate and control linear time invariant (LTI) and in the design of optimal controllers in modern control systems. Thus the state space model of a LTI system can be defined by [16,17]:

$$\left.\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned}\right\} \tag{1}$$

The constants A, B, C, and D represent the state matrix, input matrix, output matrix, and direct transition matrix respectively. The state space description of the DC servomotor position control for deep space antenna is defined in (2) and (3). Table 1 defines the parameters of the deep space DC motor based antenna system.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{B_a}{J_a} & \frac{K_T}{J_a} \\ 0 & -\frac{K_B}{L_a} & -\frac{R_a}{L_a} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L_a} \end{bmatrix} V_a \tag{2}$$

$$y = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} V_a$$

(3)

Table 1. Parameters of deep space DC motor based antenna system [15]

Physical Quantity	Definition of Quantity	Numerical Value
a	Power Amplifier Pole	100
a_m	Motor and Load Pole	1.71
B_a	Motor Dampening Constant [Nm/rad]	0.01
B_L	Load Dampening Constant [Nms/rad]	1
B_m	Equivalent Viscous Friction Coefficient [Nms/rad]	0.02
J_a	Motor Inertia constant [kgm^2]	0.02
J_L	Load Inertia constant [kgm^2]	1
J_m	Equivalent moment of Inertia [kgm^2]	0.03
K	Preamplifier Gain	-
K_1	Power Amplifier Gain	100
K_B	Back emf Constant [Vs/rad]	0.5
K_g	Gear Ratio	0.1
K_m	Motor and Load Gain	2.083
K_{pot}	Potentiometer Gain	0.318
K_T	Motor Torque Constant [Nm/A]	0.5
L_a	Motor Armature Inductance [H]	0.45
N	Turns on Potentiometer	10
N_1, N_2, N_3	Gear Teeth (Respectively	25,250,250
R_a	Motor Armature resistance [Ω]	8
V	Voltage Across Potentiometer [V]	10

The substitution of the numerical values of the DC servomotor of the antenna positioning control system in Table 1 gives:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -0.5 & 25 \\ 0 & -1.111 & -17.78 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 2.22 \end{bmatrix} u \tag{4}$$

where,

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -0.5 & 25 \\ 0 & -1.111 & -17.78 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 2.22 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix},$$

$$D = \begin{bmatrix} 0 \end{bmatrix}$$

$$y = \begin{bmatrix} 663 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} u \tag{5}$$

Equation (4) and (5) are the continuous time state-space model of the system. The discrete time state-space model is presented in subsection 3.1.

3.1 Discrete Time State-space Model

For nth order LTI system, its discrete-time state-space model is defined by:

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$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) + Du(k) \end{aligned} \quad (6)$$

In this paper, the sampling time chosen for the conversion from continuous time to discrete time is 0.01 s based on zero order hold (ZOH) sampling technique. Thus, for the DC servomotor positioning system, the discrete time model of the state-space equation is given by:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} = \begin{bmatrix} 0.9999 & 0.009977 & 3.656e-05 \\ -0.02424 & 0.9937 & 0.006258 \\ -4.149 & -1.094 & 0.3571 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + \begin{bmatrix} 1.315e-07 \\ 3.656e-05 \\ 0.006258 \end{bmatrix} u(k) \quad \ldots (7)$$

$$y(k) = \begin{bmatrix} 0 & 0 & 663 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} u(k) \quad (8)$$

3.2 Design of Discrete LQR

The design of discrete-time linear quadratic regulator (LQR) is carried out using the MATLAB command below:

$$\left.\begin{aligned} & A = sys\_d.a; \\ & B = sys\_d.b; \\ & C = sys\_d.c; \\ & D = sys\_d.d; \\ & Q = C' * C \\ & R = 1; \\ & [K] = dlqr(A,B,Q,R) \end{aligned}\right\} \quad (10)$$

where Q and R are real symmetric matrices, and K is the gain (optimal) matrix. The values of Q and K for the DC servomotor speed/position system and the antenna positioning loop are given by (11) and (12):

$$\left.\begin{aligned} & Q_{DCmotor} = \begin{bmatrix} 100 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ & K_{DCmotor} = \begin{bmatrix} 9.6702 & 2.4536 & 2.9657 \end{bmatrix} \end{aligned}\right\}$$
(11)

$$\left.\begin{aligned} & Q = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 439569 \end{bmatrix} \\ & K = \begin{bmatrix} -509.1512 & 32.6613 & 54.8124 \end{bmatrix} \end{aligned}\right\} \quad (12)$$

The closed-loop Simulink model of the discrete time LQR based deep space antenna servomotor position control system is shown in Fig. 1.

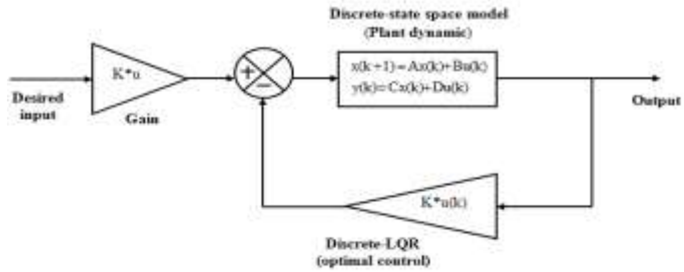

Fig. 1. *Designed LQR position control system*

4. RESULTS

The proposed discrete linear quadratic regulator (LQR) control technique for antenna positioning servo control system has been implemented by simulation in MATLAB/Simulink environment. In addition, LQR controller has been compared with PID controller so as to carry out performance evaluation. Simulation has been carried out considering uncompensated scenario (or open loop) and compensated scenario (closed loop) antenna positioning servomotor. The simulation was based on step response of uncompensated antenna positioning servo control system shown in Fig. 2. In the same vein, the positioning servo control system compensated with a LQR controller is presented Fig. 3. The numerical performance characteristics of the system obtained from simulation curves for each scenario is presented in Table 2. The step responses of the state variables of the system, namely position (x_1), speed (x_2), and current (x_3), of the system in open loop state (uncompensated) are shown in Fig. 4. Similarly, the step responses of the state variables in the closed loop state (compensated) are presented in Fig. 5. Furthermore, to validate this work, step response comparison of proportional-integral-derivative (PID) controller and the designed controller is shown in Fig. 6. Table 3 shows the numerical performance of PID and LQR aid closed loop control of the system.

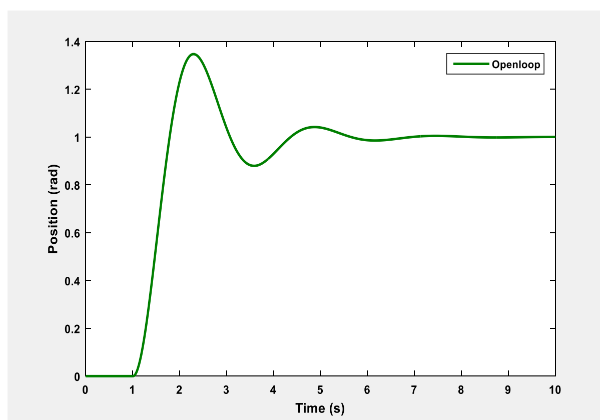

Fig. 2. *Open-loop step response (uncompensated)*

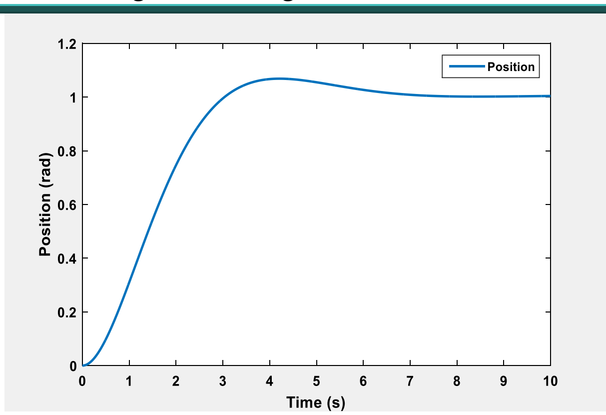

Fig. 3. *LQR control system step response*

Figure 2 represents the step response of antenna positioning servomotor control system model in the open loop when the designed discrete time linear quadratic regulator (LQR) has not been integrated into the system to realize the feedback closed loop control system. That is the condition of the system without a controller (uncompensated). The simulation curve reveals that the system has high degree of instability considering the overshoot which is 34.6% and subsequent oscillations before the system stabilizes.

Figure 3 is the simulation plot for unit step response of antenna positioning servo control system model with an LQR controller. The performance response evaluation in Table 2 indicates that the LQR control technique reduced overshoot value to 6.43% with a steady state error value of 0.

Table 2. Numerical performance analysis of open loop system and LQR system

System	Rise time (s)	Peak time (s)	Settling time (s)	Over shoot (%)	Steady state error (rad)
Open loop	0.52	2.28	5.34	34.6	0.181
LQR	2.02	4.21	6.11	6.43	0

Looking at Table 2, it can be deduced in terms of the rise time that uncompensated system outperformed the LQR system, which is 0.52s against 2.02s. The same holds in terms of peak time and settling time where the values were peak time of 2.28s against peak time of 4.21s and settling time of 5.34s against 6.11s respectively. However, in terms of overshoot and steady state error, the LQR compensated system outperformed the uncompensated system such that when the overshoot was 34.6 % in the case of uncompensated system, the overshoot was 6.43% for the LQR system. For steady state error, the uncompensated system yields 0.181 rad whereas LQR system gives 0 rad. Since the objective is design a system to ensure improved stability by reducing error or deviation with improved tracking for effective positioning, The LQR compensated will offer better antenna positioning for improved line of sight operation (considered in this case as the ability to track unit step input). Thus, with LQR the antenna tracked the satellite signal with zero error (steady state error of 0) with improved stability (overshoot of 6.43%).

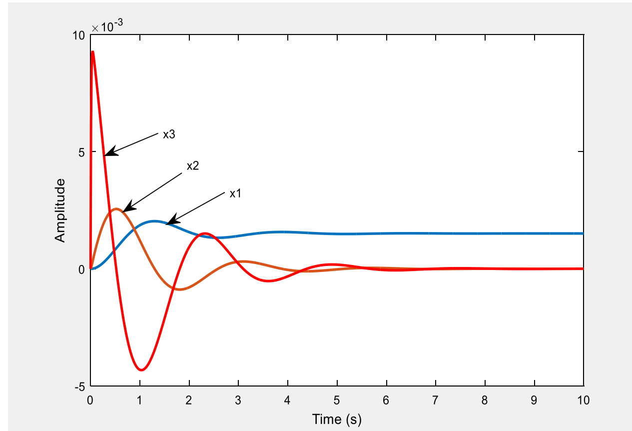

Fig. 4. *Step response performance of state-variables (uncompensated)*

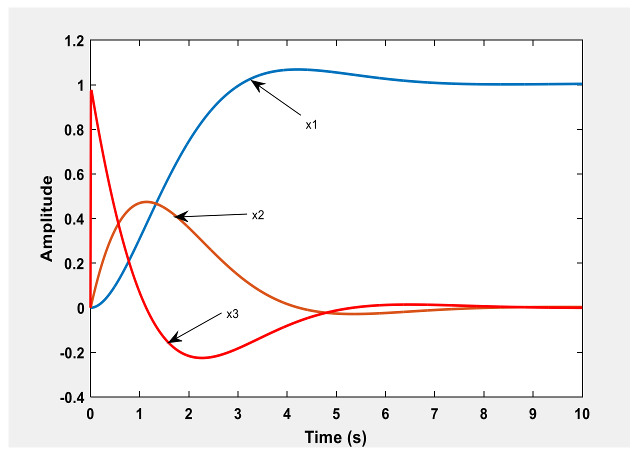

Fig. 5. *Step response performance of state variables (compensated system)*

Figures 4 and 5 are the simulation curves of the step response performances of the state-variables (or internal variable characteristics) of the DC servomotor based antenna positioning system when the controller has not being included as part of the system and when LQR was added to the system. Generally, the curves revealed that the internal characteristics variables of the system are influenced by the system configurations. Hence, it can be seen that the addition of LQR has affected the entire state variables and improved the overall system performance by reducing and eliminating oscillations of the internal variables as shown in Fig. 5.

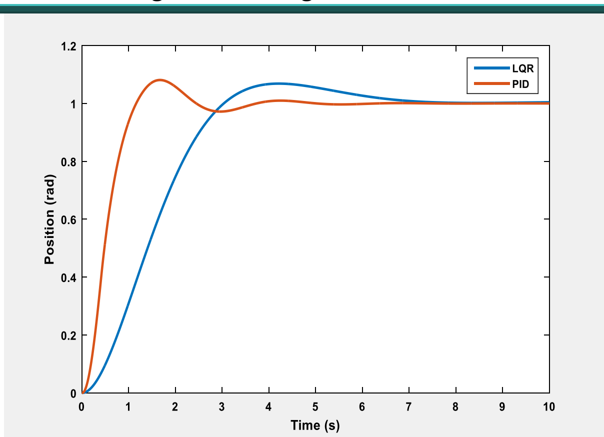

Fig. 6. Comparison of step response of PID and LQR controllers

Table 2. Numerical comparison of PID and LQR control systems

System	Rise time (s)	Peak time (s)	Settling time (s)	Over shoot (%)	Steady state error (rad)
PID	0.73	1.67	3.29	8.09	0
LQR	2.02	4.21	6.11	6.43	0

From Fig. 6 and Table 3 it can be observed considering the rise time that PID system outperformed the LQR system, which is 0.73 s against 2.02 s. The same goes for the case of peak time and settling time where the values were peak time of 1.67 s against peak time of 4.21s and settling time of 3.29s against 6.11s respectively. However, in terms of overshoot and steady state error, the LQR compensated system outperformed the PID system such that when the overshoot was 8.09 % in the case of PID compensated system, the overshoot was 6.43% for the LQR system. For steady state error, both PID and LQR yielded 0 rad. Generally, the LQR control servomotor based antenna will improve the stability of the system compare with PID controller considering the reduced over shoot it provided, which gives it edge in offering more stable system response.

5. CONCLUSION

A discrete-time Linear Quadratic Regulator (LQR) control system has been designed for Direct Current (D.C.) servomotor-based deep space antenna positioning system has been effectively achieved. This was established through simulation conducted in MATLAB/Simulink environment for Simulated output responses of antenna positioning servo control system to step input signal which met the design specifications. Also, the simulation results obtained showed that LQR control technique could be set up in driving the azimuth/elevation of D.C. servomotors so as to direct a parabolic dish antenna and ensures that it is always kept within the referenced line of sight with a particular satellite.

Generally, the performance of antenna positioning servo control system was optimized to establish a more robust and improved tracking response of satellite communication system using LQR controller.

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