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New features of soft boundary sets in soft ideal topological spaces

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Abstract— *In this research, we will study the term Soft local function, and through it we can define a new function and call it Soft **-frontier set and there is an important and closed relationship with it and the Soft operator.*

Keywords— Soft ideal space; Soft frontier set; Soft Hayashi- Samuel, Soft **- boundary; Soft **- frontier.

1. Introduction

When a scientific problem arises and there is no ability to solve it in the usual ways within a specific space. Attention turns to expanding or contracting that space and constructing the algebraic and topological properties of these spaces in order to solve those problems or contribute to the solution process . Hence the successive ideas in building new forms of sets on the basis of existing and familiar mathematical concepts, as Zadeh built in 1965 [1] the fuzzy sets. In 1999, researcher Molodtsov [2] expanded fuzzy sets and called them soft sets, depended on the parameters E of members of the universal set x such that the soft set $\mathrm{F_A} = \{(e, F(e)); F : E - P(K), A \subseteq E\}$, soft empty $\varphi_E = \{(e, \varphi); \forall\, e \in E\}$ $K_E = \{(e, F(e)); \forall\, e \in E\}$, soft union ant intersection between two soft sets $F_A \cup_E G_B = \{(e, F(e) \cup G(e); \forall\, e \in E)\}$, $F_A \cap_E G_B = \{(e, F(e) \cap G(e); \forall\, e \in E)\}$, the soft complement of $\mathrm{F_A}$, ${\mathrm{F_A}}^c = \{(e, X - F(e)); \forall\, e \in E\}$. In 2011 Shabir [3], defined the topology space on these sets in their canonical form but using soft sets. In 2014 [4], the local function was studied in its usual forms, but using soft on soft points in its three types by kandil and others as follows, ${\mathrm{F_A}}^* =$\{soft point ; ∀ soft open containing the soft point $\ni u_A \cap F_A \notin_E \hat{I}_E$\}, where $\hat{I}_E$ soft ideal on X_E. In 2014 [5], ψ- operation was studied in its usual forms, but using soft sets on soft points in its three types, by Rodynas [5] $\psi(\mathrm{F_A}) = \{soft\ point, \exists soft\ open\ u_F\ containing\ the\ \ soft\ point\ \ni u_F \cap {F_A}^e \in \hat{I}_E\} = [(F_A^e)^*]^e$. It is worth noting that we mention some of the contributions in this field, where we knew two types of soft points that we called them soft turning and bench paint [6],[7] also we defined the local function in fuzzy ideal topological space [8], where AL Razzaq [9],[10] conducted a process of merging between the two fuzzy and soft sets, and through this merging new and diverse points were identified .

2. The Main soft ideal

2.1Definition [5]: Let $(\hat{\mathcal{K}}, \hat{\mathcal{T}}, \hat{I}_{\mathcal{A}})$ be a soft ideal:

1. For any soft subset $\hat{\mathcal{K}}$, $\psi^*(\mathcal{D}_{\mathcal{A}}) = \hat{\mathcal{K}} - (\hat{\mathcal{K}} - \mathcal{D}_{\mathcal{A}})^{**}$.
2. $\mathcal{D}_{\mathcal{A}} \subseteq \hat{\mathcal{K}}$ is called $\hat{I}_{\mathcal{A}}^{\varpi}$ – soft dense iff ${\mathcal{D}_{\mathcal{A}}}^{**} = \hat{\mathcal{K}}$.

2.2Theorem: Let $(\hat{\mathcal{K}}, \hat{\mathcal{T}}, \hat{I}_{\mathcal{A}})$ be a soft ideal, the statement are equivalents:

1. $\hat{\mathcal{T}} \cap \hat{I}_{\mathcal{A}} = \hat{\varphi}$.
2. If $\mathcal{J}_{\mathcal{A}} \in \hat{I}_{\mathcal{A}}$, so $int(\mathcal{J}_{\mathcal{A}}) = \hat{\varphi}$.
3. $\mathcal{H}_{\mathcal{A}} \subset \mathcal{H}_{\mathcal{A}}^*, \forall\, \mathcal{H}_{\mathcal{A}} \subseteq \mathcal{J}_{\mathcal{A}}$.
4. $\hat{\mathcal{K}} = \hat{\mathcal{K}}^*$.

So, soft Hayashi-Samuel space, for any $\mathcal{H}_{\mathcal{A}} \subset \hat{\mathcal{T}}, \mathcal{H}_{\mathcal{A}}^{*} = \mathcal{H}_{\mathcal{A}}^{**}$. If we have any soft ideal topological spaces $(\hat{\mathcal{K}}, \hat{\mathcal{T}}, \hat{I}_{\mathcal{A}})$, we have three functions of any soft subset $\mathcal{C}_{\mathcal{A}}$ of $\hat{\mathcal{K}}$ as follows:$\mathcal{F}_{r\mathcal{A}}^{*}(\mathcal{C}_{\mathcal{A}}) \subseteq \hat{\mathcal{T}}^{*} - \mathcal{F}_{r\mathcal{A}}(\mathcal{C}_{\mathcal{A}}) \subseteq \hat{\mathcal{T}} - \mathcal{F}_{r\mathcal{A}}(\mathcal{C}_{\mathcal{A}})$ where $\mathcal{F}_{r\mathcal{A}}(\mathcal{C}_{\mathcal{A}}) = c\ell_{\hat{\mathcal{T}}^{*}}(\hat{\mathcal{K}} - \mathcal{C}_{\mathcal{A}}) \cap c\ell_{\hat{\mathcal{T}}}(\mathcal{C}_{\mathcal{A}})$. $\hat{\mathcal{T}}^{*} - \mathcal{F}_{r\mathcal{A}}(\mathcal{C}_{\mathcal{A}}) = c\ell_{\hat{\mathcal{T}}^{*}}(\mathcal{C}_{\mathcal{A}}) \cap c\ell_{\hat{\mathcal{T}}^{*}}(\hat{\mathcal{K}} - \mathcal{C}_{\mathcal{A}})$ and $\mathcal{F}_{rr\mathcal{A}}^{*}(\mathcal{C}_{\mathcal{A}}) = \mathcal{C}_{\mathcal{A}}{}^{*} \cap (\hat{\mathcal{K}} - \mathcal{C}_{\mathcal{A}})^{*}$.

2.3Definition: Let $(\hat{\mathcal{K}}, \hat{\mathcal{T}}, \hat{I}_{\mathcal{A}})$ be a soft ideal, then operator $\mathcal{F}_{r\mathcal{A}}^{**}(\mathcal{D}_{\mathcal{A}}): \mathcal{E}(\hat{\mathcal{K}}) \to \mho(\hat{\mathcal{K}})$, defined by $\mathcal{F}_{r\mathcal{A}}^{**}(\mathcal{D}_{\mathcal{A}}) = \mathcal{D}_{\mathcal{A}}^{**} \cap (\hat{\mathcal{K}} - \mathcal{K})^{**}$ is called *-soft boundary of $\mathcal{D}_{\mathcal{A}}$.

2.4proposition: Let $(\hat{\mathcal{K}}, \hat{\mathcal{T}}, \hat{I}_{\mathcal{A}})$ be a soft ideal space. For any soft subset $\mathcal{C}_{\mathcal{A}}$ of $\hat{\mathcal{K}}$, the following are carried:

1. $\mathcal{D}_{\mathcal{A}} \in \mathcal{F}_{r\mathcal{A}}^{**}(\mathcal{C}_{\mathcal{A}})$ iff $\mathcal{D}_{\mathcal{A}}^{**} - \psi^{*}(\mathcal{C}_{\mathcal{A}})$.
2. $\mathcal{F}_{r\mathcal{A}}(\mathcal{C}_{\mathcal{A}}) = \hat{\varphi}$ iff $\mathcal{C}_{\mathcal{A}}^{**} \subseteq \psi^{*}(\mathcal{C}_{\mathcal{A}})$.
3. $\mathcal{F}_{r\mathcal{A}}^{**}(\mathcal{C}_{\mathcal{A}}) = (\hat{\mathcal{K}} - \mathcal{C}_{\mathcal{A}})^{**}$ iff $\hat{\mathcal{K}} - \mathcal{C}_{\mathcal{A}}^{**} \subseteq \psi^{*}(\mathcal{C}_{\mathcal{A}})$.
4. If $\mathcal{C}_{\mathcal{A}}$ is $\hat{I}_{\mathcal{A}}^{\mathfrak{w}}$ – soft dense, then $\mathcal{F}_{r\mathcal{A}}^{**}(\mathcal{C}_{\mathcal{A}}) = (\hat{\mathcal{K}} - \mathcal{C}_{\mathcal{A}})^{**}$.

Proof 1. Let $\mathcal{D}_{\mathcal{A}} \in \mathcal{F}_{r\mathcal{A}}^{**}(\mathcal{C}_{\mathcal{A}})$ iff $\mathcal{D}_{\mathcal{A}} \in \mathcal{C}_{\mathcal{A}}^{**}$ and iff $\mathcal{D}_{\mathcal{A}} \in (\hat{\mathcal{K}} - \mathcal{C}_{\mathcal{A}})^{**}$ iff $\mathcal{D}_{\mathcal{A}} \in \mathcal{C}_{\mathcal{A}}^{**}$ and $\mathcal{D}_{\mathcal{A}} \notin \psi^{*}(\mathcal{C}_{\mathcal{A}})$ iff $\mathcal{D}_{\mathcal{A}} \in \mathcal{C}_{\mathcal{A}}^{**} - \psi^{*}(\mathcal{C}_{\mathcal{A}})$.

Proof 4. Let $\mathcal{C}_{\mathcal{A}}$ is $\hat{I}_{\mathcal{A}}^{\mathfrak{w}}$ – soft dense, then $\mathcal{C}_{\mathcal{A}}^{**} = \hat{\mathcal{K}}$, so, $\mathcal{F}_{r\mathcal{A}}^{**}(\mathcal{C}_{\mathcal{A}}) = \mathcal{C}_{\mathcal{A}}^{**} \cap (\hat{\mathcal{K}} - \mathcal{C}_{\mathcal{A}})^{**}$ =$(\hat{\mathcal{K}} - \mathcal{C}_{\mathcal{A}})^{**}$. By (proposition 2.4) pare (1) $\psi^{*}(\mathcal{C}_{\mathcal{A}}) \subseteq \mathcal{C}_{\mathcal{A}}^{**}$ for any soft subset $\mathcal{C}_{\mathcal{A}}$ in the soft Hayashi-Samuel space. There are some properties that connect ψ^{*}-operator with **- soft frontier in the soft Hayashi-Samuel space.

2.5proposition: Let $(\hat{\mathcal{K}}, \hat{\mathcal{T}}, \hat{I}_{\mathcal{A}})$ is soft Hayashi-Samuel space, the following statements are correct:

1. $\mathcal{F}_{r\mathcal{A}}^{**}(\mathcal{D}_{\mathcal{A}}) = \hat{\varphi}$ iff $\mathcal{D}_{\mathcal{A}}^{**} = \psi^{*}(\mathcal{D}_{\mathcal{A}})$.
2. For each soft closed set $\mathcal{D}_{\mathcal{A}}, \mathcal{F}_{r\mathcal{A}}^{**}(\mathcal{D}_{\mathcal{A}}) = \mathcal{D}_{\mathcal{A}}^{**} \subseteq int(\mathcal{D}_{\mathcal{A}})$.

Proof. Let $\mathcal{D}_{\mathcal{A}}$ be soft closed subset of $\hat{\mathcal{K}}, \mathcal{F}_{r\mathcal{A}}^{**}(\mathcal{D}_{\mathcal{A}}) = \mathcal{D}_{\mathcal{A}}^{**} \cap (\hat{\mathcal{K}} - \mathcal{D}_{\mathcal{A}})^{**} = \mathcal{D}_{\mathcal{A}}^{**} \cap c\ell(\hat{\mathcal{K}} - \mathcal{D}_{\mathcal{A}})$, by (Theorem 2.2), $c\ell(\hat{\mathcal{K}} - \mathcal{D}_{\mathcal{A}}) = (\hat{\mathcal{K}} - \mathcal{D}_{\mathcal{A}})^{**} = (\hat{\mathcal{K}} - \mathcal{D}_{\mathcal{A}})^{*}$, so, $\mathcal{F}_{r\mathcal{A}}^{**}(\mathcal{D}_{\mathcal{A}}) = \mathcal{D}_{\mathcal{A}}^{**} \cap (\hat{\mathcal{K}} \cap int(\mathcal{D}_{\mathcal{A}})) = \mathcal{D}_{\mathcal{A}}^{**} \subseteq int(\mathcal{D}_{\mathcal{A}})$. There are multiple properties and characteristics to **-soft boundary set, as in the following proposition.

2.6proposition : For any soft subsets $\mathcal{B}_{\mathcal{A}}, \mathcal{C}_{\mathcal{A}}$ in soft ideal $(\hat{\mathcal{K}}, \hat{\mathcal{T}}, \hat{I}_{\mathcal{A}})$, the properties are true:

1. $\mathcal{F}_{r\mathcal{A}}^{**}(\varphi) = \mathcal{F}_{r\mathcal{A}}^{**}(\hat{\mathcal{K}}) = \hat{\varphi}$.
2. For any $\mathcal{J}_{\mathcal{A}} \in \hat{I}_{\mathcal{A}}, \mathcal{F}_{r\mathcal{A}}^{**}(\mathcal{J}_{\mathcal{A}}) = \hat{\varphi}$.
3. $\mathcal{F}_{r\mathcal{A}}^{**}(\mathcal{B}_{\mathcal{A}} \cup \mathcal{C}_{\mathcal{A}}) = \mathcal{F}_{r\mathcal{A}}^{**}(\mathcal{B}_{\mathcal{A}}) \cup \mathcal{F}_{r\mathcal{A}}^{**}(\mathcal{C}_{\mathcal{A}})$.
4. $\mathcal{F}_{r\mathcal{A}}^{**}(\mathcal{B}_{\mathcal{A}}) = \mathcal{B}_{\mathcal{A}}^{**} - \psi^{*}(\mathcal{B}_{\mathcal{A}})$.
5. $\mathcal{F}_{r\mathcal{A}}^{**}(\mathcal{F}_{r\mathcal{A}}^{**}(\mathcal{B}_{\mathcal{A}})) \subseteq \mathcal{F}_{r\mathcal{A}}^{**}(\mathcal{B}_{\mathcal{A}})$.
6. $\mathcal{F}_{r\mathcal{A}}^{**}(\mathcal{B}_{\mathcal{A}}) = (\hat{\mathcal{K}} - \mathcal{B}_{\mathcal{A}})^{**} - \psi^{*}(\hat{\mathcal{K}} - \mathcal{B}_{\mathcal{A}})$.
7. $\mathcal{F}_{r\mathcal{A}}^{**}(\hat{\mathcal{K}} - \mathcal{B}_{\mathcal{A}}) = \mathcal{F}_{r\mathcal{A}}^{**}(\mathcal{B}_{\mathcal{A}})$.
8. $\hat{\mathcal{K}} - \mathcal{F}_{r\mathcal{A}}^{**}(\mathcal{B}_{\mathcal{A}}) = \psi^{*}(\hat{\mathcal{K}} - \mathcal{B}_{\mathcal{A}}) \cup \psi^{*}(\mathcal{B}_{\mathcal{A}})$.
9. $\hat{\mathcal{K}} = \psi^{*}(\hat{\mathcal{K}} - \mathcal{B}_{\mathcal{A}}) \cup \psi^{*}(\mathcal{B}_{\mathcal{A}}) \cup \mathcal{F}_{r\mathcal{A}}^{**}(\mathcal{B}_{\mathcal{A}})$.

Proof 6. Let $\mathcal{F}_{r\mathcal{A}}^{**}(\mathcal{B}_{\mathcal{A}}) = \mathcal{B}_{\mathcal{A}}^{**} \cap (\hat{\mathcal{K}} - \mathcal{B}_{\mathcal{A}})^{**} = \mathcal{B}_{\mathcal{A}}^{**} \cap \hat{\mathcal{K}} - (\hat{\mathcal{K}} - (\hat{\mathcal{K}} - \mathcal{B}_{\mathcal{A}})^{**}) = \mathcal{B}_{\mathcal{A}}^{**} \cap \hat{\mathcal{K}} - \psi^{*}(\mathcal{B}_{\mathcal{A}})$.

Proof 8. $\hat{\mathcal{K}} - \mathcal{F}_{r\mathcal{A}}^{**}(\mathcal{B}_{\mathcal{A}}) = \hat{\mathcal{K}} - (\mathcal{B}_{\mathcal{A}}^{**} \cap (\hat{\mathcal{K}} - \mathcal{B}_{\mathcal{A}})^{**}) = (\hat{\mathcal{K}} - \mathcal{B}_{\mathcal{A}}^{**}) \cup (\hat{\mathcal{K}} - (\hat{\mathcal{K}} - \mathcal{B}_{\mathcal{A}})^{**}) = \hat{\mathcal{K}} - (\hat{\mathcal{K}} - (\hat{\mathcal{K}} - \mathcal{B}_{\mathcal{A}})^{**}) \cup \psi^{*}(\mathcal{B}_{\mathcal{A}}) = \psi^{*}(\hat{\mathcal{K}} - \mathcal{B}_{\mathcal{A}}) \cup \psi^{*}(\mathcal{B}_{\mathcal{A}})$.

Proof 9. $\psi^{*}(\hat{\mathcal{K}} - \mathcal{B}_{\mathcal{A}}) \cup \psi^{*}(\mathcal{B}_{\mathcal{A}}) \cup \mathcal{F}_{r\mathcal{A}}^{**}(\mathcal{B}_{\mathcal{A}}) = \hat{\mathcal{K}} - (\hat{\mathcal{K}} - (\hat{\mathcal{K}} - \mathcal{B}_{\mathcal{A}})^{**}) \cup (\hat{\mathcal{K}} - (\hat{\mathcal{K}} - \mathcal{B}_{\mathcal{A}})^{**})\mathcal{F}_{r\mathcal{A}}^{**}(\mathcal{B}_{\mathcal{A}}) = (\hat{\mathcal{K}} - \mathcal{B}_{\mathcal{A}})^{**} \cup (\hat{\mathcal{K}} - (\hat{\mathcal{K}} - \mathcal{B}_{\mathcal{A}})^{**})\mathcal{F}_{r\mathcal{A}}^{**}(\mathcal{B}_{\mathcal{A}}) = \hat{\mathcal{K}} \cup \mathcal{F}_{r\mathcal{A}}^{**}(\mathcal{B}_{\mathcal{A}}) = \hat{\mathcal{K}}$. For the (Definition 2.1), it is simple to show that for any soft open set $\mathcal{U}_{\mathcal{A}}, \mathcal{F}_{r\mathcal{A}}^{**}(\mathcal{U}_{\mathcal{A}}) \subseteq \mathcal{U}_{\mathcal{A}}{}^{**} - \mathcal{U}_{\mathcal{A}}$.

2.7 proposition : For any soft subsets $\hat{I}_{1\mathcal{A}}, \hat{I}_{2\mathcal{A}}$ in soft ideal $(\hat{\mathcal{K}}, \hat{\mathcal{T}}, \hat{I}_{\mathcal{A}})$, the following are correct:

1. $\mathcal{F}_{r\mathcal{A}}^{**}(\hat{I}_{1\mathcal{A}} \cap \hat{I}_{2\mathcal{A}}) \subseteq \mathcal{F}_{r\mathcal{A}}^{**}(\hat{I}_{1\mathcal{A}}) \cup \mathcal{F}_{r\mathcal{A}}^{**}(\hat{I}_{2\mathcal{A}})$.
2. $\mathcal{F}_{r\mathcal{A}}^{**}(\hat{I}_{1\mathcal{A}} - \hat{I}_{2\mathcal{A}}) \subseteq \mathcal{F}_{r\mathcal{A}}^{**}(\hat{I}_{1\mathcal{A}}) \cup \mathcal{F}_{r\mathcal{A}}^{**}(\hat{I}_{2\mathcal{A}})$.

3. $\mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{1\mathcal{A}}) \cup \mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{2\mathcal{A}}) = \mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{1\mathcal{A}} - \hat{I}_{2\mathcal{A}}) \cup \mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{2\mathcal{A}} - \hat{I}_{1\mathcal{A}}) \cup \mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{1\mathcal{A}} \cap \hat{I}_{2\mathcal{A}})$.

Proof 1. By (Proposition 2.6) part (3,7) we get that $\mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{1\mathcal{A}} \cap \hat{I}_{2\mathcal{A}}) = \mathcal{F}^{**}_{r\mathcal{A}}(\hat{\mathcal{K}} - \hat{I}_{1\mathcal{A}} \cap \hat{I}_{2\mathcal{A}}) = \mathcal{F}^{**}_{r\mathcal{A}}(\hat{\mathcal{K}} - \hat{I}_{1\mathcal{A}} \cup \hat{\mathcal{K}} - \hat{I}_{2\mathcal{A}}) \subseteq \mathcal{F}^{**}_{r\mathcal{A}}(\hat{\mathcal{K}} - \hat{I}_{1\mathcal{A}}) \cup \mathcal{F}^{**}_{r\mathcal{A}}(\hat{\mathcal{K}} - \hat{I}_{2\mathcal{A}}) = \mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{1\mathcal{A}}) \cup \mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{2\mathcal{A}})$.

Proof 2. By part (1) and (Proposition 2.6) part (3,7) we get that $\mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{1\mathcal{A}} - \hat{I}_{2\mathcal{A}}) = \mathcal{F}^{**}_{r\mathcal{A}}\left(\hat{I}_{1\mathcal{A}} \cap (\hat{\mathcal{K}} - \hat{I}_{2\mathcal{A}})\right) \subseteq \mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{1\mathcal{A}}) \cup \mathcal{F}^{**}_{r\mathcal{A}}(\hat{\mathcal{K}} - \hat{I}_{2\mathcal{A}}) = \mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{1\mathcal{A}}) \cup \mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{2\mathcal{A}})$.

The concept of symmetric difference of $\hat{I}_{1\mathcal{A}}, \hat{I}_{2\mathcal{A}}$ usually denoted by $\hat{I}_{1\mathcal{A}}\Delta\hat{I}_{2\mathcal{A}}$and equal to $(\hat{I}_{1\mathcal{A}} - \hat{I}_{2\mathcal{A}})$ union to $(\hat{I}_{2\mathcal{A}} - \hat{I}_{1\mathcal{A}})$, also equal to $(\hat{I}_{1\mathcal{A}} \cup \hat{I}_{2\mathcal{A}}) - (\hat{I}_{1\mathcal{A}} \cap \hat{I}_{2\mathcal{A}})$and the important property is $\mathcal{D}_{\mathcal{A}} \cap (\hat{I}_{1\mathcal{A}}\Delta\hat{I}_{2\mathcal{A}}) = (\mathcal{D}_{\mathcal{A}} \cap \hat{I}_{1\mathcal{A}})\Delta(\mathcal{D}_{\mathcal{A}} \cap \hat{I}_{2\mathcal{A}})$. Through these observations, there are important properties that relate symmetric difference and **-soft boundary as shown by the following properties.

2.8 proposition: For any soft subsets $\hat{I}_{1\mathcal{A}}, \hat{I}_{2\mathcal{A}}$ in soft ideal $(\hat{\mathcal{K}}, \hat{\mathcal{T}}, \hat{I}_{\mathcal{A}})$, the properties are true:

1. $\mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{1\mathcal{A}}) \cup \mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{2\mathcal{A}}) = \mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{1\mathcal{A}} \cap \hat{I}_{2\mathcal{A}}) \cup \mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{1\mathcal{A}} - \hat{I}_{2\mathcal{A}}) \cup \mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{1\mathcal{A}} \cup \hat{I}_{2\mathcal{A}})$.
2. $\mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{1\mathcal{A}}) \cup \mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{2\mathcal{A}}) = \mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{1\mathcal{A}} \cup \hat{I}_{2\mathcal{A}}) \cup \mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{2\mathcal{A}} - \hat{I}_{1\mathcal{A}}) \cup \mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{1\mathcal{A}} \cap \hat{I}_{2\mathcal{A}})$.
3. $\mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{1\mathcal{A}}) \cup \mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{2\mathcal{A}}) = \mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{1\mathcal{A}} - \hat{I}_{2\mathcal{A}}) \cup \mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{2\mathcal{A}} - \hat{I}_{1\mathcal{A}}) \cup \mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{1\mathcal{A}} \cap \hat{I}_{2\mathcal{A}})$.
4. $\mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{1\mathcal{A}}) \cup \mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{1\mathcal{A}}\Delta\hat{I}_{2\mathcal{A}}) = \mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{1\mathcal{A}} - \hat{I}_{2\mathcal{A}}) \cup \mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{1\mathcal{A}} \cap \hat{I}_{2\mathcal{A}}) \cup \mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{2\mathcal{A}} \cap \hat{I}_{1\mathcal{A}})$.
5. $\mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{2\mathcal{A}}) \cup \mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{1\mathcal{A}}\Delta\hat{I}_{2\mathcal{A}}) = \mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{1\mathcal{A}} - \hat{I}_{2\mathcal{A}}) \cup \mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{1\mathcal{A}} \cap \hat{I}_{2\mathcal{A}}) \cup \mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{2\mathcal{A}} \cap \hat{I}_{1\mathcal{A}})$.

Proof 1. By (Proposition 2.4) part (2,5) we get that $\mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{1\mathcal{A}}) \cup \mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{2\mathcal{A}}) = \mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{1\mathcal{A}}) \cup \mathcal{F}^{**}_{r\mathcal{A}}(\hat{\mathcal{K}} - \hat{I}_{2\mathcal{A}}) = \mathcal{F}^{**}_{r\mathcal{A}}\left(\hat{I}_{1\mathcal{A}} - (\hat{\mathcal{K}} - \hat{I}_{2\mathcal{A}})\right) \cup \mathcal{F}^{**}_{r\mathcal{A}}\left(\hat{I}_{1\mathcal{A}} \cap (\hat{\mathcal{K}} - \hat{I}_{2\mathcal{A}})\right) \cup \mathcal{F}^{**}_{r\mathcal{A}}\left((\hat{\mathcal{K}} - \hat{I}_{2\mathcal{A}}) - \hat{I}_{1\mathcal{A}}\right) = \mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{1\mathcal{A}} \cap \hat{I}_{2\mathcal{A}}) \cup \mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{1\mathcal{A}} - \hat{I}_{2\mathcal{A}}) \cup \mathcal{F}^{**}_{r\mathcal{A}}\left(\hat{\mathcal{K}} - (\hat{I}_{1\mathcal{A}} \cup \hat{I}_{2\mathcal{A}})\right) = \mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{1\mathcal{A}} \cap \hat{I}_{2\mathcal{A}}) \cup \mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{1\mathcal{A}} - \hat{I}_{2\mathcal{A}}) \cup \mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{1\mathcal{A}} \cup \hat{I}_{2\mathcal{A}})$.

Proof 2. By (Proposition 2.4) part (2,6) we get that $\mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{1\mathcal{A}}) \cup \mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{2\mathcal{A}}) = \mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{2\mathcal{A}}) \cup \mathcal{F}^{**}_{r\mathcal{A}}(\hat{\mathcal{K}} - \hat{I}_{1\mathcal{A}}) = \mathcal{F}^{**}_{r\mathcal{A}}((\hat{\mathcal{K}} - \hat{I}_{1\mathcal{A}}) - \hat{I}_{2\mathcal{A}}) \cup \mathcal{F}^{**}_{r\mathcal{A}}\left((\hat{\mathcal{K}} - \hat{I}_{1\mathcal{A}}) \cap \hat{I}_{2\mathcal{A}}\right) \cup \mathcal{F}^{**}_{r\mathcal{A}}\left(\hat{I}_{2\mathcal{A}} - (\hat{\mathcal{K}} - \hat{I}_{1\mathcal{A}})\right) = \mathcal{F}^{**}_{r\mathcal{A}}\left(\hat{\mathcal{K}} - (\hat{I}_{1\mathcal{A}} \cup \hat{I}_{2\mathcal{A}})\right) \cup \mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{1\mathcal{A}} - \hat{I}_{2\mathcal{A}}) \cup \mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{1\mathcal{A}} \cap \hat{I}_{2\mathcal{A}}) = \mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{1\mathcal{A}} \cup \hat{I}_{2\mathcal{A}}) \cup \mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{2\mathcal{A}} - \hat{I}_{1\mathcal{A}}) \cup \mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{1\mathcal{A}} \cap \hat{I}_{2\mathcal{A}})$.

Proof 4. By (Proposition 2.7) we get that $\mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{1\mathcal{A}}) \cup \mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{1\mathcal{A}}\Delta\hat{I}_{2\mathcal{A}}) = \mathcal{F}^{**}_{r\mathcal{A}}\left(\hat{I}_{1\mathcal{A}} - (\hat{I}_{1\mathcal{A}}\Delta\hat{I}_{2\mathcal{A}})\right) \cup \left(\hat{I}_{1\mathcal{A}} \cap (\hat{I}_{1\mathcal{A}}\Delta\hat{I}_{2\mathcal{A}})\right) \cup \left((\hat{I}_{1\mathcal{A}}\Delta\hat{I}_{2\mathcal{A}}) - \hat{I}_{2\mathcal{A}}\right)$.

Since $\hat{I}_{1\mathcal{A}} - (\hat{I}_{1\mathcal{A}}\Delta\hat{I}_{2\mathcal{A}}) = \hat{I}_{1\mathcal{A}} \cap \left[(\hat{I}_{1\mathcal{A}} \cap \hat{I}_{2\mathcal{A}}{}^{c}) \cup (\hat{I}_{1\mathcal{A}}{}^{c} \cap \hat{I}_{2\mathcal{A}})\right]^{c} = \hat{I}_{1\mathcal{A}} \cap \left[(\hat{I}_{1\mathcal{A}}{}^{c} \cup \hat{I}_{2\mathcal{A}}) \cap (\hat{I}_{1\mathcal{A}}{}^{c} \cap \hat{I}_{2\mathcal{A}})\right]^{c} = \hat{I}_{1\mathcal{A}} \cap (\hat{I}_{1\mathcal{A}}{}^{c} \cap \hat{I}_{2\mathcal{A}}) = \hat{I}_{1\mathcal{A}} \cap \hat{I}_{2\mathcal{A}}$. Then $\mathcal{F}^{**}_{r\mathcal{A}}\left(\hat{I}_{1\mathcal{A}} - (\hat{I}_{1\mathcal{A}}\Delta\hat{I}_{2\mathcal{A}})\right) = \mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{1\mathcal{A}} \cap \hat{I}_{2\mathcal{A}})$. But $\hat{I}_{1\mathcal{A}} - (\hat{I}_{1\mathcal{A}}\Delta\hat{I}_{2\mathcal{A}}) = \hat{I}_{1\mathcal{A}} \cap \hat{I}_{1\mathcal{A}}\Delta\hat{I}_{2\mathcal{A}} \cap \hat{I}_{2\mathcal{A}} = \hat{I}_{1\mathcal{A}}\Delta(\hat{I}_{1\mathcal{A}} \cap \hat{I}_{2\mathcal{A}}) = [\hat{I}_{1\mathcal{A}} \cap (\hat{\mathcal{K}} - \hat{I}_{1\mathcal{A}} \cup \hat{\mathcal{K}} - \hat{I}_{2\mathcal{A}})] \cup [(\hat{I}_{1\mathcal{A}} \cap \hat{I}_{2\mathcal{A}}) \cap (\hat{\mathcal{K}} - \hat{I}_{1\mathcal{A}})] = \hat{I}_{1\mathcal{A}} \cup (\hat{\mathcal{K}} - \hat{I}_{2\mathcal{A}}) = \hat{\mathcal{K}} - (\hat{I}_{1\mathcal{A}} - \hat{I}_{2\mathcal{A}})$. Then $\mathcal{F}^{**}_{r\mathcal{A}}\left(\hat{I}_{1\mathcal{A}} \cap (\hat{I}_{1\mathcal{A}}\Delta\hat{I}_{2\mathcal{A}})\right) = \mathcal{F}^{**}_{r\mathcal{A}}\left(\hat{\mathcal{K}} - (\hat{I}_{1\mathcal{A}} - \hat{I}_{2\mathcal{A}})\right) = \mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{1\mathcal{A}} - \hat{I}_{2\mathcal{A}})$. Finally, since $(\hat{I}_{1\mathcal{A}}\Delta\hat{I}_{2\mathcal{A}}) - \hat{I}_{2\mathcal{A}} = (\hat{I}_{1\mathcal{A}}\Delta\hat{I}_{2\mathcal{A}}) \cap (\hat{\mathcal{K}} - \hat{I}_{2\mathcal{A}}) = [\hat{I}_{1\mathcal{A}} \cap (\hat{\mathcal{K}} - \hat{I}_{2\mathcal{A}})]\Delta[\hat{I}_{2\mathcal{A}} \cap (\hat{\mathcal{K}} - \hat{I}_{1\mathcal{A}})] = (\hat{I}_{1\mathcal{A}} - \hat{I}_{2\mathcal{A}})\Delta\varphi = \hat{I}_{1\mathcal{A}} - \hat{I}_{2\mathcal{A}}$, so $\mathcal{F}^{**}_{r\mathcal{A}}\left((\hat{I}_{1\mathcal{A}}\Delta\hat{I}_{2\mathcal{A}}) - \hat{I}_{2\mathcal{A}}\right) = \mathcal{F}^{**}_{r\mathcal{A}}(\hat{I}_{1\mathcal{A}} - \hat{I}_{2\mathcal{A}})$.

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