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Artin's character table of the group($Q_{2m}\times C_4$) When $m=2^h$, $h\in Z^+$

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Abstract— *Let Q_{2m} be the Quaternion group of order 4m when $m=2^h$, $h\in Z^+$ and C4 be the cyclic group of order 4. Let ($Q_{2m}\times C_4$) The direct product of Q_{2m} and C_4 such that ($Q_{2m}\times C_4$) = {(q,c):q$\in Q_{2m}$,c $\in C_4$} and $|Q_{2m}\times C_4|=|Q_{2m}|.|C_4|=4m.4=8m$. In this paper, we prove that the general form of Artin's characters table of the group ($Q_{2m}\times C_4$) this table depends on Artin's characters table of a quaternion group of order 4m when $m=2^h$, $h\in Z^+$. which is denoted by Ar($Q_{2m}\times C_4$).*

Keywords— Q_{2m}; C_4; group; Artin's characters.

INTRODUCTION

let G be a finite group, two elements of G are said to be Γ-conjugate if the cyclic subgroups they generate are conjugate in G and this defines an equivalence relation on G and its classes are called Γ-classes. let R(G) denotes the abelian group generated by Z - valued characters of G under the operation of point wise addition. Inside this group there is a subgroup generated by Artin characters (The characters induced form the principal characters of cyclic subgroups of G). In 1967, T.Y.Lam [9] proves a sharp form of Artin theorem and he determines the least positive integer A(G) such that[$\overline{R}$ (G) :T (G)]=A(G).In1976, I. M.Isaacs [4] studied Character Theory of Finite Groups. In 2008, A.H.Abdul-Mun'em[1] studied the Artin cokernel of the Quaternion Group Q_{2m} when m is an Odd Number.

The aim of this paper is to find the general from of the Artin's characters table of the group ($Q_{2m}\times C_4$) When $m=2^h$, $h\in Z^+$.

1.Preliminaries

This section introduce some important definitions and basic concepts of the group ($Q_{2m}\times C_4$), the Artin characters and the Artin characters table.

1.2 Definition:[11]

Let G be a finite group, all characters of G induced from a principal character of cyclic subgroups of G are called **Artin's characters of G.**

1.3 Proposition:[2]

The number of all distinct Artin's characters on a group G is equal to the number of Γ-classes on G Furthermore, Artin's characters are constant on each Γ-classes.

1.4 Definition: [1]

Artin's characters of finite group G can be displayed in a table **called Artin's characters table of G** which is denoted by Ar (G). The first row is the Γ- conjugate classes, the second row is the number of elements in each conjugate classes, the third row is the size of the centralize $|C_G(CL_\alpha)|$ and the rest rows contain the values of Artin's characters.

1.5 Proposition: [10]

The Artin's characters table of the Quaternion group Q_{2m} when $m=2^h$, $h\in Z^+$ is given as follows

Ar($Q_{2^{h+1}}$)=

Table (1)

Γ- classes	**Γ- classes of C_{2m}**					
	[1]	$[x^{2^h}]$			**[y]**	**[xy]**
$\lvert CL_\alpha \rvert$	**1**	**1**	**2 2 …**	**2**	$\mathbf{2^h}$	$\mathbf{2^h}$
$\lvert C_{Q_{2^{h+1}}}(CL_\alpha) \rvert$	$\mathbf{2^{h+2}}$	$\mathbf{2^{h+2}}$	$\mathbf{2^{h+1}}$ $\mathbf{2^{h+1}}$ **…**	$\mathbf{2^{h+1}}$	**4**	**4**
Φ $_1$	**2Ar($C_{2^{h+1}}$)**				**0**	**0**
Φ $_2$					**0**	**0**
⋮					⋮	⋮
Φ $_l$					**0**	**0**
Φ $_{l+1}$	$\mathbf{2^h}$	$\mathbf{2^h}$	**0 0 …**	**0**	**2**	**0**
Φ $_{l+2}$	$\mathbf{2^h}$	$\mathbf{2^h}$	**0 0 …**	**0**	**0**	**2**

where l is the number of Γ- classes of C_{2m} and Φ_j ; $1 \le j \le l+2$ are the Artin characters of the Quaternion group Q_{2m} when $m=2^h$, $h \in Z^+$

2. The main results

In this section we find the general form of Artin's characters of the group ($Q_{2m} \times C_4$) when $m=2^h$, $h \in Z^+$

2.1 Proposition:

The general from of the Artin's characters table of the group $(Q_{2^{h+1}} \times C_4)$ when $m=2^h$, $h \in Z^+$ is give as follows:

$Ar(Q_{2^{h+1}} \times C_4) =$

	Γ- classes of $(Q_{2m})\times\{I\}$						Γ- classes of $(Q_{2m})\times\{z^2\}$						Γ- classes of $(Q_{2m})\times\{z\}$					
Γ- classes	$[1,I]$	$[x^m,I]$	…	$[x,I]$	$[y,I$	$[xy,I$	$[I,z^2$	$[x^m,z^2]$	…	$[x,z^2$	$[y,z^2]$	$[xy,z^2]$	$[I,z\ ]$	$[x^m,z]$	…	$[x,z]$	$[y,z$	$[xy,z]$
$\lvert CL_\alpha \rvert$	**1**	**1**	…	**2**	**m**	**m**	**1**	**1**	…	**2**	**m**	**m**	**1**	**1**	…	**2**	**m**	**m**
$\lvert C_{Q_{2m}\times C_3}(CL_\alpha) \rvert$	**16 m**	**16 m**	…	**8m**	**16**	**16**	**16 m**	**16 m**	...	**8m**	**16**	**16**	**16m**	**16 m**	...	**8m**	**16**	**16**
Φ(1,1) ***Φ(2,1)*** ⋮ ***Φ(l ,1)*** ***Φ(l+1,1)*** ***Φ(l+2,1)***	**4Ar(Q2m)**						**0**						**0**					
Φ(1,2) ***Φ(2,2)*** ⋮ ***Φ(l ,2)*** ***Φ(l+1,2)*** ***Φ(l+2,2)***	**2Ar(Q2m)**						**2Ar(Q2m)**						**0**					
Φ(1,3) ***Φ(2,3)*** ⋮ ***Φ(l ,3)*** ***Φ(l+1,3)*** ***Φ(l+2,3)***	**Ar(Q2m)**						**Ar(Q2m)**						**Ar(Q2m)**					

Table (2)

Proof :

Let $g \in (Q_{2m}\times C_4)$;$g=(q,I)$ or $g=(q,z)$or $g=(q,z^2)$or $g=(q,z^3)$ $q \in Q_{2m}$,$I,z,z^2,z^3 \in C_4$

Case (I):

If H is a cyclic subgroup of $Q_{2m} \times \{I\}$,then:

1.H=⟨(x, I)⟩ 2. H=⟨(y, I)⟩ 3. H=⟨(xy, I)⟩

And φ the principal character of H, Φ_j Artin characters of Q_{2m} where $1 \le j \le l+2$ then by using Theorem (1.8)

1. H=⟨(x, I)⟩

(i) If g=(1,I) and g∈H

$$\Phi_{(j,1)}((1,I)) = \frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g) = \frac{16m}{|C_H(I,1)|}.1 = \frac{4.4m}{|C_H(I,1)|}.1 = \frac{4|C_{Q_{2m}}(1)|}{|C_{\langle x\rangle}(1)|}.\varphi(1) = 4.\Phi_j(1)$$

since H ∩CL(1,I)={(1,I)}

(ii) if g= (x^m, I) and g∈H

$$\Phi_{(j,1)}(g) = \frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g) = \frac{16m}{|C_H(g)|}.1 = \frac{4.4m}{|C_H(g)|}.1 = \frac{4|C_{Q_{2m}}(x^m)|}{|C_{\langle x\rangle}(x^m)|}.\varphi(g) = 4.\Phi_j(x^m)$$

since H ∩CL(g)={g},φ(g)=1

(iii) if g= $(x^i, I), i \ne m$ and $i \ne 2m$ and g∈H

$$\Phi_{(j,1)}(g) = \frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1})) = \frac{8m}{|C_H(g)|}(1+1) =$$

$$\frac{4.2m}{|C_H(g)|}.(1+1) = \frac{4|C_{Q_{2m}}(q)|}{|C_{\langle x\rangle}(q)|}.(\varphi(g)+\varphi(g^{-1})) = 4.\Phi_j(q)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g^{-1})=1,g=(q,I),q∈ Q_{2m} and q≠ x^m ,q≠1

(iv) if g ∉ H

$\Phi_{(j,1)}(g) = 4.0 = 4.\Phi_j(q)$ Since H∩CL(g)= ϕ

2. H=⟨(y, I)⟩ ={(1,I),(y,I),(y^2,I),(y^3,I)}

(i) If g=(1,I) H∩CL(1,I)={(1,I)}

$$\Phi_{(l+1,1)}(g) = \frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g) = \frac{16m}{4}.1 = 4m = 4.\Phi_{l+1}(1)$$

(ii) If g= (x^m, I)=(y^2,I) and g∈ H

$$\Phi_{(l+1,1)}(g) = \frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g) = \frac{16m}{4}.1 = 4m = 4.\Phi_{l+1}(x^m)$$

Since H∩CL(g)={g} ,φ(g)=1

(iii) g=(y,I) or g=(y^3,I) and g∈H

$$\Phi_{(l+1,1)}(g) = \frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1})) = \frac{16}{4}.(1+1) = 4.2 = 4.\Phi_{l+1}(y)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g^{-1})=1

Otherwise

$\Phi_{(l+1,1)}(g) = 0$ since H∩CL(g)=∅

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3- H=⟨(xy, I)⟩ ={(1,I),(xy,I),($(xy)^2$,I),($(xy)^3$,I)}

(i) If g=(1,I) H∩CL(1,I)={(1,I)}

$$\Phi_{(l+2,1)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g)=\frac{16m}{4}.1=4m=4.\Phi_{l+2}(1)$$

(ii) If g= (x^m, I)=($(xy)^2$,I)=(y^2,I) and g∈ H

$$\Phi_{(l+2,1)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g)=\frac{16m}{4}.1=4m=4.\Phi_{l+2}(x^m)$$

Since H∩CL(g)={g} ,φ(g)=1

(iii) If g=(xy,I) or g=($(xy)^3$,I) =(xy^3,I) and g∈H

$$\Phi_{(l+2,1)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1}))=\frac{16}{4}.(1+1)=4.2=4.\Phi_{l+2}(xy)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g^{-1})=1

Otherwise

$\Phi_{(l+2,1)}(g)=0$ since H∩CL(g)=∅

<u>Case (II):</u>

If H is a cyclic subgroup of Q_{2m} × {z^2},then:

1.H=⟨(x, I)⟩=$\langle (x,z^2)\rangle$ 2. H=⟨(y, I)⟩ = $\langle (y,z^2)\rangle$ 3. H=⟨(xy, I)⟩ = $\langle (xy,z^2)\rangle$

And φ the principal character of H, Φ_j Artin characters of Q_{2m} where $1\le j\le l+2$ then by using Theorem (1.8)

1. H=⟨(x, I)⟩ = $\langle (x,z^2)\rangle$

(i) If g=(1,I) or g=(1,z^2) and g∈H

$$\Phi_{(j,2)}((1,I))=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g)=\frac{16m}{|C_H(I,1)|}.1=\frac{4.4m}{|C_H(I,1)|}.1=\frac{4|C_{Q_{2m}}(1)|}{2|C_{\langle x\rangle}(1)|}.\varphi(1)=2.\Phi_j(1)$$ since H∩CL(1,I)={(1,I),(1,z^2)}

(ii) if g= (x^m,I) and g∈H

$$\Phi_{(j,2)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g)=\frac{16m}{|C_H(g)|}.1=\frac{4.4m}{|C_H(g)|}.1=\frac{4|C_{Q_{2m}}(x^m)|}{2|C_{\langle x\rangle}(x^m)|}.\varphi(g)=2.\Phi_j(x^m)$$

since H ∩CL(g)={g},φ(g)=1

(iii) if g= $(x^i,I), i\neq m$ and $i\neq 2m$ and g∈H

$$\Phi_{(j,2)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1}))=\frac{8m}{|C_H(g)|}(1+1)=\frac{4.2m}{|C_H(g)|}.(1+1)=\frac{4|C_{Q_{2m}}(q)|}{2|C_{\langle x\rangle}(q)|}.(\varphi(g)+\varphi(g^{-1}))=2.\Phi_j(q)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g^{-1})=1,g=(q,I),q∈ Q_{2m} and q≠ x^m ,q≠1

(iv) if g ∉ H

$\Phi_{(j,2)}(g)=2.0=2.\Phi_j(q)$ Since H∩CL(g)= ϕ

2. H=⟨(y, I)⟩ ={(1,I),(y,I),(y^2,I),(y^3,I), (1, z^2),(y, z^2),(y^2, z^2),(y^3,z^2)}

(i) If g=(1,I) or g=$(1,z^2)$ H∩CL(1,I)={(1,I), $(1,z^2)$}

$$\Phi_{(l+1,2)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g)=\frac{16m}{8}.1=2m=2.\Phi_{l+1}(1)$$

(ii) If g= (x^m, I)=(y^2,I) or g=(y^2,z^2) and g∈ H

$$\Phi_{(l+1,2)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g)=\frac{16m}{8}.1=2m=2.\Phi_{l+1}(x^m)$$

Since H∩CL(g)={g} ,φ(g)=1

(iii) g=(y,I) or g=(y^3,I) or g=(y,z^2) or g=(y^3, z^2) and g∈H

$$\Phi_{(l+1,2)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1}))=\frac{16}{8}.(1+1)=2.2=2.\Phi_{l+1}(y)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g^{-1})=1

Otherwise

$$\Phi_{(l+1,2)}(g)=0 \qquad \text{since } H\cap CL(g)=\emptyset$$

3- H=⟨(xy, I)⟩ ={(1,I),(xy,I),$((xy)^2,I)$,$((xy)^3,I)$, $(1, z^2)$,(xy, z^2),$((xy)^2, z^2)$,$((xy)^3,z^2)$}

(i) If g=(1,I) or g=$(1,z^2)$ H∩CL(1,I)={(1,I),$(1,z^2)$}

$$\Phi_{(l+2,2)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g)=\frac{16m}{8}.1=2m=2.\Phi_{l+2}(1)$$

(ii) If g= (x^m, I)=$((xy)^2,I)$=(y^2,I) or g= (x^m, z^2)=$((xy)^2,z^2)$=(y^2,z^2) and g∈ H

$$\Phi_{(l+2,2)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g)=\frac{16m}{8}.1=2m=2.\Phi_{l+2}(x^m)$$

Since H∩CL(g)={g} ,φ(g)=1

(iii) If g=(xy,I) or g=$((xy)^3,I)$ =(xy^3,I) or g=(xy,z^2) or g=$((xy)^3, z^2)$ =(xy^3, z^2) and g∈H

$$\Phi_{(l+2,2)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1}))=\frac{16}{8}.(1+1)=2.2=2.\Phi_{l+2}(xy)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g^{-1})=1

Otherwise

$$\Phi_{(l+2,2)}(g)=0 \qquad \text{since } H\cap CL(g)=\emptyset$$

<u>Case (III):</u>

If H is a cyclic subgroup of (Q_{2m}× {z}),then:

1.H=$\langle(x,z)\rangle=\langle(x,z^2)\rangle=\langle(x,z^3)\rangle$ *2. H=*$\langle(y,z)\rangle=\langle(y,z^2)\rangle=\langle(y,z^3)\rangle$

3. H=$\langle(xy,z)\rangle=\langle(xy,z^2)\rangle=\langle(xy,z^3)\rangle$

And φ the principal character of H, Φ_j Artin characters of Q_{2m} where $1\le j\le l+2$ then by using Theorem (1.8)

1.*H=*$\langle(x,z)\rangle$

(i) If g=(1,I) or g=(1,z)or g=$(1,z^2)$ or g=$(1,z^3)$ and g∈H

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$$\Phi_{(j,3)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(1,I)|}\cdot\varphi(g)=\frac{16m}{|C_H(1,I)|}.1=\frac{4.4m}{|C_{\langle(x,z)\rangle}(1,I)|}.1=\frac{4|C_{Q_{2m}}(1)|}{4|C_{\langle x\rangle}(1)|}.\varphi(1)=\Phi_j(1)$$

since H ∩CL(g)={(1,I),(1,z),$(1,z^2)$,$(1,z^3)$}

(ii) If g=(1,I) or g=(x^m,I) or g=(x^m,z) or g=(1,z)or g=(x^m,z^2) or g=(1,z^2)or g=(1,z^3) or g=(x^m,z^3) and g∈H

(a) if g=(1,I) or g=(1,z) or g=(1,z^2) or g=(1,z^3) and g∈H.

$$\Phi_{(j,3)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g)=\frac{16m}{|C_H(g)|}.1=\frac{4.4m}{|C_{\langle(x,z)\rangle}(g)|}.1=\frac{4|C_{Q_{2m}}(1)|}{4|C_{\langle x\rangle}(1)|}.\varphi(1)=\Phi_j(1)$$

since H ∩CL(g)={g},φ(g)=1

(b)If g=(x^m,I) or g=(x^m,z) or g=(x^m,z^2) or g=(x^m,z^3) and g∈H

$$\Phi_{(j,3)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g)=\frac{16m}{|C_H(g)|}.1=\frac{4.4m}{|C_H(g)|}.1=\frac{4|C_{Q_{2m}}(x^m)|}{4|C_{\langle x\rangle}(x^m)|}.\varphi(x^m)=\Phi_j(x^m)$$

since H ∩CL(g)={g},φ(g)=1

(iii) If g $=\{(x^i,I),(x^i,z),(x^i,z^2),(x^i,z^3)\}$,$i\neq m, i\neq 2m$ and g∈H

$$\Phi_{(j,3)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1}))=\frac{8m}{|C_H(g)|}(1+1)\ \frac{4.2m}{|C_H(g)|}.(1+1)=\frac{4|C_{Q_{2m}}(q)|}{4|C_{\langle x\rangle}(q)|}.(\varphi(g)+\varphi(g^{-1}))=\Phi_j(q)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g^{-1})=1,g=(q,z)=(q,z^3),q∈ Q_{2m} and q≠ x^m ,q≠1

(iv) if g∉H

$\Phi_{(j,)}(g)=0$ Since H∩CL(g)= ϕ

2. H= $\langle(y,z)\rangle$ ={(1,I),(y,I),(y^2,I),(y^3,I),(1,z),(y,z),(y^2,z),(y^3,z), (1,z^2),(y,z^2),(y^2,z^2),(y^3,z^2), (1,z^3),(y,z^3),(y^2,z^3),(y^3,z^3)}

(i) If g=(1,I) or g=(1,z) or g=(1,z^2) or g=(1,z^3)and g∈H H∩CL(g)={(1,I),(1,z),(1,z^2) ,(1,z^3)}

$$\Phi_{(l+1,3)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g)=\frac{16m}{16}.1=m=.\Phi_{l+1}(1)$$

(ii) If g= (x^m,I)=(y^2,I) or g= (y^2,z) or g=(y^2,z^2) or g=(y^2,z^3) and g∈ H

$$\Phi_{(l+1,3)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g)=\frac{16m}{16}.1=m=\Phi_{l+1}(x^m)$$

Since H∩CL(g)={g} ,φ(g)=1

(iii) g=(y,I)or g= (y,z)or g=(y,z^2) or g=(y,z^3) or g=(y^3,I_2)or g= (y^3,z) or g= (y^3,z^2) or g= (y^3,z^3) and g∈H

$$\Phi_{(l+1,3)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1}))=\frac{16}{16}.(1+1)=2=\Phi_{l+1}(y)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g^{-1})=1

Otherwise

$\Phi_{(l+1,3)}(g)=0$ since H∩CL(g)=∅

3.H= $\langle(xy,z)\rangle$ ={(1,I),(xy,I),($(xy)^2$,I)=(y^2,I),($(xy)^3$,I)=(xy^3,I),(1,z),(xy,z),

$((xy)^2,z)),((xy)^3,z),(1,z^2),(xy,z^2),((xy)^2,z^2)),((xy)^3,z^2),(1,z^3)(xy,z^3),((xy)^2,z^3)),((xy)^3,z^3)\}$

(i) If g=(1,I) or g=(1,z) or g=$(1,z^2)$ or g=$(1,z^3)$ H∩CL(g)={g}

$$\Phi_{(l+2,3)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g)=\frac{16m}{16}.1=m=\Phi_{l+2}(1)$$

(ii) If g= (x^m, I)=$((xy)^2,I)=(y^2,I)$or g=$((xy)^2,z)=(y^2,z)$or g=$((xy)^2,z^2)=(y^2,z^2)$ or g=$((xy)^2,z^3)=(y^2,z^3)$ and g∈ H

$$\Phi_{(l+2,3)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g)=\frac{16m}{16}.1=m=\Phi_{l+2}(x^m)$$

Since H∩CL(g)={g} ,φ(g)=1

(iii) If g=(xy,I) or g= $((xy)^3,I)$ or g= (xy,z) or g= $((xy)^3,z)$ or g= (xy,z^2) or

(iv) g= $((xy)^3,z^2)$ or g= (xy,z^3) or g= $((xy)^3,z^3)$ and g∈H

$$\Phi_{(l+2,3)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1}))=\frac{16}{16}.(1+1)=2=\Phi_{l+2}(xy)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g^{-1})=1

Otherwise

$$\Phi_{(l+2,3)}(g)=0 \qquad \text{since } H\cap CL(g)=\emptyset$$

2.2 Example:

To construct Ar($Q_{16}\times C_4$) by using the theorem (2.1) we get the following table:

Ar($Q_2^{16}\times C_4$)=

-classes	[1,I]	[x^8,I]	[x^4,I]	[x^2,I]	[x,I]	[y,I]	[xy,I]	[1, z^2]	[x^8, z^2]	[x^4, z^2]	[x^2, z^2]	[x, z^2]	[y, z^2]	[xy, z^2]	[1, z]	[x^8,z]	[x^4,z]	[x^2,z]	[x, z]	[y, z]	[xy,z]
$\|CL_\alpha\|$	1	1	2	2	2	8	8	1	1	2	2	2	8	8	1	1	2	2	2	8	8
$\|C_Q(CL_\alpha)\|$	128	128	64	64	64	16	16	128	128	64	64	64	16	16	128	128	64	64	64	16	16
$\Phi_{(1,1)}$	128	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(2,1)}$	64	64	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(3,1)}$	32	32	32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(4,1)}$	16	16	16	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(5,1)}$	8	8	8	8	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(6,1)}$	32	32	0	0	0	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(7,1)}$	32	32	0	0	0	0	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(1,2)}$	64	0	0	0	0	0	0	64	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(2,2)}$	32	32	0	0	0	0	0	32	32	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(3,2)}$	16	16	16	0	0	0	0	16	16	16	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(4,2)}$	8	8	8	8	0	0	0	8	8	8	8	0	0	0	0	0	0	0	0	0	0
$\Phi_{(5,2)}$	4	4	4	4	4	0	0	4	4	4	4	4	0	0	0	0	0	0	0	0	0
$\Phi_{(6,2)}$	16	16	0	0	0	4	0	16	16	0	0	0	4	0	0	0	0	0	0	0	0
$\Phi_{(7,2)}$	16	16	0	0	0	0	4	16	16	0	0	0	0	4	0	0	0	0	0	0	0
$\Phi_{(1,3)}$	32	0	0	0	0	0	0	32	0	0	0	0	0	0	32	0	0	0	0	0	0

$\Phi_{(2,3)}$	16	16	0	0	0	0	0	16	16	0	0	0	0	0	16	16	0	0	0	0	0
$\Phi_{(3,3)}$	8	8	8	0	0	0	0	8	8	8	0	0	0	0	8	8	8	0	0	0	0
$\Phi_{(4,3)}$	4	4	4	4	0	0	0	4	4	4	4	0	0	0	4	4	4	4	0	0	0
$\Phi_{(5,3)}$	2	2	2	2	2	0	0	2	2	2	2	2	0	0	2	2	2	2	2	0	0
$\Phi_{(6,3)}$	8	8	0	0	0	2	0	8	8	0	0	0	2	0	8	8	0	0	0	2	0
$\Phi_{(7,3)}$	8	8	0	0	0	0	2	8	8	0	0	0	0	2	8	8	0	0	0	0	2

Table (3)

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