

Performance Comparison of State Variable Based Controllers for Single-Stage Refrigeration System

Chukwudi Christian Chigozie¹ Osinachi Bright Abika² Martin Ngwaldi Dillum³ Okhueleigbe Vincent Onos⁴ Chisom Victory Onyenagubo⁵ Iwu Phinehas Ikechukwu⁶ Jeremiah Henry Chijioke⁷

¹Department of Mechanical Engineering, Imo State University, Nigeria
²Electrical and Electronics Engineering, Louisiana Tech University, USA
³Marshall School of Business, University of Southern California, USA
⁴Electrical and Computer Engineering, Lagos State University, Nigeria
⁵Electrical Engineering, Southern University and A&M College, USA
⁶Department of Mechanical Engineering, Imo State University, Nigeria
⁷Software Engineering, University of Bolton, United Kingdom

Abstract: Refrigeration systems need control mechanisms to maintain efficient operation so that the cycle can go ON and OFF to keep a fixed (setpoint) temperature. In this paper, the performance of different state variable based controllers for control of temperature in a vapour-compression single-stage refrigeration system has been examined. The mathematical expressions representing the thermal dynamic characteristics of a vapour-compression system were determined and represented as multivariable state space model. Simulink state space model was developed to represent the system. Different state variable based controllers such as full state feedback controller, Linear Quadratic Regulator (LQR), and hybrid PID-LQR were implemented. Each of the controllers was separately incorporated into the closed loop control system for temperature in a single-stage refrigeration system. The control system was subjected to unit step forcing function in MATLAB/Simulink simulation environment considering different scenarios so as to evaluate and obtain the time domain parameters that characterized the transient and steady state response performances. The simulation results obtained showed that the addition of the various controllers provided improved temperature response performance such that the desired temperatures for evaporator, compressor, condenser, and expansion valve were achieved when unit step temperature input was applied. The time domain performance characteristics obtained with full state feedback controller, LQR, and PID-LQR controller with respect to evaporator, compressor, condenser, and expansion valve temperatures in terms of rise time, settling time, overshoot, and steady-state error revealed that the full state feedback controller offered the most efficient and smooth response with zero overshoot for all the temperatures in various unit.

Keywords: LQR, PID-LQR, Full feedback control, Single-stage refrigeration system, Temperature control, Vapour compression refrigeration

1. Introduction

Refrigeration system can be used for many purposes such as food processing, product preservation and more. It is one of the largest energy consumers in a facility. There are certain challenges in refrigeration systems such as proper control of superheat for efficient and safe operation of the system, and also maintaining the temperature of the refrigerated space or foodstuff within the desired requirement, for example, to prevent possible deterioration of foodstuff. The use of ON-OFF thermostat or relay based control drive the system temperature on limited cycle. The time duration and magnitude of such cycle affect a number of relevant characteristics of the system. Particularly, an excessive changes in temperature, may affect food preservation properties. Considering the high content variability of refrigeration system, a predetermined control approach may serve well as a solution if fixed requirements on food preservation and energy consumption are to be achieved.

Refrigeration systems are generally designed for predetermined capacity to achieve cooling capacity in terms of the maximum demand at the highest ambient temperature. Conventionally, it is designed with an ON and OFF control so as to be able to adjust the cooling demand [1]. In recent times, control schemes with different design approach have been well developed for improved effectiveness and reliability of the refrigeration systems so as to regulate its internal temperature more effectively and accurately. The control strategy used in refrigeration system is an essential device in terms of thermal performance [2].

This work presents control schemes that take into consideration the states of the refrigeration system known as the internal characteristics to effectively control the indoor temperatures for proper food preservation in domestic refrigeration. Compared to other methods, the state variable control techniques offer best performance in terms of settling time and oscillation [3,4], state-space model can be used to solve the problem of time-varying plants [5], or plants whose operating conditions are many [6], including multiple inputs and outputs signal required systems [6], and can shape closed-loop system dynamic to meet design criteria due to its flexibility properties [7].

2. Literature Review

In this section empirical review of related works that have been done regarding temperature control in refrigerating systems are presented. For instance, to address control problem in one-stage refrigeration system, a decentralized control method based on Generalized-Proportional-Integral (GPI) observers was employed for internal loops and Proportional Integral Derivative (PID) controller tuned by Quantitative Feedback Theory (QFT) was used for each outer loop [8]. A modified conventional PID controller called Nonlinear PID (NPID) was used to maintain desired cooling temperature in a refrigeration system [9]. Decentralized PID controllers tuned by a method called Competition Over Resources (COR) optimization algorithm were used to solve the problem of vapour compression in refrigeration system, which resulted in performance index of 68.39% [10]. The application of decentralized Linear Active Disturbance Rejection Control (LADRC) method to a benchmark refrigeration system yielded better tracking and disturbance rejection than two benchmark PID control models [11]. The closed-loop control performance of a benchmark refrigeration system was enhanced using data-driven control design based on Optimal Controller Identification (OCI) plus anti-windup technique [12]. The transient performance and power efficiency of a variable-speed compressor refrigeration system was improved using anti-windup control system [13]. The control problem associated with for vapour compression refrigeration system was solved using Multi-Objective Optimization Design (MOOD) decentralized controller [14]. Recursive Integration Optimal Trajectory Solver (RIOTS) optimized Model Predictive Control (MPC) applied to one-stage refrigeration system provided significant improvement in performance compared to discrete decentralized control and multi-variable PID controllers [15]. In [16], lumped parameter approach was used to analyze a single-stage VCR system. Robust control of temperature in a single-stage refrigeration system was achieved using PID optimized Linear Quadratic Regulator (PID-LQR) controller with improved rise time and settling time compared with LQR [17].

The review has shown from the studies considered that in literature, the focus is usually on addressing control problem in vapour-compression of refrigeration system. The objective has generally been to reduced control cost with improved performance index in terms of term time domain transient parameters such as rise time, peak time, peak percentage overshoot, settling time, and steady state error. Further, it can be seen that most of studies have been concerned with the use of conventional PID and its improved or advanced form using optimization scheme. Despite the benefits provided by these approaches, concern have not been given to the internal non-measurable system states of refrigeration system, whose dynamic can affect the smooth running of the process in the temperature control loop of vapour-compression refrigeration systems. This can be solved by implementing refrigeration system as state-space model.

3. System Design

In this section, the various steps taken to implement the full state feedback temperature control system, LQR, and PID-LQR controller for single-stage refrigerating system are highlighted as follows. Design Specifications: The time response of control system is characterized by transient response and steady state response, which together describes the response performance of the control system. Hence, the time response performance parameters of control system are: rise time (t_r), settling time (t_s), peak percentage overshoot (M_p) and steady state error. The design specifications or performance requirement adopted in this work are: $t_r \leq 4s$, $t_s \leq 5s$ and $M_p \leq 10\%$, which conforms to the standards of a practical plant [18].

3.1 Single-Stage Refrigeration System Modelling

Figure 1 shows a simplified model of single-stage VCR system. The system has basically four units to achieve its temperature demands. These are evaporator, compressor, condenser, and expansion valve.

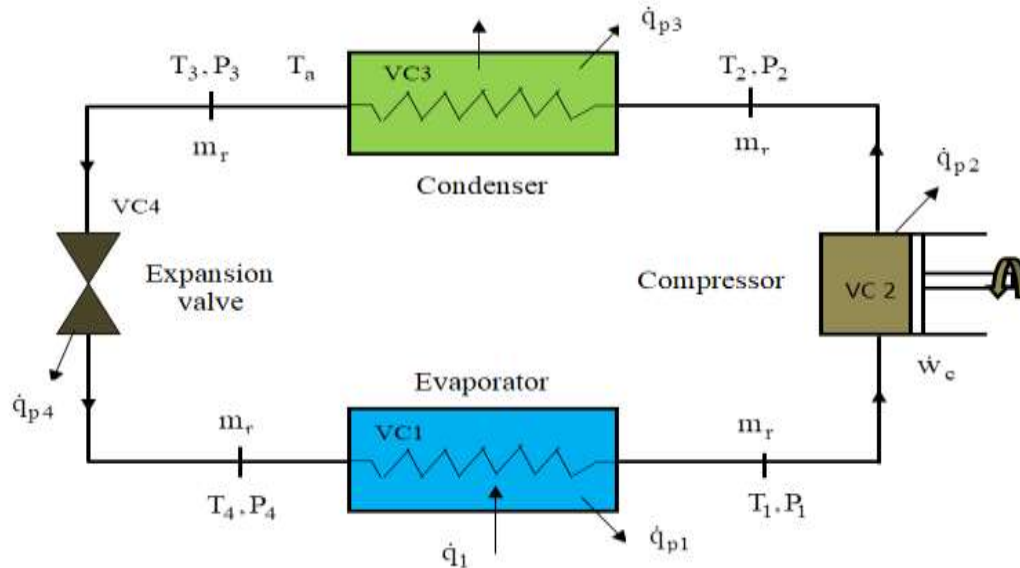


Figure 1. Single-stage VCR system model [17]

The modeling method used conforms to that of a single-stage VCR system using lumped parameters proposed by [16]. The following assumptions are made [16]: the refrigeration cycle is considered a closed process, it operates in steady-state, same mass flow rates is assumed for refrigerant flow through the four units on the basis of continuity principle for mass and energy steadiness, refrigerant flows continuously without variation in properties over time, uniformity of all variables along the control volume, thermal equilibrium exists between the refrigerant liquid and vapour phases, ideal thermal insulations of the heat exchangers, and negligible axial heat condition in the pipes.

Generally, energy balance equation can be established for system with multiple mass flows of the same properties in and out and can be defined[16]:

$$\dot{q} + \dot{w} = \dot{m} \left[(h_f - h_o) + \frac{v_f^2 - v_o^2}{2} + g(z_f - z_o) \right] \quad (1)$$

where $\dot{q}$ represents amount of heat transferred from the surrounding heat flow to the system, $\dot{w}$ work done by compressor's electric motor, $\dot{m}$ stands for mass flow rate, $h = (h_f - h_o)$ is the change in enthalpy between final and initial value, $v_f^2 - v_o^2 / 2$ and $g(z_f - z_o)$ are expressions for change in kinetic and potential energies respectively, g and Z respectively stands for acceleration due to gravity and height. Simplification of Eq. (1) with respect to the uniformity assumption for variables along the control volume, gives [16]:

$$\dot{q} + \dot{w} + \dot{q}_{pi} = \dot{m}_r c_{pi} (T_f - T_o) \quad (2)$$

$$\dot{q}_{pi} = C_{Ti} \frac{dT_i}{dt} + \frac{(T_i - T_a)}{R_{Ti}} \quad (3)$$

where $\dot{m}_r$ is circulating refrigerant mass flow rate, c_{pi} is the refrigerant's specific heat capacity at constant room temperature, $T_f - T_o$ increase temperature, $\dot{q}_{pi}$ is heat generated by system or heat loss, C_{Ti} stands for heat capacity of confined space that is interior to evaporator element and other element of the system, $C_{Ti} = m c_{p,m}$ where m is elements' mass, $c_{p,m}$ stands as material's specific thermal capacity, T_i temperature inside system, T_a is the room (or ambient) temperature, and dT_i/dt is the cooling rate of the interior.

From Figure 1, the thermal equations in the evaporator, compressor, condenser, and expansion valve are given by [16]:

$$\frac{dT_1}{dt} = T_1 \left(\frac{c_p \dot{m}_r}{C_{T1}} - \frac{1}{C_{T1} R_{T1}} \right) - T_4 \frac{c_p \dot{m}_r}{C_{T1}} + \frac{T_a}{C_{T1} R_{T1}} - \frac{\dot{q}_1}{C_{T1}} \quad (4)$$

$$\frac{dT_2}{dt} = -T_1 \frac{c_p \dot{m}_r}{C_{T2}} + T_2 \left(\frac{c_p \dot{m}_r}{C_{T2}} - \frac{1}{C_{T2} R_{T2}} \right) + \frac{T_a}{C_{T2} R_{T2}} - \frac{\dot{w}_c}{C_{T2}} \quad (5)$$

$$\frac{dT_3}{dt} = -T_2 \frac{c_p \dot{m}_r}{C_{T3}} + T_3 \left(\frac{c_p \dot{m}_r}{C_{T3}} - \frac{1}{C_{T3} R_{T3}} \right) + \frac{T_a}{C_{T3} R_{T3}} - \frac{\dot{q}_3}{C_{T3}} \quad (6)$$

$$\frac{dT_4}{dt} = -T_3 \frac{c_p \dot{m}_r}{C_{T4}} + T_4 \left(\frac{c_p \dot{m}_r}{C_{T4}} - \frac{1}{C_{T4} R_{T4}} \right) + \frac{T_a}{C_{T4} R_{T4}} \quad (7)$$

where C_{pi} in Eq. (2) is replaced by C_p for $i=1, 2, 3, 4$, $\dot{q}_1, \dot{q}_3$ are the evaporator heat rate and condenser heat rate, $Q_{p1}, Q_{p2}, Q_{p3}, Q_{p4}$, are the rate of transfer of heat to evaporator, compressor, condenser, and expansion valve surrounding respectively, the thermal resistance R_{Ti} in Eq. (3) has been replaced with R_{T1}, R_{T2}, R_{T3} , and R_{T4} , where $i = 1, 2, 3, 4$. T_1, T_2, T_3 , and T_4 are the evaporator temperature, compressor temperature, condenser temperature, and expansion valve temperature respectively.

The state-space representation of the VCR system in terms of Eq. (4) to (7) can be represented in canonical form defined in Eq. (8) [19,20] and is given by Eq. (9) and (10). Substituting the values of VCR system parameter defined Table 1 gives the numerical expression for state matrix A, input matrix B, and output matrix. Note $D = 0$.

$$\begin{aligned} \dot{T}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned} \quad (8)$$

$$\begin{aligned} \begin{bmatrix} \dot{T}_1 \\ \dot{T}_2 \\ \dot{T}_3 \\ \dot{T}_4 \end{bmatrix} &= \begin{bmatrix} \left(\frac{c_{p1} \dot{m}_r}{C_{T1}} - \frac{1}{C_{T1} R_{T1}} \right) & 0 & 0 & -\frac{c_{p1} \dot{m}_r}{C_{T1}} \\ -\frac{c_{p2} \dot{m}_r}{C_{T2}} & \left(\frac{c_{p2} \dot{m}_r}{C_{T2}} - \frac{1}{C_{T2} R_{T2}} \right) & 0 & 0 \\ 0 & -\frac{c_{p3} \dot{m}_r}{C_{T3}} & \left(\frac{c_{p3} \dot{m}_r}{C_{T3}} - \frac{1}{C_{T3} R_{T3}} \right) & 0 \\ 0 & 0 & -\frac{c_{p4} \dot{m}_r}{C_{T4}} & \left(\frac{c_{p4} \dot{m}_r}{C_{T4}} - \frac{1}{C_{T4} R_{T4}} \right) \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} \\ &+ \begin{bmatrix} \frac{1}{C_{T1} R_{T1}} & -\frac{1}{C_{T1}} & 0 & 0 \\ \frac{1}{C_{T2} R_{T2}} & 0 & -\frac{1}{C_{T2}} & 0 \\ \frac{1}{C_{T3} R_{T3}} & 0 & 0 & -\frac{1}{C_{T3}} \\ \frac{1}{C_{T4} R_{T4}} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} T_a \\ \dot{q}_1 \\ \dot{w}_c \\ \dot{q}_3 \end{bmatrix} \end{aligned} \quad (9)$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} + 0 \quad (10)$$

$$A = \begin{bmatrix} -2.4 \times 10^{-3} & 0 & 0 & -1.03 \times 10^{-5} \\ -1.96 \times 10^{-5} & -0.0160 & 0 & 0 \\ 0 & -3.186 \times 10^{-5} & -0.0167 & 0 \\ 0 & 0 & 9.23 \times 10^{-5} & -5.327 \times 10^{-4} \end{bmatrix}, x(t) = \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix}$$

$$B = \begin{bmatrix} 2.47 \times 10^{-3} & -2.22 \times 10^{-4} & 0 & 0 \\ 0.016 & 0 & -4 \times 10^{-4} & 0 \\ 0.0167 & 0 & 0 & -8 \times 10^{-4} \\ 6.25 \times 10^{-4} & 0 & 0 & 0 \end{bmatrix}, u(t) = \begin{bmatrix} T_a \\ \dot{q}_1 \\ \dot{w}_c \\ \dot{q}_3 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, D = 0$$

Table 1 parameters of single-stage VCR system [16,17]

Description	Symbol	Value	Unit
Specific heat of refrigerant (evaporator)	C_{p1}	1330	J/kg - K
Specific heat of refrigerant (compressor)	C_{p2}	1400	J/kg - K
Specific heat of refrigerant (condenser)	C_{p3}	1138	J/kg - K
Specific heat of refrigerant (expansion valve)	C_{p4}	1318	J/kg - K
Thermal capacity (evaporator)	C_{T1}	4500	J/K
Thermal capacity (compressor)	C_{T2}	2500	J/K
Thermal capacity (condenser)	C_{T3}	1250	J/K
Thermal capacity (expansion valve)	C_{T4}	500	J/K
Mass flow rate	m_r	0.000035	kg/s
Thermal resistance (evaporator)	R_{T1}	0.090	K/W
Thermal resistance (compressor)	R_{T2}	0.025	K/W
Thermal resistance (condenser)	R_{T3}	0.048	K/W
Thermal resistance (expansion valve)	R_{T4}	3.20	K/W
Evaporator heat rate	q_1	195	W
Condenser heat rate	q_3	-200	W
Ambient	T_a	298	K
Compressor power input	T_R, w_c	5	W

3.2 Full State Variable Feedback

Figure 2 shows the closed loop plant incorporating a feedback gain that provides the control law. In designing full state feedback control system, an initial assumption is made taking the reference input to be zero as state in Eq. (11) so as to show that the method can place pole in any required location.

$$u = -Kx \quad (11)$$

where each of u and K represents control input and feedback gain., and x stands for state variable. Putting (11) in Eq. (8) yields:

$$\dot{x} = (A - BK)x \quad (12)$$

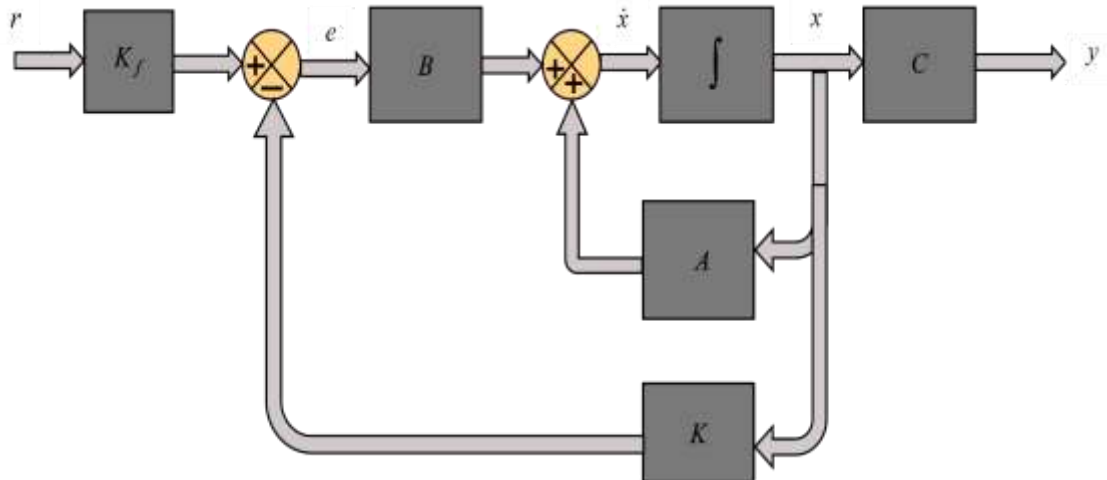


Figure 2.Full state feedback control system

The closed-loop of the state space system given by Eq. (8) and whose input is given by Equation (11) can be expressed as:

$$u = K_f r - Kx \quad (13)$$

$$\dot{x} = (A - BK)x + BK_f r \quad (14)$$

where u, k_f, r are the control input, forward gain and reference value respectively. The computed gain matrix is given by:

$$K = 1.0 \times 10^5 \begin{bmatrix} -0.0000 & -0.0000 & -0.0000 & -0.0480 \\ -0.0900 & -0.0000 & -0.0000 & -0.5340 \\ 0.0000 & -0.0496 & -0.0001 & -1.9197 \\ 0.0000 & 0.0000 & -0.0373 & -1.0018 \end{bmatrix} \quad (15)$$

The expression for K matrix above is called the called full state feedback gain and multiplying it with the state variable (that is $u = -Kx$) gives the control law. The designed forward gain is:

$$K_f = \begin{bmatrix} 890 & 0 & 0 & 0 \\ 0 & 129 & 0 & 0 \\ 0 & 0 & 189 & 0 \\ 0 & 0 & 0 & 4890 \end{bmatrix} \text{ or } K_f = [890;129;189;4890] \quad (16)$$

3.3 LQR and PID-LQR Control Systems

LQR is considered an optimal controller whose design is based on state feedback control law that aims to minimize the quadratic cost function given by:

$$J = \frac{1}{2} \int_0^t (\mathbf{x}^T(t) \mathbf{Q} \mathbf{x}(t) + \mathbf{u}^T(t) \mathbf{R} \mathbf{u}(t)) dt \quad (17)$$

where $\mathbf{Q}$ and $\mathbf{R}$ are the weighting matrices of states and control variable respectively. The value of $\mathbf{Q}$ and $\mathbf{R}$ can be determined by repeated iteration wherein $\mathbf{Q}$ is varied and $\mathbf{R}$ is fixed. Also, the LQR uses a control law defined by:

$$\mathbf{u} = \mathbf{K} \mathbf{x} \quad (18)$$

The control law matrix $\mathbf{K}$ is obtained from Eq. (19), which is computed by initially solving the Algebraic Riccati Equation (ARE) computed using Eq. (20). Figure 3 shows the system block model of LQR controlled VCR.

$$\mathbf{K} = \mathbf{R}^{-1} \mathbf{B}^T \mathbf{P} \quad (19)$$

$$\mathbf{P} \mathbf{A} + \mathbf{A}^T \mathbf{P} - \mathbf{P} \mathbf{B} \mathbf{R}^{-1} \mathbf{B}^T \mathbf{P} + \mathbf{Q} = 0 \quad (20)$$

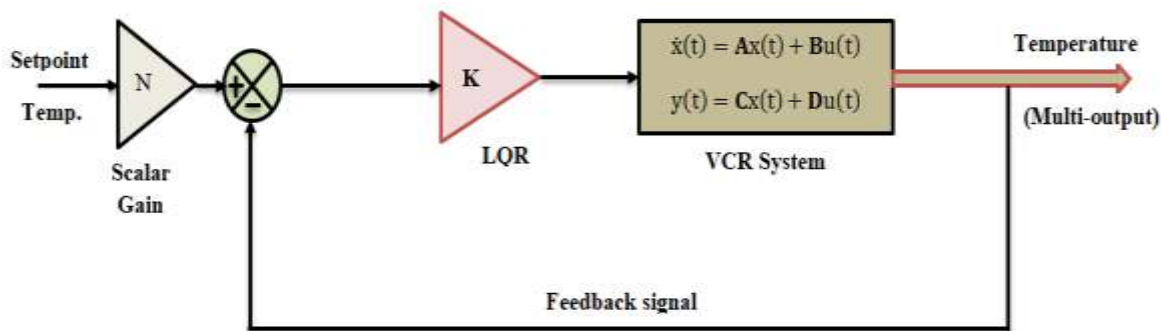


Figure 3. Block diagram of LQR

The values of $\mathbf{Q}$, $\mathbf{R}$, and $\mathbf{K}$ computed in MATLAB are [17]:

$$\mathbf{Q} = \begin{bmatrix} 10000 & 0 & 0 & 0 \\ 0 & 10000 & 0 & 0 \\ 0 & 0 & 10000 & 0 \\ 0 & 0 & 0 & 10000 \end{bmatrix}, \quad \mathbf{R} = 1; \quad \mathbf{P} = [-0.0015 \quad -0.0229 \quad -0.0647 \quad -2.3277]^T$$

$$\text{and } \mathbf{K} = \begin{bmatrix} 16.3295 & 68.2268 & 69.5506 & 26.4429 \\ -89.2227 & 8.0669 & 4.1615 & 17.2451 \\ 14.5349 & -29.5418 & 22.6250 & 50.6223 \\ 14.9963 & 45.2501 & -50.2648 & 36.3840 \end{bmatrix}$$

PID control action is defined by the mathematical expression given in Eq. (21). It uses three elements to achieve simultaneously coordinated control signal in order to perform correctional action that drives the plant into a new state [21]. The PID-LQR controller designed for the VCR system has a control action defined in Eq. (22) by [17].

$$\mathbf{u}(t) = \mathbf{K}_p \mathbf{e}(t) + \mathbf{K}_i \int_0^t \mathbf{e}(t) dt + \mathbf{K}_d \frac{d\mathbf{e}(t)}{dt} \quad (21)$$

where the proportional, integral and derivative gains are $\mathbf{K}_p$, $\mathbf{K}_i$ and $\mathbf{K}_d$ respectively. The block diagram of the PID-LQR is shown in Figure 4.

$$u_{\text{PID-LQR}} = u_{\text{PID}}K = K_{pk}e_i - K_{dk}\dot{e}_i + K_{ik}\int_0^t e_i dt \quad (22)$$

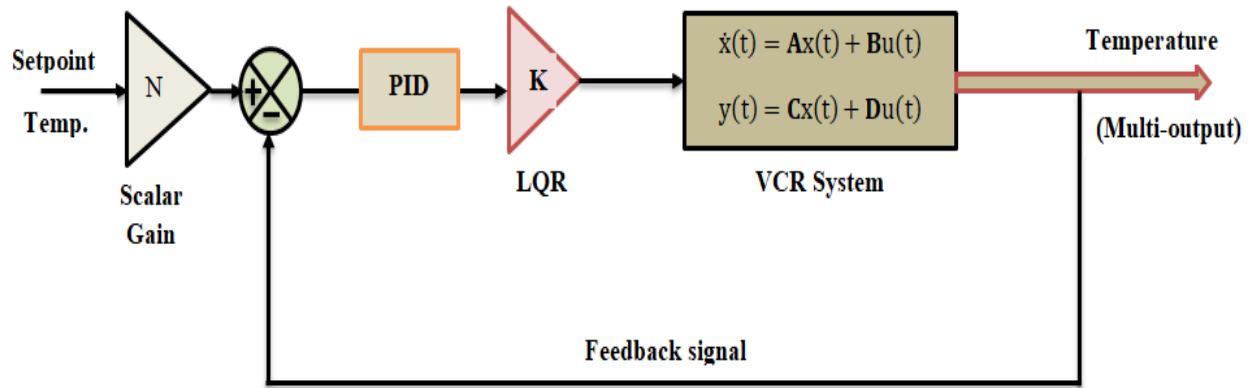


Figure 4. Block diagram of PID-LQR VCR system

Hence, the gains of the PID-LQR controller for the VCR system are $K_pK = K_{pk}$, $K_iK = K_{ik}$, and $K_dK = K_{dk}$. Where $K_p = 50$, $K_i = 0.1$, and $K_d = 2$ are tuned parameters of the PID.

4. Result and Discussion

This section presents the result of simulation conducted in MATLAB/Simulink for the analysis of the various control systems. The step responses for the full state feedback controller, LQR, and the PID-LQR are presented in Figures 5 to 8 in terms of evaporator temperature, compressor temperature, condenser temperature, and expansion valve temperature. In Tables 2 to 5, the numerical values of time domain parameters of the step responses are presented.

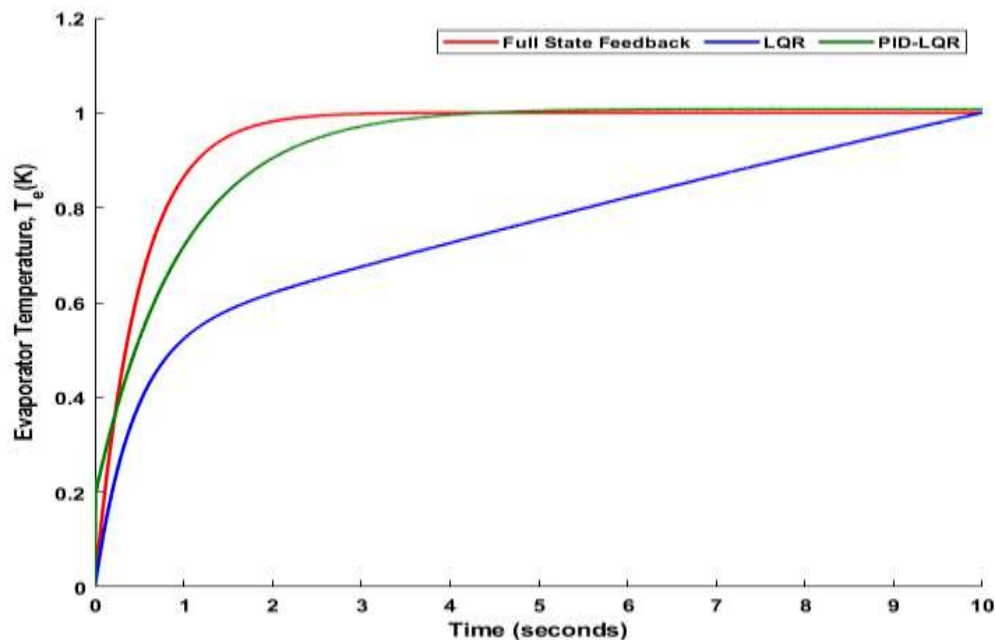


Figure 5. Step responses of evaporator temperatures

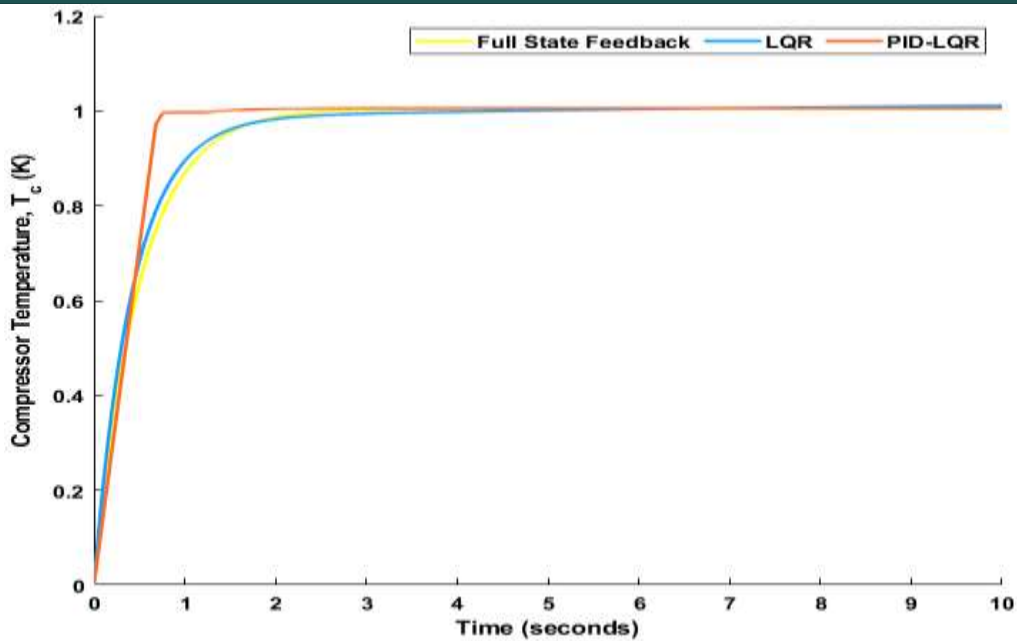


Figure 6. Step responses of compressor temperatures

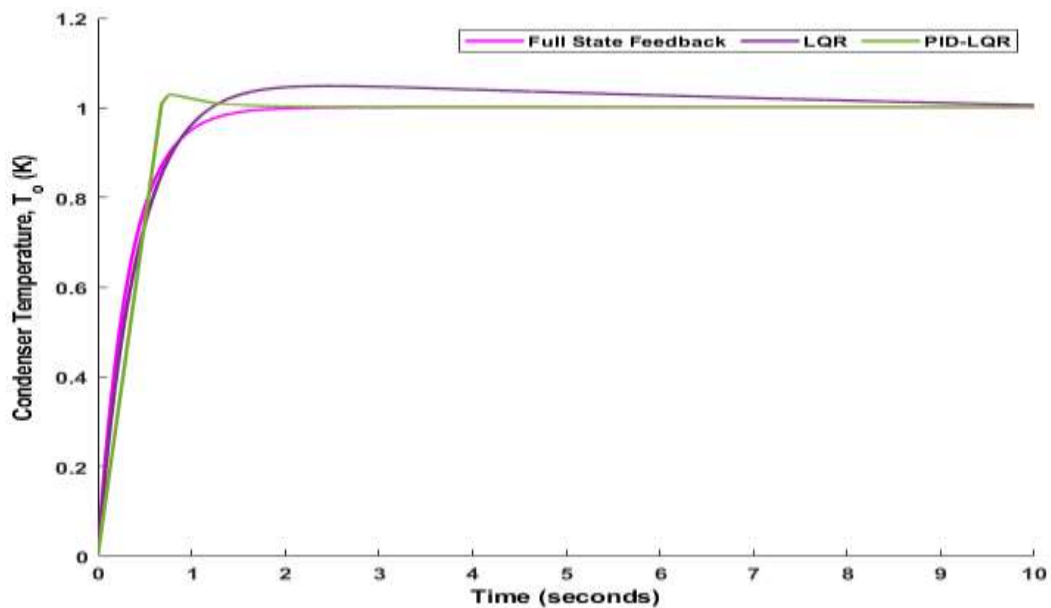


Figure 7. Step responses of condenser temperatures

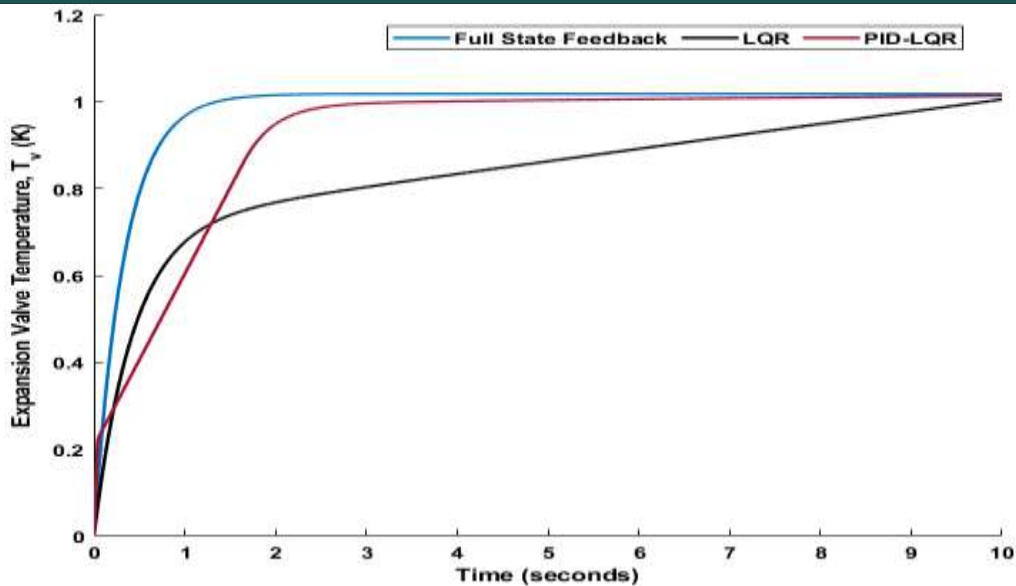


Figure 8. Step responses of expansion valve temperatures

Table 2. Time domain analysis of evaporator temperature response

Controller	Rise time (s)	Settling time (s)	Peak overshoot (%)	Steady-state error
Full state feedback	1.1010	1.9605	0	0
LQR	7.6278	9.5326	0	0
PID-LQR	2.0242	3.5550	0.033	0

Table 3. Time domain analysis of compressor temperature response

Controller	Rise time (s)	Settling time (s)	Peak overshoot (%)	Steady-state error
Full state feedback	1.1004	1.9597	0	0
LQR	1.0301	2.5231	0	0
PID-LQR	0.5643	0.7247	0.01	0

Table 4. Time domain analysis of condenser temperature response

Controller	Rise time (s)	Settling time (s)	Peak overshoot (%)	Steady-state error
Full state feedback	0.7347	1.3081	0	0
LQR	0.7663	6.2899	4.2533	0.048
PID-LQR	0.5414	0.9676	2.7707	0.029

Table 5. Time domain analysis of expansion valve temperature response

Controller	Rise time (s)	Settling time (s)	Peak overshoot (%)	Steady-state error
Full state feedback	0.7343	1.3075	0	0
LQR	6.3908	9.2836	0	0
PID-LQR	1.7911	2.7711	0	0

The time domain parameter analysis in Table 2 of Fig. 5, for the evaporator temperature, the responses revealed that in terms of rise time and settling time, the full state feedback method provided the fastest response and transient time to steady-state compare to the LQR and PID-LQR methods. In contrast, the LQR exhibit the worst performance in this regard, showing a very poor rise time of 7.6278 s and settling time of 9.5326 s. All the methods showed smooth responses with little or no overshoot.

In Table 3 of Fig. 6, the step response evaluation of the compressor showed that the temperature of the unit will exhibit the fastest response and transient to steady state to setpoint command when PID-LQR is allied to it. Thus, the PID-LQR outperformed the other two methods. However, the LQR control system slightly demonstrated faster response to setpoint command than the full state feedback scheme at the expense of poorest rise time 2.5231 s).

The PID-LQR exhibited the fastest response and transition time to steady-state compared to other controllers for the condenser unit temperature looking at Table 4 and Fig. 7. Nevertheless, its response showed slight degree of overshoot together with the LQR system. Thus, looking Table 4 and Fig. 7, the full state feedback system offer the most smooth response with zero overshoot.

As for the expansion valve, from Table 5 and Fig. 8, it is obvious that the full state feedback system provided the best temperature control for the unit. On the other hand, the LQR exhibited the worst performance with rise time of 6.3908 s and settling time of 9.2836 s. All the methods exhibit smooth responses.

5. Conclusion

This paper has presented performance comparison of state variable based controllers for single-stage refrigeration system. Full state feedback control system was designed and compared with LQR and PID-LQR previously implemented in [17]. With step input applied to the various system at time $t = 0$, the step responses of the various temperatures in the evaporator, compressor, condenser, and expansion valve revealed that the three methods tracked the setpoint temperature. The designed full state feedback controller achieved fastest response and transient time to steady state for the evaporator and expansion valve units. The PID-LQR yielded the most result according to the response time (or rise time) and transient time. Generally, the analysis indicated that the designed full state feedback controller exhibited smooth responses in all cases unlike the LQR and PID-LQR that showed some level of oscillations while controlling the temperature in the condenser with overshoot of 4.2533 % and 2.7707 s respectively. Though the simulated control systems provided good performance for the VCR system, the use of intelligence based control system is recommended with either of the state variable technique, which could prove worthwhile.

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