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On Intuitionistic Fuzzy BD-ideals in BD-algebra

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Abstract— *This study introduces new forms of intuitionistic fuzzy structures within AB-algebras, focusing on multiplication-based fuzzy subalgebras and AB-ideals. It examines their interconnections with extension-based counterparts and explores how these fuzzy constructs behave under algebraic homomorphisms.*

Keywords: **fuzzy AB -ideal, intuitionistic fuzzy AB -ideal, multiplication intuitionistic fuzzy AB -ideal, image (pre-image) of AB -ideal.**

1. Introduction

Since the foundational introduction of fuzzy sets [2**5**], numerous extensions and refinements of the concept have emerged, enriching various mathematical and logical frameworks. Among these, the theory of intuitionistic fuzzy sets—proposed [24]—stands out as a significant generalization. While classical fuzzy sets associate each element with a degree of membership in the interval [0,1], intuitionistic fuzzy sets go a step further by incorporating both degrees of membership and non-membership, with the constraint that their combined values do not exceed unity. This dual-assessment provides a more nuanced way of handling uncertainty and vagueness in set theory and algebraic structures.

In parallel, BCK/ BCI-algebras—introduced [**20**]—have played an influential role in the de$velo$pment of non-classical logic and algebraic logic systems. These structures have facilitated the study of various types of fuzzy and intuitionistic fuzzy subsets, including subalgebras and ideals. Within this framework, researchers have explored the behavior and properties of fuzzy and intuitionistic fuzzy constructs, particularly in BCK/BCI-algebras [**24**]. Notably, T.Bantaojai and et. cl. introduceed the notion of a BD-A [2]. Their work laid the groundwork for defining intuitionistic fuzzy subalgebras and BD-ideals in this setting and analyzing their structural attributes.

Building on this foundation, the present paper introduces and investigates new types of intuitionistic fuzzy subsets in BD-algebras, specifically focusing on multiplication-based intuitionistic fuzzy fuzzy BD-ideals. Furthermore, intuit$ioni$stic fuzzy sets is considered, and the interrelationships among these different constructions are thoroughly analyzed. The paper delves into their algebraic properties, including their behavior under certain operations and mappings. In addition, the study examines how these fuzzy structures are preserved under homomorphic images and pre-images within BD-algebras, contributing to a deeper understanding of their theoretical significance and potential applications in algebraic logic and fuzzy systems.

2. Preliminaries

Def. 2.1. ([1-3]).
A *BD*-algebra (BD-A) is a non-empty set ζ with a constant o and a binary " $\diamond$ " satisfying the following axioms hold $\forall \varepsilon, \eta \in \zeta$, if :
(1) $\varepsilon \diamond o = \varepsilon, \forall \varepsilon \in \zeta$,
(2)$(\varepsilon \diamond \eta) = o \; and \; \eta \diamond \varepsilon = o \,, then \; \varepsilon = \eta$.
Remark **2.2[1-3].**
A *BD-A* can be (partially) order by $\varepsilon \leq \eta$ if and only if $(\varepsilon \diamond \eta) = 0$, $\forall \varepsilon, \eta \in \zeta$.
Prop. **2.3[1-3].**
In any *BD-A* $(\zeta; \diamond, o)$, the following hold: $\forall \varepsilon, \eta, \iota, \kappa \in \zeta$
(1) $\varepsilon \diamond \varepsilon = o$,
(2) $\varepsilon \diamond o = \varepsilon$,
(3)$(\varepsilon \diamond \eta) \diamond \iota = (\varepsilon \diamond \iota) \diamond \eta$,
(4)$(\varepsilon \diamond \eta) \diamond (\iota \diamond u) = (\varepsilon \diamond \iota) \diamond (\eta \diamond \kappa)$,
(5) $(\varepsilon \diamond (\varepsilon \diamond \eta)) \diamond \eta = o,$
(6) $(\varepsilon \diamond \ (\varepsilon \diamond \iota) \diamond \eta) \ \diamond \eta = o,$
Remark **2.4[1-3].**
Let $(\zeta; \diamond, o)$ be a *BD-A*, then
(1) If $\varepsilon \leq o, \forall \varepsilon \in \zeta$, then ζ contains only .
(2) If $\varepsilon \leq \eta$, then $\varepsilon * (\varepsilon * (\varepsilon * \eta)) = o, \forall \varepsilon, \eta \in \zeta$.
(3) If $\varepsilon \leq \eta$ such that $\varepsilon * \iota \leq \eta$, then $o \leq \iota, \forall \varepsilon, \eta, \iota \in \zeta$.

Def. 2.5. ([1-3]).

A subset S of a BD-A $(\zeta;\diamond,\epsilon)$ is Name **subalgebra of ζ (SA)** if $\varepsilon \diamond \iota \in S$ whenever $\varepsilon, \iota \in S$.

Def. 2.6. ([1-3]).

A non-empty subset γ of a BD-A $(\zeta;\diamond,\epsilon)$ is Name ***BD*-ideal of ζ (BD -I)** if for $\varepsilon, \iota, \omega \in \zeta$

(I-1) $\epsilon \in \gamma$

(I-2) $(\varepsilon \diamond \iota) \diamond \omega \in \gamma$ and $\in \gamma \Rightarrow \varepsilon \diamond \omega \in \gamma$.

Prop. 2.7. ([1-3]).

Every BD-I of BD-A $(\zeta;\diamond,\epsilon)$ is BD -SA of ζ.

Prop. 2.8. ([1-3]).

Let $\{\ _i \mid i\in\Lambda\}$ be a family of BD-Is of BD-A $(\zeta;\diamond,\epsilon)$. The intersection of them of BD-Is of ζ is an BD-I of ζ.

Def. 2.9. ([25]).

Let $(\zeta;\diamond,\epsilon)$ be a nonempty set, a map. $\xi: \zeta \rightarrow [0,1]$.is name a fuzzy set ξ of ζ .

Def. 2.10. ([25]).

Let ξ be a fuzzy subset (FS) of a set . For t ∈ [0, 1], the set

$\xi t = U(\xi, t) = \{\varepsilon \in \zeta \mid \xi(\varepsilon) \geq t\}$, is name **upper level of ξ** and the set $L(\xi, t) = \{\varepsilon \in \zeta \mid \xi(\varepsilon) \leq t\}$ is name **lower level of ξ**.

Def. 2.11. ([25]).

Let $f: (\zeta;\diamond,\epsilon) \rightarrow (\zeta';\diamond',\epsilon')$ be a map. Non-empty sets ζ and ζ' resp.

If ξ is FS of ζ, then FS β of ζ' by:

$$f(\xi)(\iota) = \begin{cases} sup\{\xi(\varepsilon): \varepsilon \in f^{-1}(\iota)\} & if \quad f^{-1}(\iota) = \{\varepsilon \in \zeta, f(\varepsilon) = \iota\} \neq \emptyset \\ 0 & otherwise \end{cases}$$

is name the image of ξ under f.

Simil., if β is FS of ζ' , then FS $\xi = (\beta \circ f)$ of ζ (i.e FS by $\xi(\varepsilon) = \beta(f(\varepsilon))\ \forall \varepsilon \in \zeta$) is name the pre-image of β under f .

Def. 2.12. ([25]).

A FS ξ of set ζ **has sup property** if for set T of ζ, $\exists t_0 \in T \ni \xi(t_0) = sup\{\xi(t) \mid t \in T\}$.

Def. 2.13. ([1-3]).

Let $(\zeta;\diamond,\epsilon)$ be BD-A, a FS ξ of ζ is Name a **fuzzy subalgebra of ζ (F-SA)** if $\forall \varepsilon, \iota \in \zeta$, $\xi(\varepsilon \diamond \iota) \geq min\{\xi(\varepsilon), \xi(\iota)\}$.

Prop. 2.14. ([1-3]).

Let ξ be a FS of BD-A $(\zeta;\diamond,\epsilon)$. If ξ is a F-SA of , then for any t ∈ [0,1], ξ_t is a SA of ζ .

Def. 2.15. [1-3].

Let $(\zeta;\diamond,\epsilon)$ be an BD-A. A FS ξ of ζ is Name **a fuzzy BD -ideal of ζ (F- BD -I)** if $\forall \varepsilon, \iota \in \zeta$,

(1) $\xi(\epsilon) \geq \xi(x)$.

(2) $\xi(\varepsilon \diamond \omega) \geq min\{\xi((\varepsilon \diamond \iota) \diamond \omega), \xi(\iota)\}$.

Prop. 2.16. [1-3].

Every F- BD -I of BD-A is F-SA.

Prop. 2.17. ([1-3]).

Let $f: (\zeta;\diamond,\epsilon) \rightarrow (\zeta';\diamond',\epsilon')$ be a homo. from BD-As ζ to ζ' resp.

1- If F- BD -I β of ζ', then $f^{-1}(\beta)$ is a F- BD -I of ζ.

2- If F- BD -I ξ of ζ with sup property, then $f(\xi)$ is a F- BD -I of ζ', where f is onto.

Def. 2.18.([24]).

An intuitionistic fuzzy subset A (IFS) in a non-empty set ζ is form $A = \{(\varepsilon, \zeta_A(\varepsilon), \eta_A(\varepsilon)) \mid \varepsilon \in \zeta\}$ which $\xi_A: \zeta \rightarrow [0,1]$ and $\eta_A: \zeta \rightarrow [0,1]$ and

$0 \leq \xi_A(\varepsilon) + \eta_A(\varepsilon) \leq 1\ \forall \varepsilon \in \zeta$.

Remark 2.19.([24]).

If IFS A of a non-empty set , then $\xi_A(\varepsilon) + \nu_A(\varepsilon) = 1$, i.e., $\nu_A(\varepsilon) = 1 - \xi_A(\varepsilon) = \xi_A^c(\varepsilon)\ \forall \varepsilon \in \zeta$. Now ξ_A is fuzzy set while $\eta_A = \xi_A^c$ is complement of ξ_A.

Prop. 2.20.

If IFS $A = \{(\varepsilon, \xi_A(\varepsilon), \eta_A(\varepsilon)) \mid \varepsilon \in \zeta\}$ of BD-A $(\zeta;\diamond,\epsilon)$ satisfies the inequalities $\xi_A(\epsilon) \geq \xi_A(\varepsilon)$ and $\eta_A(\epsilon) \leq \eta_A(\varepsilon), \forall \varepsilon \in \zeta$.

Def. 2.21.

Let $A = \{(\varepsilon, \xi_A(\varepsilon), \eta_A(\varepsilon)) \mid \varepsilon \in \zeta\}$ be an IFS of BD-A $(\zeta;\diamond,\epsilon)$. A is named **an intuitionistic fuzzy BD -ideal of ζ (IF- BD -I)** if $\forall \varepsilon, \iota, \omega \in \zeta$,

(IFI1) $\xi_A(\epsilon) \geq \xi_A(\varepsilon)$ and $\eta_A(\epsilon) \leq \eta_A(\varepsilon)$.

(IFI 2) $\xi_A(\varepsilon \diamond \omega) \geq \min\{\xi_A((\varepsilon \diamond \iota) \diamond \omega), \xi_A(\iota)\}$ and

$\eta_A(\varepsilon \diamond \omega) \leq \max\{\eta_A((\varepsilon \diamond \iota) \diamond \omega), \eta_A(\iota)\}$.

$\Rightarrow \xi_A$ is a F- BD -I & η_A is a doubt F- BD -I.

Theorem 2.22.

An IFS $A = \{(\varepsilon, \xi_A(\varepsilon), \eta_A(\varepsilon)) \mid \varepsilon \in \zeta\}$ is an IF- BD -I of BD-A $(\zeta;\diamond,\epsilon) \Leftrightarrow \forall$ $t \in [0,1]$, $U(\xi_A, t)$ & $L(\eta_A, s)$ are BD -I in ζ.

Prop. 2.23.

Let $A = \{(\varepsilon, \xi_A(\varepsilon), \eta_A(\varepsilon)) \mid \varepsilon \in \zeta\}$ be an IF- BD -I of BD-A $(\zeta;\diamond,\epsilon)$, then A is an IF-SA of ζ.

Theorem 2.24.

An IFS $A = \{(\varepsilon, \xi_A(\varepsilon), \eta_A(\varepsilon)) \mid \varepsilon \in \zeta\}$ is IF- BD -I of BD-A $(\zeta;\diamond,\epsilon) \Leftrightarrow$ the fuzzy sets ξ_A is a F- BD -I of ζ and η_A is a doubt F- BD -I of ζ.

3. β-multiplication intuitionistic of fuzzy BD -ideals.

Def. 3.1.

Let $A_\beta^M = \{(\varepsilon, (\xi_A)_\beta^M, (\eta_A)_\beta^M) \mid \varepsilon \in \zeta\}$ be MI of FS of BD-A $(\zeta;\diamond,\epsilon)$. A_β^M is **multiplication intuitionistic of fuzzy BD -ideal of ζ (MI-F- BD -I)** if $\forall \varepsilon, \iota, \omega \in \zeta$,

(IFAB1) $(\xi_A)_\beta^M(\epsilon) = \beta.\xi_A(\epsilon) \geq (\xi_A)_\beta^M(\varepsilon) = \beta.\xi_A(\varepsilon)$ and

$(\eta_A)_\beta^M(\epsilon) = \beta.\eta_A(\epsilon) \leq (\eta_A)_\beta^M(\varepsilon) = \beta.\eta_A(\varepsilon)$.

(IFAB2) $(\xi_A)_\beta^M(\varepsilon \diamond \omega) = \beta.\xi_A(\varepsilon \diamond \omega) \geq \min\{\beta.\mu_A((\varepsilon \diamond \iota) \diamond \omega), \beta.\xi_A(\iota)\}$ and

$(\eta_A)_\beta^M(\varepsilon \diamond \omega) = \beta.\eta_A(\varepsilon \diamond \omega) \leq \max\{\beta.\eta_A((\varepsilon \diamond \iota) \diamond \omega), \beta.\eta_A(\iota)\}$.

That mean $(\xi_A)_\beta^M$ is a F- BD -I of ζ and $(\eta_A)_\beta^M$ is a doubt F- BD -I of ζ.

Theorem 3.2.

If $A = \{(\varepsilon, \xi_A(\varepsilon), \eta_A(\varepsilon)) \mid \varepsilon \in \zeta\}$ is an IF- BD -I of BD-A $(\zeta;\diamond,\epsilon)$, then the MI $A_\beta^M = \{(\varepsilon, (\xi_A)_\beta^M, (\eta_A)_\beta^M) \mid \varepsilon \in \zeta\}$ of A is an IF- BD -I of ζ, $\forall \beta \in (0,1)$.

Proof:

Let $A = \{(\varepsilon, \xi_A(\varepsilon), \eta_A(\varepsilon)) \mid \varepsilon \in \zeta\}$ be an IF- BD -I of ζ and $\beta \in (0,1)$, then $(\xi_A)_\beta^M(\epsilon) = \beta.\xi_A(\epsilon) \geq \beta.\xi_A(\varepsilon) = (\xi_A)_\beta^M(\varepsilon)$ and $(\eta_A)_\beta^M(\epsilon) = \beta.\eta_A(\epsilon) \leq \beta.\eta_A(\varepsilon) = (\eta_A)_\beta^M(\varepsilon)$, $\forall \varepsilon \in \zeta$.

$$(\xi_A)_\beta^M(\varepsilon \diamond \omega) = \beta.\xi_A(\varepsilon \diamond \omega) \geq \beta.\min\{\xi_A((\varepsilon \diamond \iota) \diamond \omega), \xi_A(\iota)\}$$
$$= \min\{\beta.\xi_A((\varepsilon \diamond \iota) \diamond \omega), \beta.\xi_A(\iota)\}$$
$$= \min\{(\xi_A)_\beta^M((\varepsilon \diamond \iota) \diamond \omega), (\xi_A)_\beta^M(\iota)\}$$ and

$$(\eta_A)_\beta^M(\varepsilon \diamond \omega) = \beta.\eta_A(\varepsilon \diamond \omega) \leq \beta.\max\{\eta_A((x * y) * z), \eta_A(\iota)\}$$
$$= \max\{\beta.\eta_A((\varepsilon \diamond \iota) \diamond \omega), \beta.\eta_A(\iota)\}$$
$$= \max\{(\eta_A)_\beta^M((\varepsilon \diamond \iota) \diamond \omega), (\eta_A)_\beta^M(\iota)\}, \forall \varepsilon, \iota, \omega \in \zeta.$$

Hence, theMI A_β^M of A is an IF- BD -I of ζ. ■

Theorem 3.3.

Let $A = \{(\varepsilon, \xi_A(\varepsilon), \eta_A(\varepsilon)) \mid \varepsilon \in \zeta\}$ be an IFS of BD-A $(\zeta;\diamond,\epsilon)$ $\ni$the MI $A_\beta^M = \{(\varepsilon, (\xi_A)_\beta^M, (\eta_A)_\beta^M) \mid \varepsilon \in \zeta\}$ of A is IF- BD -I of ζ. $\beta \in (0,1)$, $\Rightarrow$ A is IF- BD -I in ζ.

Proof:

Let $A_\beta^M = \{(\varepsilon, (\xi_A)_\beta^M, (\eta_A)_\beta^M) \mid \varepsilon \in \zeta\}$ is an IF- BD -I of ζ, let $\varepsilon, \iota, \omega \in \zeta$,

$\beta.\xi_A(\epsilon) = (\xi_A)_\beta^M(\epsilon) \geq (\xi_A)_\beta^M(\varepsilon) = \beta.\xi_A(\varepsilon)$,

$\beta.\eta_A(\epsilon) = (\eta_A)_\beta^M(\epsilon) \leq (\eta_A)_\beta^M(\varepsilon) = \beta.\eta_A(\varepsilon)$.

which implies $\xi_A(\epsilon) \geq \xi_A(\varepsilon)$ and $\eta_A(\epsilon) \leq \eta_A(\varepsilon)$.

Now, we have

$$\beta.\xi_A(\varepsilon \diamond \omega) = (\xi_A)_\beta^M(\varepsilon \diamond \omega) \geq \min\{(\xi_A)_\beta^M((\varepsilon \diamond \iota) \diamond \omega), (\xi_A)_\beta^M(\iota)\}$$
$$= \min\{\beta.\xi_A((\varepsilon \diamond \iota) \diamond \omega), \beta.\xi_A(\iota)\}$$
$$= \beta.\min\{\xi_A((\varepsilon \diamond \iota) \diamond \omega), \xi_A(\iota)\}\}$$

and $\beta.\eta_A(\varepsilon \diamond \omega) = (\eta_A)_\beta^M(\varepsilon \diamond \omega) \leq \max\{(\eta_A)_\beta^M((\varepsilon \diamond \iota) \diamond \omega), (\eta_A)_\beta^M(\iota)\}$

$$= \max\{\beta.\eta_A((\varepsilon \diamond \iota) \diamond \omega), \beta.\eta_A(\iota)\}$$

$= \beta.\max\{\eta_A((\varepsilon \diamond \iota) \diamond \omega), \eta_A(\iota)\}$,

which implies that $\xi_A(\varepsilon \diamond \omega) \geq \min\{\xi_A((\varepsilon \diamond \iota) \diamond \omega), \xi_A(\iota)\}$

and $\eta_A(\varepsilon \diamond \omega) \leq \max\{\eta_A((\varepsilon \diamond \iota) \diamond \omega), \eta_A(\iota)\}$, $\forall \varepsilon, \iota, \omega \in \zeta$.

Hence, $A = \{(\varepsilon, \xi_A(\varepsilon), \eta_A(\varepsilon)) \mid \varepsilon \in \zeta\}$ is an IF- BD -I of ζ.■

Theorem 3.4

If the MI $A_\beta^M = \{(\varepsilon, (\xi_A)_\beta^M, (\eta_A)_\beta^M) \mid \varepsilon \in \zeta\}$ of A is an IF- BD -I of AB-A $(\zeta; \diamond, \epsilon)$, $\forall \beta \in (0,1)$, then A_β^M must be an IF-SA of ζ.

Proof:

Let the MI $A_\beta^M = \{(\varepsilon, (\xi_A)_\beta^M, (\eta_A)_\beta^M) \mid \varepsilon \in \zeta\}$ of A be IF- BD -I of .

We have $\forall \varepsilon, \iota, \omega \in \zeta$,

$(\xi_A)_\beta^M(\varepsilon \diamond \omega) \geq \min\{(\xi_A)_\beta^M((\varepsilon \diamond \iota) \diamond \omega), (\xi_A)_\beta^M(\iota)\}$ and

$(\eta_A)_\beta^M(\varepsilon \diamond \omega) \leq \max\{(\eta_A)_\beta^M((\varepsilon \diamond \iota) \diamond \omega), (\eta_A)_\beta^M(\iota)\}$, then

$(\xi_A)_\beta^M(\varepsilon \diamond \omega) \geq \min\{(\xi_A)_\beta^M((\varepsilon \diamond \iota) \diamond \omega), (\xi_A)_\beta^M(\iota)\}$

$= \min\{(\xi_A)_\beta^M(\varepsilon \diamond \epsilon), (\xi_A)_\beta^M(\iota)\}$

$= \min\{(\xi_A)_\beta^M(\varepsilon), (\xi_A)_\beta^M(\iota)\}$ and

$(\eta_A)_\beta^M(\varepsilon \diamond \omega) \leq \max\{(\eta_A)_\beta^M((\varepsilon \diamond \iota) \diamond \omega), (\nu_A)_\beta^M(\iota)\}$

$= \max\{(\eta_A)_\beta^M(\varepsilon \diamond \epsilon), (\eta_A)_\beta^M(\iota)\}$

$= \max\{(\eta_A)_\beta^M(\varepsilon), (\eta_A)_\beta^M(\iota)$

Hence, A_β^M is of IF-SA of ζ. ■

Theorem 3.5.

If $A = \{(\varepsilon, \xi_A(\varepsilon), \eta_A(\varepsilon)) \mid \varepsilon \in \zeta\}$ is an IFS of BD-A $(\zeta; \diamond, \epsilon) \ni$ the MI $A_\beta^M = \{(\varepsilon, (\xi_A)_\beta^M, (\eta_A)_\beta^M) \mid \varepsilon \in \zeta\}$ of A is an IF- BD -I of ζ, for $\beta \in (0,1)$, then the sets I_{ξ_A} and I_{η_A} are BD -Is of ζ.

Proof:

Let $A_\beta^M = \{(\varepsilon, (\xi_A)_\beta^M, (\eta_A)_\beta^M) \mid \varepsilon \in \zeta\}$ is an IF- BD -I of ζ, so $(\xi_A)_\beta^M$ is F- BD -I of ζ and $(\eta_A)_\beta^M$ is doubt F- BD -I of ζ.

Clearly $\epsilon \in I_{\xi_A}, I_{\eta_A}$, suppose $\varepsilon, \iota, \omega \in \zeta \ni$

$((\varepsilon \diamond \iota) \diamond \omega) \in I_{\xi_A}$ and $\iota \in I_{\xi_A}$, hence $(\xi_A)_\beta^M((\varepsilon \diamond \iota) \diamond \omega) = (\xi_A)_\beta^M(\epsilon) = (\xi_A)_\beta^M(\iota)$ and

$(\xi_A)_\beta^M(\varepsilon \diamond \omega) \geq \min\{(\xi_A)_\beta^M((\varepsilon \diamond \iota) \diamond \omega), (\xi_A)_\beta^M(\iota)\} = (\xi_A)_\beta^M(\epsilon)$.

Since, $(\xi_A)_\beta^M$ is F- BD -I of ζ, then

$(\xi_A)_\beta^M(\varepsilon \diamond \omega) = (\xi_A)_\beta^M(\epsilon)$.

Hence, $\beta.\xi_A(\varepsilon \diamond \omega) = \beta.\xi_A(\epsilon)$ or $\xi_A(\varepsilon \diamond \omega) = \xi_A(\epsilon)$ and $(\varepsilon \diamond \omega) \in I_{\xi_A}$ then I_{ξ_A} is BD -I of ζ.

Suppose u, v, w, $\in \zeta \ni ((u * v) * w) \in I_{\eta_A}$ & $v \in I_{\eta_A}$. Hence

$(\eta_A)_\beta^M((u * v) * w) = (\eta_A)_\beta^M(\epsilon) = (\eta_A)_\beta^M(v)$ &

$(\eta_A)_\beta^M(u * w) \leq \max\{(\eta_A)_\beta^M((u * v) * w), (\eta_A)_\beta^M(v)\}$

$= (\eta_A)_\beta^M(\epsilon)$.

Since, $(\eta_A)_\beta^M$ is F- BD -I of ζ, then $(\eta_A)_\beta^M(u * w) = (\eta_A)_\beta^M(\epsilon)$.

Hence $\beta.\eta_A(u * w) = \beta.\eta_A(\epsilon), \eta_A(u * v) = \eta_A(\epsilon)$ & $(u * v) \in I_{\eta_A}$, $\Rightarrow I_{\eta_A}$ is BD -I of ζ. ■

Prop. 3.6.

Let the MI $A_\beta^M = \{(\varepsilon, (\xi_A)_\beta^M, (\eta_A)_\beta^M) \mid \varepsilon \in \zeta\}$ of A be IF- BD -I of BD-A $(\zeta; \diamond, \epsilon)$ for $\beta \in (0,1)$. If $\varepsilon \leq \iota$ then $(\xi_A)_\beta^M(\varepsilon) \geq (\xi_A)_\beta^M(\iota)$ and $(\eta_A)_\beta^M(\varepsilon) \leq (\eta_A)_\beta^M(\iota)$, that is, $(\xi_A)_\beta^M$ is reverse the system and $(\eta_A)_\beta^M$ is Maintain order.

Proof:

Let $\varepsilon, \iota \in \zeta$ & $\varepsilon \leq \iota, \Rightarrow \varepsilon \diamond \iota = \epsilon$ and

$(\xi_A)_\beta^M(\varepsilon) = (\xi_A)_\beta^M(\varepsilon \diamond \epsilon) \geq \min\{(\xi_A)_\beta^M((\varepsilon \diamond \iota) \diamond \epsilon), (\xi_A)_\beta^M(\iota)\}$

$= \min\{(\xi_A)_\beta^M(\varepsilon \diamond \iota), (\xi_A)_\beta^M(\iota)\}$

$= \min\{(\xi_A)_\beta^M(\epsilon), (\xi_A)_\beta^M(\iota)\} = (\xi_A)_\beta^M(\iota)$ and

$(\eta_A)_\beta^M(\varepsilon) = (\eta_A)_\beta^M(\varepsilon \diamond \epsilon) \leq \max\{(\eta_A)_\beta^M((\varepsilon \diamond \iota) \diamond \epsilon), (\eta_A)_\beta^M(\iota)\}$

$= \max\{(\eta_A)_\beta^M(\varepsilon \diamond \iota), (\eta_A)_\beta^M(\iota)$

$= \max\{(\eta_A)_\beta^M(\epsilon), (\eta_A)_\beta^M(\iota)\} = (\eta_A)_\beta^M(\iota)$. ■

Theorem 3.7.

The MI $A_\beta^M = \{(\varepsilon, (\xi_A)_\beta^M, (\eta_A)_\beta^M) |\ \varepsilon \in \zeta\}$ of A is a F- BD -I of BD-A $(\zeta; \diamond, \epsilon)$, then $\forall t, s \in [0,1]$, the set $U_\beta(\xi_A, t)$ and $L_\beta(\eta_A, s)$ are BD -Is of ζ.

Proof.

Let $A_\beta^M = \{(\varepsilon, (\xi_A)_\beta^M, (\eta_A)_\beta^M) |\ \varepsilon \in \zeta\}$ be F- BD -I of ζ and $U_\beta(\xi_A, t) \neq \emptyset \neq L_\beta(\eta_A, s)$.

Since $\beta.\xi_A(\epsilon) \geq t$ and $\beta.\eta_A(\epsilon) \leq s$, let $\varepsilon, \iota, \omega \in \zeta$ be $\ni ((\varepsilon \diamond \iota) \diamond \omega) \in U_\beta(\xi_A, t)$, $(\iota) \in U_\beta(\xi_A, t)$, then $\beta.\xi_A((\varepsilon \diamond \iota) \diamond \omega) \geq t$ and $\beta.\xi_A(\iota) \geq t$, it follows that $\beta.\xi_A(\varepsilon \diamond \omega) \geq \min\{\beta.\xi_A((\varepsilon \diamond \iota) \diamond \omega), \beta.\xi_A(\iota)\} \geq t$, so that $(\varepsilon \diamond \omega) \in U_\beta(\xi_A, t)$.

Hence $U_\beta(\xi_A, t)$ is BD -I in ζ.

In a similar way, $L_\beta(\eta_A, s)$ is BD -I in ζ. ■

Theorem 3.8.

The MI $A_\beta^M = \{(\varepsilon, (\xi_A)_\beta^M, (\eta_A)_\beta^M) |\ \varepsilon \in \zeta\}$ of A is a FS of BD-A $(\zeta; \diamond, \epsilon) \ni \forall t, s \in [0,1]$, the set $U_\beta(\xi_A, t)$ and $L_\beta(\eta_A, s)$ are BD -Is of ζ, then A_β^M is a F- BD -I of ζ.

Proof.

Assume that for each , t , s $\in [0,1]$, the sets $U_\beta(\xi_A, t)$ and $L_\beta(\eta_A, s)$ are BD -Is of ζ. For any $\in \zeta$, let $\beta.\xi_A(\varepsilon) = t$ and $\beta.\eta_A(\varepsilon) = s$, then $\varepsilon \in U_\beta(\xi_A, t) \cap L_\beta(\eta_A, s)$ and so $U_\beta(\xi_A, t) \neq \emptyset \neq L_\beta(\eta_A, s)$.

Since $U_\beta(\xi_A, t)$ and $L_\beta(\eta_A, s)$ are BD -Is of ζ, therefore $0 \in U_\beta(\xi_A, t) \cap L_\beta(\eta_A, s)$.

Hence $\beta.\xi_A(\epsilon) \geq t = \xi_A(\varepsilon)$ and $\beta.\eta_A(\epsilon) \leq s = \eta_A(\varepsilon)$, $\forall \varepsilon \in \zeta$.

If $\exists\ x', y', z' \in \zeta$ be $\ni$

$\beta.\xi_A(x' * z') < \min\{\beta.\xi_A((x' *' y') * z'), \beta.\xi_A(y')\} \Rightarrow$

$t_0 = \frac{1}{2}\{\beta.\xi_A(x' * z') + \min\{\beta.\xi_A((x' *' y') * z'), \beta.\xi_A(y')\}\}, \Rightarrow$

$\beta.\xi_A(x' * z') < t_0 < \min\{\beta.\xi_A(x' *' (y' * z'), \beta.\xi_A(y')\} \Rightarrow$

$(x' * z') \in U_\beta(\xi_A, t_0)$, $((x' *' y') * z') \in U_\beta(\xi_A, t_0)$, $(y') \in U_\beta(\xi_A, t_0)$.

i.e., $U_\beta(\xi_A, t_0)$, is not BD -I of $\zeta \Rightarrow$ C!.

Finally $\exists\ x', y', z' \in \zeta \ni$

$\beta.\eta_A(x' * z') > \max\{\beta.\eta_A((x' *' y') * z'), \beta.\eta_A(y')\} \Rightarrow$

$s_0 = \frac{1}{2}\{\beta.\eta_A(x' * z') + \max\{\beta.\eta_A((x' *' y') * z'), \beta.\eta_A(y')\}\} \Rightarrow \max\{\beta.\eta_A((x' *' y') * z'), \beta.\eta_A(y')\} > s_0 > \eta_A(x' * z') \Rightarrow (x' * z') \in U_\beta(\eta_A, s_0)$, $((x' *' y') * z') \in L_\beta(\eta_A, s_0)$, $(y') \in L_\beta(\eta_A, s_0)$.

i.e., $L_\beta(\eta_A, s_0)$, is not AB-I of $\zeta \Rightarrow$ C!.

Hence $A_\beta^M = \{(\varepsilon, (\xi_A)_\beta^M, (\eta_A)_\beta^M) |\ \varepsilon \in \zeta\}$ is F- BD -I of ζ.■

Theorem 3.9.

Let A and B be two MI of F- BD -Is of BD-A $(\zeta; \diamond, \epsilon)$. Then $A \cap B$ is $MI$of F- BD -I of ζ.

Proof.

Let $\varepsilon, \iota \in A \cap B$, then x, y$\in$ A & B. Now,

$$
\begin{aligned}
(\xi_{A\cap B})_\beta^M(\epsilon) &= (\xi_{A\cap B})_\beta^M(\varepsilon \diamond \varepsilon) \\
&\geq \min\{(\xi_{A\cap B})_\beta^M(\varepsilon), (\xi_{A\cap B})_\beta^M(\varepsilon)\} \\
&= (\xi_{A\cap B})_\beta^M(\varepsilon) \quad \&
\end{aligned}
$$

$$
\begin{aligned}
(\eta_{A\cap B})_\beta^M(\epsilon) &= (\eta_{A\cap B})_\beta^M(\varepsilon \diamond \varepsilon) \\
&\leq \max\{(\eta_{A\cap B})_\beta^M(x), (\eta_{A\cap B})_\beta^M(\varepsilon)\} \\
&= (\eta_{A\cap B})_\beta^M(\varepsilon).
\end{aligned}
$$

$$
\begin{aligned}
&(\xi_{A\cap B})_\beta^M(\varepsilon \diamond \omega) = \min\{(\xi_A)_\beta^M(\varepsilon \diamond \omega), (\xi_B)_\beta^M(\varepsilon \diamond \omega)\} \\
&\geq \min\{\min\{(\xi_A)_\beta^M((\varepsilon \diamond \iota) \diamond \omega), (\xi_A)_\beta^M(\iota)\}, \min\{(\xi_B)_\beta^M(\varepsilon \diamond \iota) \diamond \omega), (\xi_B)_\beta^M(\iota)\}\} \\
&= \min\{\min\{(\xi_A)_\beta^M(\varepsilon \diamond \iota) \diamond \omega), (\xi_B)_\beta^M((\varepsilon \diamond \iota) \diamond \omega)\}, \min\{(\xi_A)_\beta^M(\iota), (\xi_B)_\beta^M(\iota)\}\} \\
&\quad = \min\{(\xi_{A\cap B})_\beta^M(\varepsilon \diamond \iota) \diamond \omega), (\xi_{A\cap B})_\beta^M(\iota)\} \text{ and}
\end{aligned}
$$

$$
\begin{aligned}
&(\eta_{A\cap B})_\beta^M(\varepsilon \diamond \omega) = \max\{(\eta_A)_\beta^M(\varepsilon \diamond \omega), (\eta_B)_\beta^M(\varepsilon \diamond \omega)\} \\
&\leq \max\{\max\{(\eta_A)_\beta^M((\varepsilon \diamond \iota) \diamond \omega), (\eta_A)_\beta^M(\iota)\}, \max\{(\eta_B)_\beta^M((\varepsilon \diamond \iota) \diamond \omega), (\eta_B)_\beta^M(\iota)\}\} \\
&= \max\{\max\{(\eta_A)_\beta^M((\varepsilon \diamond \iota) \diamond \omega), (\eta_B)_\beta^M((\varepsilon \diamond \iota) \diamond \omega)\}, \max\{(\eta_A)_\beta^M(\iota), (\eta_B)_\beta^M(\iota)\}\} \\
&\quad = \max\{(\eta_{A\cap B})_\beta^M((\varepsilon \diamond \iota) \diamond \omega), (\eta_{A\cap B})_\beta^M(\iota)\}
\end{aligned}
$$

Hence, $A \cap B$ is MI of F- BD -I of ζ. ■

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Theorem 3.10.

The MI of fuzzy $A_{\beta}^{M} = \{(\varepsilon, (\xi_A)_{\beta}^{M}, (\eta_A)_{\beta}^{M}) \mid \varepsilon \in \zeta\}$ of A is F- BD -I of BD-A $(\zeta; \diamond, \epsilon) \Leftrightarrow$ the FSs $(\xi_{\mathrm{A}})_{\beta}^{\mathrm{M}}$ and $(\bar{\eta}_{\mathrm{A}})_{\beta}^{\mathrm{M}}$ are F- BD -Is of ζ.

Proof.

Let $A_{\beta}^{M} = \{(\varepsilon, (\xi_A)_{\beta}^{M}, (\eta_A)_{\beta}^{M}) \mid \varepsilon \in \zeta\}$ be MI of F- BD -I of ζ.

Cleary, ξ_{A} is a F- BD -I of ζ, $\exists \varepsilon, \iota \in \zeta \Rightarrow$

$(\bar{\eta}_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\epsilon)$=1−$(\eta_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\epsilon) \geq$1−$(\eta_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\varepsilon)$=$(\bar{\eta}_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\varepsilon)$ and

$(\bar{\eta}_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\varepsilon \diamond \omega)$=1−$(\eta_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\varepsilon \diamond \omega) \geq 1 - \max\{(\eta_{\mathrm{A}})_{\beta}^{\mathrm{M}}((\varepsilon \diamond \iota) \diamond \omega), (\eta_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\iota)\}$

=min{1−$(\eta_{\mathrm{A}})_{\beta}^{\mathrm{M}}((\varepsilon \diamond \iota) \diamond \omega)$,$1 - (\eta_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\iota)\}$=min$\{(\bar{\eta}_{\mathrm{A}})_{\beta}^{\mathrm{M}}((\varepsilon \diamond \iota) \diamond \omega), (\bar{\eta}_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\iota)\}$.

Hence,$(\bar{\eta}_{\mathrm{A}})_{\beta}^{\mathrm{M}}$ is a F-AB-I of ζ.

Conversely, assume that$(\xi_{\mathrm{A}})_{\beta}^{\mathrm{M}}$ and $(\bar{\eta}_{\mathrm{A}})_{\beta}^{\mathrm{M}}$are two MI of F- BD -Is of ζ, for every $\varepsilon, \iota \in \zeta$, we get

$(\xi_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\epsilon) \geq (\xi_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\varepsilon)$ and $(\bar{\eta}_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\epsilon) \geq (\bar{\eta}_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\varepsilon)$.

$\Rightarrow$ 1−$(\eta_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\epsilon) \geq$ 1−$(\eta_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\epsilon) \Rightarrow (\eta_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\epsilon) \leq (\eta_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\varepsilon)$.

$(\eta_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\varepsilon \diamond \omega) \geq \min\{(\eta_{\mathrm{A}})_{\beta}^{\mathrm{M}}((\varepsilon \diamond \iota) \diamond \omega), (\eta_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\iota)\}$& 1−$(\eta_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\varepsilon \diamond \omega) = (\bar{\eta}_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\varepsilon \diamond \omega)$

$\geq \min\{(\bar{\eta}_{\mathrm{A}})_{\beta}^{\mathrm{M}}((\varepsilon \diamond \iota) \diamond \omega), (\bar{\eta}_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\iota)$

$= \min\{1 - (\eta_{\mathrm{A}})_{\beta}^{\mathrm{M}}((\varepsilon \diamond \iota) \diamond \omega), 1 - (\eta_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\iota)\}$

=1−max$\{(\eta_{\mathrm{A}})_{\beta}^{\mathrm{M}}((\varepsilon \diamond \iota) \diamond \omega), (\eta_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\iota)\}$, that is,

$(\eta_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\varepsilon \diamond \omega) \leq \max\{(\eta_{\mathrm{A}})_{\beta}^{\mathrm{M}}((\varepsilon \diamond \iota) \diamond \omega), (\eta_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\iota)\}$.

Hence $A_{\beta}^{M} = \{(\varepsilon, (\xi_A)_{\beta}^{M}, (\eta_A)_{\beta}^{M}) \mid \varepsilon \in \zeta\}$ is F- BD -I of ζ. ■

4- Homomorphism of Multiplication intuitionistic of BD -algebra.

Def. 4.1.[11].

Let $(\zeta; \diamond, \epsilon)$ & $(\zeta'; \diamond', \epsilon')$ be two non-empty sets. A map. $f: \zeta \to \zeta'$ is **homomorphism** (**homo.**)if $f(\varepsilon \diamond \iota) = f(\varepsilon) *' f(\iota)$, $\forall \varepsilon, \iota \in \zeta$ &$f(\epsilon) = \epsilon'$.

Def. 4.2.

Let $f : (\zeta; \diamond, \epsilon) \to (\zeta'; \diamond', \epsilon')$ be a homo. of BD -As for any

$\mathrm{A}_{\beta}^{\mathrm{M}} = \{(\iota, (\xi_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\iota), (\eta_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\iota)) \mid \iota \in \zeta'\}$ in ζ' and $\beta \in (0,1)$ we define

$(\mathrm{A}_{\beta}^{\mathrm{M}})^{f}$ =$\{(\varepsilon, ((\xi_{\mathrm{A}})_{\beta}^{\mathrm{M}})^{f}(\varepsilon), ((\eta_{\mathrm{A}})_{\beta}^{\mathrm{M}})^{f}(\varepsilon)) \mid \varepsilon \in \zeta\}$ in ζ by

$((\xi_{\mathrm{A}})_{\beta}^{\mathrm{M}})^{f}(\varepsilon) = (\xi_{\mathrm{A}})_{\beta}^{\mathrm{M}}(f(\varepsilon))$ and $((\eta_{\mathrm{A}})_{\beta}^{\mathrm{M}})^{f}(\varepsilon) = (\eta_{\mathrm{A}})_{\beta}^{\mathrm{M}}(f(\varepsilon))$, $\forall \varepsilon \in \zeta$.

Theorem 4.3.

Let $f : (\zeta; \diamond, \epsilon) \to (\zeta'; \diamond', \epsilon')$ be a homo. of BD -As. If

$\mathrm{B}_{\beta}^{\mathrm{M}} = \{(\varepsilon, (\xi_{\mathrm{B}})_{\beta}^{M}, (\eta_{\mathrm{B}})_{\beta}^{M}) \mid \varepsilon \in \zeta\}$ is MI of F- BD -I of ζ', ⇒the pre-image

$f^{-1}(\mathrm{B}_{\beta}^{\mathrm{M}}) = \{(\varepsilon, f^{-1}((\xi_{\mathrm{B}})_{\beta}^{\mathrm{M}})(\varepsilon), f^{-1}((\eta_{\mathrm{B}})_{\beta}^{\mathrm{M}})(\varepsilon)) \mid \varepsilon \in \zeta\}$ of $\mathrm{B}_{\beta}^{\mathrm{M}}$ under f in ζ is MI of F- BD -I of ζ.

Proof.

$\forall \epsilon \in \zeta$,

$$
\begin{aligned}
f^{-1}((\xi_{\mathrm{B}})_{\beta}^{\mathrm{M}})(\varepsilon) &= (\xi_{\mathrm{B}})_{\beta}^{\mathrm{M}}(f(\varepsilon)) \\
&\leq \left((\xi_{\mathrm{B}})_{\beta}^{\mathrm{M}}(\epsilon) = (\xi_{\mathrm{B}})_{\beta}^{\mathrm{M}}(f(\epsilon))\right) \\
&= f^{-1}((\xi_{\mathrm{B}})_{\beta}^{\mathrm{M}})(\epsilon) \text{ and}
\end{aligned}
$$

$$
\begin{aligned}
f^{-1}((\eta_{\mathrm{B}})_{\beta}^{\mathrm{M}})(\varepsilon) &= (\eta_{\mathrm{B}})_{\beta}^{\mathrm{M}}(f(\varepsilon)) \\
&\geq (\eta_{\mathrm{B}})_{\beta}^{\mathrm{M}}(\epsilon) = (\eta_{\mathrm{B}})_{\beta}^{\mathrm{M}}(f(\epsilon))) \\
&= f^{-1}((\eta_{\mathrm{B}})_{\beta}^{\mathrm{M}})(\epsilon).
\end{aligned}
$$

Let $, \iota, \omega \in \zeta$, then

$$
\begin{aligned}
(f^{-1}((\xi_{\mathrm{B}})_{\beta}^{\mathrm{M}})(\varepsilon \diamond \omega) &= (\xi_{\mathrm{B}})_{\beta}^{\mathrm{M}}(f(\varepsilon \diamond \omega)) \\
&\geq \min\{(\xi_{\mathrm{B}})_{\beta}^{\mathrm{M}}\left((f(\varepsilon) \diamond' f(\iota)) \diamond' f(\omega)\right), (\xi_{\mathrm{B}})_{\beta}^{\mathrm{M}}(f(\iota))\} \\
&\geq \min\{(\xi_{\mathrm{B}})_{\beta}^{\mathrm{M}}(f((\varepsilon \diamond \iota) \diamond \omega), ((\xi_{\mathrm{B}})_{\beta}^{\mathrm{M}}(f(\iota))\}
\end{aligned}
$$

$= \min\{(f^{-1}((\xi_B)^M_\beta)((\varepsilon \diamond \iota) \diamond \omega), (f^{-1}((\xi_B)^M_\beta)(\iota)\}$ and

$(f^{-1}((\eta_B)^M_\beta)(\varepsilon \diamond \omega) = ((\eta_B)^M_\beta\big(f(\varepsilon \diamond \omega)\big)$

$\leq \max\{(\eta_B)^M_\beta\Big(\big(f(\varepsilon) \diamond' f(\iota)\big) \diamond' f(\omega)\Big), (\eta_B)^M_\beta(f(\iota))\}$

$\geq \max\big\{(\eta_B)^M_\beta(f((\varepsilon \diamond \iota) \diamond \omega), (\eta_B)^M_\beta\big(f(\iota)\big)\big\}$

$= \max\{f^{-1}((\eta_B)^M_\beta)((\varepsilon \diamond \iota) \diamond \omega), f^{-1}((\eta_B)^M_\beta)(\iota)\}$.

Hence, $f^{-1}(B^M_\beta) = \{(\varepsilon, f^{-1}((\xi_B)^M_\beta)(\varepsilon), f^{-1}((\eta_B)^M_\beta)(\varepsilon))|\ \varepsilon \in \zeta\}$ is MI of F- BD -I of ζ. ■

Theorem 4.4.

Let $f:(\zeta;\diamond,\epsilon) \to (\zeta';\diamond',\epsilon')$ be an epimorphism of BD -As. If $A^M_\beta = \{(\varepsilon, (\xi_A)^M_\beta, (\eta_A)^M_\beta)|\ \varepsilon \in \zeta\}$ is MI of F- BD -I of ζ, then $f(A^M_\beta) = \{(\varepsilon, f((\xi_A)^M_\beta)(\varepsilon), f((\eta_A)^M_\beta)(\varepsilon))|\ \varepsilon \in \zeta\}$ of A is F- BD -I in ζ'.

Proof.

Let a$\in \zeta$, $\exists$y $\in \zeta' \ni f(\mathrm{a}) = \mathrm{y} \Rightarrow$

$f((\xi_A)^M_\beta)(\mathrm{y}) = f((\xi_A)^M_\beta)\big(f(\mathrm{a})\big)$

$= f^{-1}\big(f((\xi_A)^M_\beta)(\mathrm{a})$

$= (\xi_A)^M_\beta(\mathrm{a}) \leq (\xi_A)^M_\beta(\epsilon)$

$=f^{-1}\big(f((\xi_A)^M_\beta)(\epsilon)$

$= f((\xi_A)^M_\beta)\big(f(\epsilon)\big) = f((\xi_A)^M_\beta)(\epsilon')$ &

$f((\eta_A)^M_\beta)(\mathrm{y}) = f((\eta_A)^M_\beta)\big(f(\mathrm{a})\big)$

$= f^{-1}\big(f((\eta_A)^M_\beta)(\mathrm{a})$

$= (\eta_A)^M_\beta(\mathrm{a}) \geq (\eta_A)^M_\beta(\epsilon)$

$= f^{-1}\big(f((\eta_A)^M_\beta)(\epsilon)$

$= f((\eta_A)^M_\beta)\big(f(\epsilon)\big) = f((\eta_A)^M_\beta)(\epsilon')$.

Let $\varepsilon, \iota, \omega \in \zeta'$, $\Rightarrow f$ (a)= ε & f (b)= ι & f (c) =ω $\ni$ a, b, c $\in \zeta$, thus

$f((\xi_A)^M_\beta)(\varepsilon \diamond' \omega) = f((\xi_A)^M_\beta)\big(f(\mathrm{a}) \diamond' f(\mathrm{c})\big)$

$= f^{-1}\big(f((\xi_A)^M_\beta)(\mathrm{a} \diamond \mathrm{c})$

$= (\xi_A)^M_\beta(\mathrm{a} \diamond \mathrm{c})$

$\geq \{\ (\xi_A)^M_\beta\big((\mathrm{a} \diamond \mathrm{b}) \diamond \mathrm{c}\big), (\xi_A)^M_\beta(\mathrm{b})\}$

$=\ \{f^{-1}\big(f((\xi_A)^M_\beta)\big)\big((\mathrm{a} \diamond \mathrm{b}) \diamond \mathrm{c}\big), f^{-1}\big(f((\xi_A)^M_\beta)(\mathrm{b})\}$

$=\ \{\ \big(f((\xi_A)^M_\beta)\ \Big(\big(f(\mathrm{a}) \diamond' f(\mathrm{b})\big) \diamond' f(\mathrm{c})\Big), \big(f((\xi_A)^M_\beta)\big(f(\mathrm{b})\big)\}$

$=\ \{f((\xi_A)^M_\beta)((\varepsilon \diamond' \iota) \diamond' \omega), f((\xi_A)^M_\beta(\iota)\}$ and

$f((\eta_A)^M_\beta)(\varepsilon \diamond' \omega) = f((\eta_A)^M_\beta)\big(f(\mathrm{a}) \diamond' f(\mathrm{c})\big)$

$= f^{-1}\big(f((\eta)^M_\beta)\big)(\mathrm{a} \diamond \mathrm{c}) = (\eta_A)^M_\beta(\mathrm{a} \diamond \mathrm{c})$

$\leq \{\ (\eta_A)^M_\beta\big((\mathrm{a} \diamond \mathrm{b}) \diamond \mathrm{c}\big), (\eta_A)^M_\beta(\mathrm{b})\}$

$=\ \{f^{-1}\big(f((\eta_A)^M_\beta)\big((\mathrm{a} \diamond \mathrm{b}) \diamond \mathrm{c}\big), f^{-1}\big(f((\eta_A)^M_\beta)(\mathrm{b})\}$

$=\ \{f((\eta_A)^M_\beta\ \Big(\big(f(\mathrm{a}) \diamond' f(\mathrm{b})\big) \diamond' f(\mathrm{c})\Big), f((\eta_A)^M_\beta\big(f(\mathrm{b})\big)\}$

$=\ \{f((\eta_A)^M_\beta((\varepsilon \diamond' \iota) \diamond' \omega), f((\eta_A)^M_\beta(\iota)\}$.

Hence $f(A^M_\beta) = \{(\varepsilon, f((\xi_A)^M_\beta)(\varepsilon), f((\eta_A)^M_\beta)(\varepsilon))|\ \varepsilon \in \zeta\}$ is a F- BD -I of ζ'. ■

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