

Modeling the Effect of Thermal Treatment on Hardness of Small Cashew Nut Shells

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Abstract: The mechanical modification of agricultural by-products through thermal processing has become an important pathway for value addition and sustainable utilization. Cashew nut shell, an abundant lignocellulosic residue in Nigeria, possesses engineering properties that can be enhanced through controlled thermal treatment. This study investigates the effect of temperature and time interactions on the hardness of small cashew nut shell accessions and developed a predictive model using the Buckingham Pi theorem. Small cashew nut shells obtained from the Cocoa Research Institute of Nigeria (CRIN) were thermally treated at 200°C, 250°C, and 300°C for 30, 60, and 90 minutes. Hardness and compressive strength measurements were conducted using a universal testing machine (UTM), while accompanying physical and thermal properties including density, moisture content, shrinkage, thermal conductivity, and specific heat capacity were determined using standard engineering methods. Dimensional analysis generated three fundamental π terms that formed the basis of a hardness prediction model expressed as: $H = \Phi \left(\frac{\rho \theta K}{T \sigma} \right)$. The generalized reduced gradient (GRG) nonlinear Excel solver was used to estimate the model constant (Φ) by minimizing the sum of squared errors. Model validation revealed strong agreement between measured and predicted hardness values, with low RMSE (0.6963) and a high residual prediction deviation (RPD) = 21.58, indicating excellent predictive performance. Findings demonstrated that thermal treatment significantly modifies the hardness of small cashew nut shells and that the developed model reliably captures these changes. This model provides a useful tool for optimizing thermal processes in cashew shell valorization and related agro-processing applications.

Keywords— Buckingham Pi theorem; cashew nut shell; GRG nonlinear optimization; predictive modeling; thermal treatment; thermo-mechanical properties

1. INTRODUCTION

Agricultural by-products are increasingly being explored for value added applications, particularly as global interest grows in renewable materials and sustainable processing. Cashew nut shells, despite their potential, are typically underutilized and often discarded as waste in Nigeria. However, recent research has shown that with proper thermal treatment, the mechanical, physical, and thermal properties of cashew nut shells can be improved [1, 2] Kadjo *et al.*, [1](2024); Adeleke *et al.*, [2](2020). Thermal treatments such as roasting or pyrolysis alter the structural properties of cashew nut shells, affecting strength, hardness, moisture content, density, and thermal properties [3, 4] Kuppusamy, (2014); Fuwat, (2023).

Hardness is a critical mechanical property that determines how cashew nut shell behaves during milling, pyrolysis, briquetting, thermal conversion, and composite

reinforcement. It is strongly influenced by the arrangement of lignin, cellulose, and hemicellulose within the lignocellulosic matrix [5, 6] Nwagwu, (2021); Singh & Heldman, (2014). Small cashew nut shell accessions also exhibit structural differences such as smaller thickness, surface-area-to-volume ratio, and variation in CNSL content, which influence thermal response and mechanical strength [7, 8] Adeleke *et al.*, {2022}; Zhang *et al.*, (2019).

Models provide tools that represent physical systems and enable quantitative predictions. In engineering, modeling assists in predicting future system behavior from initial and boundary conditions [9] Ndirika, (2019). The Buckingham Pi theorem, widely used in agricultural and food engineering, provides a systematic framework for dimensional analysis, and has been applied to a wide range of thermal and mechanical modeling problems [10, 11] Barenblatt, (2021); Hunter, (2009).

2. MATERIALS AND METHODS

2.1 Sample Collection

Small cashew nut accessions were collected from Cocoa Research Institute of Nigeria (CRIN), Ibadan germplasm. The nuts were sorted out in order of their weights and sizes in agronomy department of CRIN, the nuts were manually split into two equal parts with a manual cashew nut splitter in value addition department of CRIN in order to remove the kernel from the shells. Small cashew nut accessions were collected from the Cocoa Research Institute of Nigeria (CRIN), Ibadan germplasm. Similar procedures for cashew sample preparation have been documented in previous works [12, 13] Adeleke *et al.*, (2014); Oladipo, (2011) as shown in plate 1.



Plate 1: Drying of Cash Nut Shell

2.2 Thermal Treatment

Samples were thermally treated at 200°C, 250°C, and 300°C for 30, 60, and 90 minutes, following procedures consistent with thermal modification studies of lignocellulosic shells [14, 15] Barth-Plange *et al.*, (2012); Singh & Heldman, (2014).

2.3 Determination of Mechanical Properties of Cashew Nut Shells

Hardness and compressive strength were measured using a universal testing machine (UTM) following standard agricultural engineering practice [15, 11] Akinmoladun *et al.*, (2019); Hunter, (2009) as shown in Plate 2. The UTM applied a gradually increasing compressive load, the machine records the maximum force applied before the sample failed, and the compressive strength was recorded as force at peak while the hardness was recorded as force at break the results were displayed on a digital control panel (Desktop computer).



Plate 2: UTM

2.3.1 Determination of hardness

Hardness of cashew nut shell is the property of the material that measures resistance to external force before deformation. The hardness of cashew nut shell was determined using UTM fitted with compression platen and was recorded as force at breaking point. Samples were positioned such that compression force was applied perpendicular to the shell surface as shown in plate 3.



Plate 3: Determination of the Hardness of Cashew Nut Shell

2.3.2 Determination of compressive strength

The compressive strength is the maximum compressive stress a material can withstand before failure; the compressive strength was determined using UTM fitted with compression platen. Samples were positioned such that compression force was applied perpendicular to the shell surface as shown in plate 4.



Plate 4: Determination of compressive strength of cashew nut shell

2.4 Determination of Physical Properties of Cashew Nut Shell

Density, was determined following standard methods [16, 8] Callister *et al.*, (2018); Zhang *et al.*, (2019).

2.4.1 Determination of density of cashew nut shell

The densities of cashew nut shells were determined using a standard formula equation 1 [16] Callister *et al.*, (2018), and the Volume of the cashew nut shells were determine using the displacement method.

$$\rho = \frac{M}{V} \quad (1)$$

Where:

ρ is Density (g/cm^3)

M is Mass (g)

V is Volume (cm^3)

2.5 Determination of Thermal Properties of Cashew Nut Shell

Thermal properties of cashew nut shells are important for understanding their behavior during any thermal treatment. Thermal conductivity and specific heat capacity were computed using validated empirical equations [14, 15] Bart-Plange *et al.*, (2012); Singh & Heldman, (2014). The thermal properties that will be considered include; thermal conductivity, specific heat capacity.

2.5.1 Determination of thermal conductivity of cashew nut shell

Thermal conductivity is defined as the rate at which a product conducts heat, and the thermal conductivity was calculated using equation 4 and 5. A probe of length 25mm was dipped into the mass of the cashew nut shells with a steady current of 1A and a varying voltage of 2.04v to 6.68v respectively, the variation in voltage was due to temperature difference and increase in load. Thermal conductivity was

determined using equations 2 and 3 [14] (Bart-Plange *et al.*,2012)

$$k = \frac{Q \ln(t_1/t_2)}{4\pi(T_2-T_1)} \quad (2)$$

k is the thermal conductivity (W/m.K)

Q is the heat input rate per unit length

$$Q = \frac{VI}{L} \quad (3)$$

V is the electrical voltage (v)

I is the electric current (A)

L is the probe Length (m)

$\ln \frac{t_1}{t_2}$ Logarithm of time ratio

t_1 and t_2 is initial time and final time

T_1, T_2 is the final and initial temperature

2.6 Development of Predictive Model Using Buckingham Pi Theorem

Models are devices that mirror nature by embodying empirical knowledge in forms that permit quantitative inferences to be derived from them. It is also a device that predicts the future state of a system from initial conditions, boundary conditions and a set of rules [9] Ndirika, (2019). Dimensional analysis followed the classical Buckingham Pi methodology as described by Barenblatt [10] (2021) and Hunter [20] (2009). A fundamental system of units of the variables (M, L, T) in mechanics and quantity that is measured in respect to the system Hunter, [11] (2009) as shown in Table 1 and 2.

Table 1: Dimensions of the variables affected during heating

Variables	Symbol	Dimensions (M, L, T)
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Stress	σ	$ML^{-1}T^{-2}$
Hardness	H	$ML^{-1}T^{-2}$
Density	ρ	ML^{-3}
Thermal Conductivity	k	$MLT^{-3}\theta^{-1}$
Specific Heat Capacity	C_p	$L^2T^{-2}\theta^{-1}$
Temperature	T	θ
Time	t	T

Table 2: Dimensional matrix of the variables

Dimensions	Parameters						
	σ	H	ρ	k	C_p	T	θ
M	1	1	1	1	0	0	0
L	-1	-1	-3	1	2	0	0
T	-2	-2	0	-3	-2	1	0
θ	0	0	0	-1	-1	0	1

2.6.1 Identifying variables

- A. Independent variables:
 - i. Temperature (T),
 - ii. Duration (t)
- B. Dependent variable:
 - i. Compressive strength
 - ii. Hardness
 - iii. Density
 - iv. Thermal conductivity
 - v. Specific heat capacity

2.6.2 Dimensional analysis

The variables were express in terms of fundamental dimensions (M, L, T, θ). Table 1 represents the summary of dimensions of the variables considered and Table 2 represents the dimensional matrix of the fundamental variables.

2.6.3 Constructing Pi Terms

Dimensionless groups will be derived using the relationship: $\pi_1=f(\pi_2,\pi_3, \dots \pi_n)$ to obtain the pi values. Applying Buckingham Pi theorem to identify the dimensionless group to be formed:

Total number of variables (n) = 7

Number of fundamental dimensions (m) = 4

$$\pi_{terms} = n - m \rightarrow \pi_{terms} = 7 - 4 = 3$$

Assuming the variables influenced by heat treatment for the desired duration of heating (Temperature and time) are Compressive strength(σ), Hardness(H), Density (ρ), Thermal conductivity(K), Heat capacity(C_p) then,

$$\theta, T \propto f(\sigma, H, \rho, K, C_p)$$

$\pi_1 = \phi f(\pi_2, \pi_3)$ With regards to the total number of Pi groups to be formed

$$\phi = \frac{\pi_1}{(\pi_2, \pi_3)} \quad (4)$$

Where ϕ = constant Temperature

$$\pi_1 = [\theta]^{C1} \cdot [\sigma]^{C2} \cdot [H]^{C3} \cdot [\rho]^{C4} \cdot [K]^{C5} \cdot [C_p]^{C6} \cdot [T]^{C7} \quad (5)$$

To obtain π_1 ,

Assuming an arbitrary values for $C1 = 1, C2 = 0, C3 = 0$ (Ndirika, 2019)

$$\pi_1 = \frac{\theta \rho T C_p^2}{K} \quad (6)$$

To obtain π_2 ,

Assume arbitrary values for $C1 = 0, C2 = 1, C3 = 0$

$$\pi_2 = \frac{\sigma T C_p}{K} \quad (7)$$

To obtain π_3 ,

Assume arbitrary values for $C1 = 0, C2 = 0, C3 = 1$

$$\pi_3 = \frac{HTC_p}{K} \quad (8)$$

From (4),

$$\phi = \frac{\pi_1}{(\pi_2, \pi_3)} \quad (9)$$

Substitute for π_1, π_2, π_3 to obtain the final model

$$\phi = \left[\frac{(\theta \rho T C_p) K^{-1}}{(\sigma T C_p) K^{-1} \times (HTC_p) K^{-1}} \right] \quad (10)$$

$$\theta = \phi \left[\frac{(\sigma T C_p) K^{-1} \cdot (HTC_p) K^{-1}}{(\rho T C_p) K^{-1}} \right] \quad (11)$$

$$H = \phi \left(\frac{\rho \theta K}{T \sigma} \right) \quad (12)$$

These three terms (π_1, π_2, π_3) describe the model of the system considering stress, hardness, density, thermal, conductivity, heat capacity, time and temperature.

2.7 Establishment of the model constant

The generalized reduced gradient (GRG) nonlinear method of excel solver was adopted to established hardness of the cashew nut shells at different combinations of operating conditions in equations (34) and experimental results, respectively. This is similar to Ekemube *et al.*, [17, 18] (2022a, 2022b); Ekemube and Atta [19] (2023) that uses GRG to established constant in their model of fuel consumption during ploughing, harrowing, and ridging operation developed using Buckingham Pi theorem. GRG in its most basic form, this solver method looks at the gradient or slope of the objective function as the input values or decision variables, change and determines that it has reached an optimum solution when the partial derivatives equal to zero.

2.8 Validation of the model

Model performance was evaluated using root mean square error (RMSE) and residual prediction deviation (RPD). Model validation was carried out by predicting hardness (*H*) using developed model and comparing the experimental data with the model results. The prediction ability of the model was evaluated on the bases of the root-mean-square-error (RMSE) equation 12, and residual prediction deviation equation 14.

$$RMSE = \sqrt{\frac{\sum_{i=1}^n (Y_i - X_i)^2}{n}} \quad (13)$$

Where:

n = number of samples

Y_i = predicted *i*th values,

X_i = measured *i*th values

$$RPD = \frac{SD}{RMSE} \quad (14)$$

Where:

SD = standard deviation of the measured experimental data.

Model prediction ability was described as excellent if $RPD > 2.0$, and almost good if $1.4 < RPD < 2.0$, and unreliable if $RPD < 1.4$ Chang *et al.*, (2001)

3. RESULTS AND DISCUSSION

3.1 Development and Validation of the Predicted Model

The predictive model developed using the Buckingham Pi theorem effectively described the interrelationship between temperature, time, and selected engineering properties (Equation 15). The observed low sum of squared errors (SSE = 4.363946) confirmed the models' reliability (Table 3). Root mean square error (RMSE) values was low (0.696335), while residual prediction deviation (RPD) value exceeded 20 for small cashew nut shell accessions, indicating excellent predictive capacity Chang *et al.*, [20] (2001).

The small cashew nut shell accessions exhibited high RPD (21.57619), implying a stable model response. This strong correlation between experimental and predicted values supports the validity of the Buckingham Pi based model and its suitability for extrapolating the effect of thermal treatment on cashew nut shells.

$$H = \phi \left(\frac{\rho \theta K}{T \sigma} \right) \quad (15)$$

Table 3: Result of the Effect of Thermal Treatment on the Selected Engineering Properties of Small Cashew Nut Shell

ϕ	$\rho, g/cm^3$	$\theta, ^\circ C$	$K, W/m.k$	σ, N	T, min	H (Measured) N	H (Predicted) N	Error ²
637.7092	0.934	200	0.137	34	30	16	16	1.26E-29
637.7092	0.732	200	0.124	38	60	40	40.94765	0.898034
637.7092	0.4	200	0.117	18	90	31	31.49181	0.241882
637.7092	1.25	250	0.127	157	30	41	42.31086	1.718348
637.7092	0.85	250	0.113	41	60	55	55.08667	0.007512
637.7092	1.4	250	0.107	39	90	63	63.58924	0.347205

637.7092	0.566	300	0.117	55	30	65	65.62608	0.391975
637.7092	0.75	300	0.107	45	60	54	53.14244	0.735414
637.7092	0.7	300	0.105	39	90	38	38.15354	0.023576

SSE = 4.363946

4. CONCLUSION

This study of modeling the effect of thermal treatment on hardness of small cashew nut shells has been carried out using Buckingham Pi theorem in Dimensional analysis. The predictive model based on Buckingham Pi theorem accurately described these interactions, validated by high RPD and low RMSE values. Consequently, thermally treated small cashew nut shells can serve as viable raw materials for sustainable energy production, composite reinforcement, and thermal insulation in engineering applications.

5. ACKNOWLEDGMENT (HEADING 5)

The preferred spelling of the word “acknowledgment” in America is without an “e” after the “g.” Avoid the stilted expression “one of us (R. B. G.) thanks .”. Instead, try “R. B. G. thanks.”. Put sponsor acknowledgments in the unnumbered footnote on the first page.

6. REFERENCES

The template will number citations consecutively within
 braEason, G., Noble, B., & Sneddon, I. N. (1995). On certain integrals of Lipschitz-Hankel type involving products of Bessel functions, *Phil. Trans. Roy. Soc. London*, vol. A247, pp. 529-551.

[1] Kadjo, A., Bello, R., & Adebisi, M. (2024). Influence of thermal modification on cashew shell properties. *West African Journal of Engineering*, vol. 41, No. 1, pp. 77 – 91.

[2] Adeleke, B. S., Olatunji, O. O., & Akinoso, R. (2020). Mechanical and biochemical characteristics of cashew nut sizes. *Journal of Food Engineering*, vol. 275, pp. 109–123.

[3] Kuppusamy, P. (2014). Structural analysis of cashew nut shell and its thermal transformation. *International Journal of Biomass Research*, vol. 9, No. 2, pp. 87–99.

[4] Fuwat, M. (2023). Heat treatment effects on lignocellulosic agricultural residues. *Journal of Renewable Biomaterials*, 4(2), 67–79

[5] Nwagwu, A. C. (2021). Mechanical behavior of thermally treated cashew shells. *Journal of Materials in Agriculture*, 15(3), 118–130.

[6] Singh, R. P., & Heldman, D. R. (2014). *Introduction to food engineering* (5th ed.). Elsevier.

[7] Adeleke, B. S., Ibrahim, J., & Olawale, S. (2022). Effect of heat treatment on engineering properties of cashew nutshells. *Journal of Agricultural Technology*, vol. 18, No. 3, pp. 112–129.

[8] Zhang, Y., Wang, L., & Xu, H. (2019). Shrinkage behavior of thermally processed biomass. *BioResources*, vol. 14 No. 1, pp. 215–229.

[9] Ndirika, V. I. (2019). *Fundamentals of modeling in agricultural engineering*. University of Nigeria Press.

[10] Barenblatt, G. I. (2021). *Scaling, self-similarity, and intermediate asymptotics*. Cambridge University

[11] Hunter, J. S. (2009). Dimensional analysis in engineering. *Engineering Science Journal*, vol. 12, pp. 9 – 16.

[12] Adeleke, B. S., Adekunle, A. A., & Ogunmola, O. O. (2014). Physical and mechanical characterization of cashew nut shell. *Nigerian Journal of Agricultural Engineering*, vol. 45, No. 2, pp. 55–62.

[13] Oladipo, F. O. (2011). Drying characteristics of cashew shells. *Nigerian Food Journal*, vol. 29, No. 1, pp. 62–70.

[14] Barth-Plange, L., Dzisi, K. A., & Ofori, E. (2012). Determination of thermal properties of agricultural materials. *Journal of Food Processing & Engineering*, vol. 35, No. No. 4, pp. 515–525.

[15] Akinmoladun, F. O., Ojediran, J. O., & Olayanju, T. M. (2019). Evaluation of moisture-dependent physical properties of agricultural biomass. *Agricultural Engineering International*, vol. 21, No. 4, pp. 45–57.

[16] Callister, W. D., & Rethwisch, D. G. (2018). *Materials science and engineering: An introduction* (10th ed.). Wiley.

[17] Ekemube, R. A., Atta, A. T., & Okogbule-Wonodi, A. (2022). The Use of Dimensional Analysis for Modeling Tractor Fuel Consumption for Harrowing Operation. *Nigerian Journal of Technology (NIJOTECH)*, vol. 41, No. 5, pp. 913 – 919.

[18] Ekemube, R. A., Atta, A. T., Nkakini, S. O., & Amgbara, T. O. (2022). Mathematical modeling of tractor fuel consumption for ridging operation in sandy loamy soil. *Nigerian Research Journal of Engineering and Environmental Sciences*, vol.7, No. 1, pp. 309 – 318.

[19] Ekemube, R. A., & Atta, A. T. (2023). Application of Buckingham Pi Theorem in development of tractor fuel consumption model for ploughing operation. *Nigerian Journal of Agricultural Mechanization (AGRIMECH)*, vol. 3, pp. 57 – 70.

- [20] 20 Chang, C. W. Laid, D. A. Mausbach, M. J. Hurburgh, C. R., Jr. (2001). Near-infrared reflectance spectroscopy-principal components regression analysis of soil properties. Soil Science Society of American Journal, vol. 65, No. 2, pp. 480 – 490.