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Periodic points and sensitivity of initial conditions in the dissection of the dynamics of complex systems (Review)

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Abstract: *Two interconnected components of comprehending the dynamics of complex systems, from mathematical models to natural phenomena, are periodicity and sensitivity to initial conditions. The heart's cycle or planetary orbits are examples of periodic points that show recurring and predictable behavior within the system, indicating hidden regularities amidst complexity. Our capacity to accurately predict even mathematically specified models is limited by sensitivity to initial conditions, which is exemplified by the "butterfly effect," which shows that even minor changes in initial conditions can result in significant variations in long-term outcomes.*

Keyword: sensitivity, dynamics

Introduction

In the past few decades, chaos theory the science that describes how dynamical systems behave has attracted a lot of attention. Chaos theory has applications in a number of scientific domains, including biology, astronomy, population, economics, and metrology. In the 1880s, Henri Poincaré's study of the n-body problem included references to the first instances of chaotic behavior. Determinism and randomness are somewhat harmonious, according to Poincaré, because of long-term unpredictability. Poincare discovered that

We can make precise predictions about the future state of the universe if we know the precise laws governing it and its initial state of truth. However, even a tiny mistake in estimating the initial state can result in random events and make the universe's future state uncertain. This was the first time that sensitivity to initial conditions was observed. According to some, Edward Lorentz was the first to discover chaotic behavior.

chaos theory as well. He initially came across the phenomenon in 1961 while conducting experiments and computations on weather forecasting using nonlinear dynamical models. In his era, mathematicians were aware that slight changes in computations lead to slight variations in the outcomes. What transpired was that he used 6-digit numbers in the first experiment trial and 3-digit numbers in the second, which did not yield the same answers. It was Lorentz's work that created the phrase "Buttery effect," which refers to the appearance of buttery wings today[5] may eventually result in a storm. The first use of the term chaos was in the mathematician's paper. The work "Period Three Implies Chaos" by James A. Yorke. Yorke outlined the and demonstrated that for a continuous map f on a closed interval, I is a chaotic function if there is a periodic point in I of period 3.The logistic map was first presented in 1976 by biologist Robert M. May as a basic equation with intricate dynamics. Feigenbaum Mitchell Jay made use of the logistic diagram to illustrate how regular dynamics give way to chaos.

1.predic point

Definition(1.1)[3][4]:- In the phase space of a dynamical system, a periodic point is a special case where the system repeats the same state after a predetermined number of time iterations. In mathematics, a point x is referred to as a periodic point with period n if it meets the following criteria, assuming that the function f represents the system's evolution (i.e., its transformation over time):

$F^n(x)=x$

Where:(x) indicates that the point x has been subjected to the function f^n times in succession. This equation's fundamental period, denoted by n, is the smallest positive integer that satisfies it.

1.2 Periodic Points: the Regular Structure With in Chaos

A system state that returns to its initial state after a predetermined number of steps (iterations) is known as a periodic point. A point x is said to be periodic with a periodic time n in mathematics if it satisfies

where f is the defining function of the evolution of the system, and f(x)=x.

Similar to a pendulum that swings back to its starting position at regular intervals, they symbolize "recurring" or "stable" states in the system. Periodic points are present even in chaotic systems, but they are frequently unstable (near-solutions are ejected from them).By stabilizing these points, they create an ordered structure within chaos that aids in the analysis of systems' controllability (such as the OGY method of controlling chaos).

Example(1.2.1): The phenomenon of period-doubling of the bifurcations causes periodic points to appear in a logistic map, and as a parameter increases, this results in chaotic behavior.

Definition(1.2.2) [2]:- If slight variations in the system's initial state result in radically different outcomes over time, then the dynamical system is sensitive to initial conditions. If there is a positive constant δ>0 such that for any point x in the state space and any arbitrarily small neighborhood around x, there exists another point y in the neighborhood and a time t such that: formally, a system is said to be sensitive to initial conditions.

$|f^t (x) – f^t(y)| > \delta$

In chaotic systems, even minute changes in the initial state can result in radically different trajectories, a phenomenon known as the "butterfly effect" that makes long-term prediction impossible.

2.Sensitivity: The random seed in a given system.

Sensitivity is defined as the sensitive dependence on initial conditions, so that over time, a small change in the inputs causes a large change in the results. Devaney defines it mathematically as follows: for every point x and every neighborhood surrounding it, there exists a point y in that neighborhood whose trajectories, after a finite number of steps, diverge from x by ϵ.

2.1 Importance

In spite of the system's determinism, the "butterfly effect" in chaotic systems makes long-term predictions impossible. Lyapunov exponents, which calculate the rate of trajectory divergence, are frequently used to measure it.

For instance, in a Lorenz system, a slight variation in the starting temperature results in drastically different climate paths.

2.2.The Integrative Role in Chaotic Systems

Three characteristics must exist for chaos to be defined by Devinney:

1. Exponential path divergence is sensitivity.

2. The existence of dense periodic points, or periodic points in every neighbourhood, is known as periodic point density.

3. The existence of a dense path that crosses any area of space is known as topological transitivity.

2.3 The interaction between the two concepts

Dense periodic points offer an unstable "regular structure," and randomness is introduced by sensitivity.

Because order is both deterministic and unpredictable, with hidden order beneath the surface of randomness, this combination produces a special balance between chaos and order.

2.4 Bifurcations

Changes in system parameters (such as the growth coefficient in a logistic map) lead to the appearance or disappearance of periodic points, or changes in their stability.

Importance: Links quantitative changes in parameters with qualitative transformations in system behavior (such as the transition from stability to chaos via cyclic multiplication).

Example(2.4.1): Hopf bifurcation in nonlinear systems

Bifurcation in the heart's electrical system, which results from a change in the conduction coefficient, can cause an arrhythmia..

2.5. Chaotic attractors.

Unstable periodic points and susceptibility combine to form these attractors, which result in intricate yet distinct pathways.

2.6 Applications in Real-World Systems

1. Ecological Models: Population models, such as the logistic map, show steady population cycles with periodic points, and abrupt and unexpected population booms or crashes can be explained by sensitivity to initial conditions It is also applied to the dynamics of asteroids and how sensitivity affects their paths.

2. Climate Science: Long-term weather forecasting is notoriously challenging because climate models frequently exhibit sensitivity to initial conditions. Seasons and cycles like El Niño are examples of periodic phenomena that can be viewed as periodic points in the system.

3. Financial Markets: Although markets can display cyclical patterns, such as business cycles, they are also extremely sensitive to initial circumstances, which means that even minor adjustments to outside variables can cause significant and erratic changes in the markets.

Conclusion

. Two essential concepts for comprehending chaotic systems are periodicity and sensitivity, which highlight the fundamental conflict between apparent randomness and mathematical determinism. Periodicity gives the system a structure that can be studied, while sensitivity limits our knowledge of the trajectory Temporal, which together makes them key to decoding the complexity in nature..

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