International Journal of Academic and Applied Research (IJAAR)
ISSN: 2643-9603
Vol. 9 Issue 3 March - 2025, Pages: 74-79

A Review of Weighted Statistics Distributions

Nahla Hadi Abdul Sahib

University of Babylon, College of Engineering Al-Musayab
Department of Automobiles Engineering
nahla.hadiabdul@uobabylon.edu.iq

Abstract: *Weighted statistical distributions and the most significant derivative characteristics were reviewed in this report along with earlier studies that addressed the topic.*

Keyword: weighted statistical distributions, pdf, CDF.

Introduction

Weight in statistical distributions is the value given to a specific variable in the group of variables, so weights are of great importance in representing variables in statistical accounts. for a more accurate depiction of data.

In science and scientific disciplines, we can use balanced statistical distributions to show the importance of data that are more valuable than others, and among these sciences are analysis of surveys, economics, and others, and thus obtaining a more accurate representation of the results.

- **Some Paper on weighted statistic Distributions**
  The topic of weighted statistical distributions has been covered by a large number of researchers due to the importance of the topic. We can discuss most of them as follows:
  1. In (2020) Mahmood A. Shamran introduced Modified Weighted Pareto Distribution Type I by used the Azzallini's method, and derived properties of new distribution, the pdf of new distribution is:

$$f(x) = \frac{2\alpha\theta^{\alpha}k^{\alpha}x^{-\alpha-1}\left(1-\frac{k^{\alpha}}{\theta^{\alpha}k^{\alpha}}\right)}{(2\theta^{\alpha}-1)}$$

And the CDF of this distribution is:

$$F(x) = \frac{(\theta^{\alpha}x^{\alpha}-k^{\alpha})^2}{\theta^{2\alpha}x^{2\alpha}}, x \geq \frac{k}{\theta} >, \alpha > 0, \theta \geq 1$$

The Azzallini's method law is:

$$f(x) = \frac{1}{p_r(x_2 < \theta x_1)} f(x_1)F(\theta x_1)$$

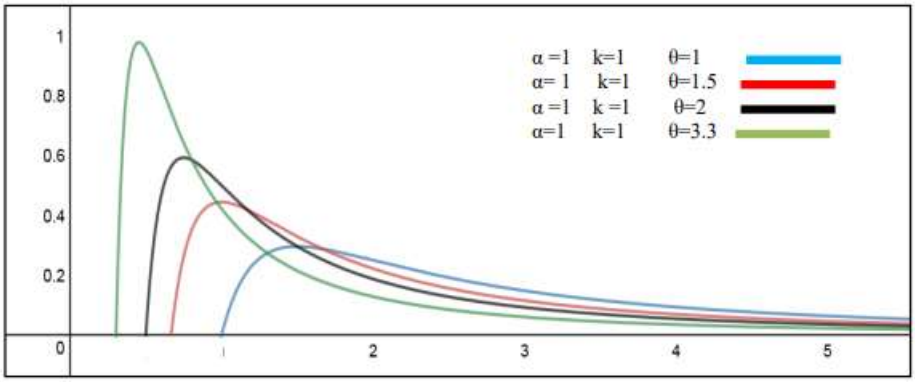


Fig. (1): The pdf of Modified Weighted Pareto Distribution Type I.

2. In (2020) Brijesh P., and Utpal Dhar introduced "The weighted exponential distribution" is mainly used in various real life fields such as ecology, reliability engineering, medical science etc, by the weighted distribution is defined as:

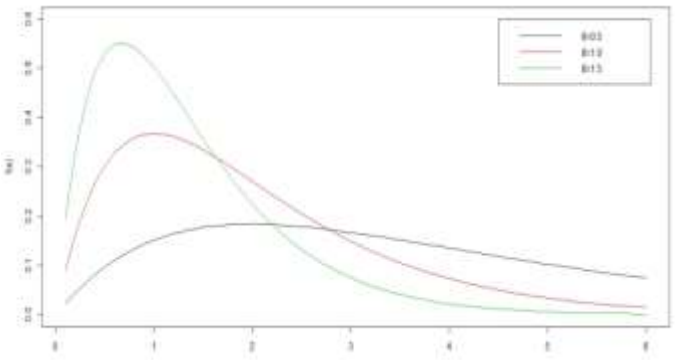
$$f(x) = \frac{w(x)f^*(x)}{E[w(x)]}; x \in R, \theta > 0$$

The pdf of new distribution is:

$$f(x) = (\lambda - w)e^{-(\lambda - w)x}; x > 0, \lambda > 0, \lambda > w, 0 < \lambda < 1$$

And the CDF for the new distribution:

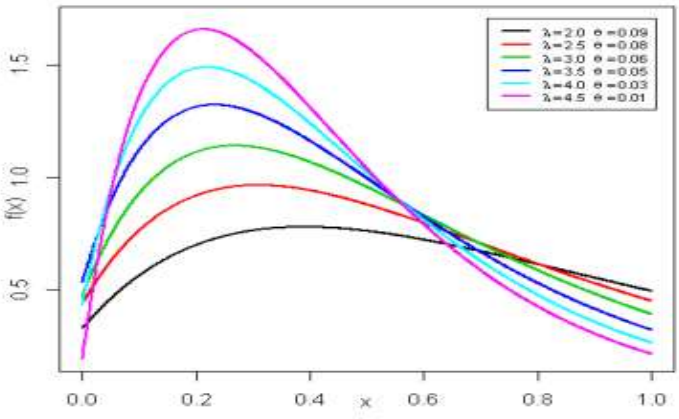
$$F(x) = 1 - (1 + \theta x)e^{-\theta x}$$



Fig(2): The pdf of Weighted exponential distribution.

3. In (2022) Tamer S. Helal et al suggested new distribution about Weighted Shanker Distribution, the pdf and CDF for this distribution are:

$$f(x) = \frac{(\lambda + \theta)^2}{\theta^2 + \lambda\theta + 1}(\theta + x)e^{-(\lambda+\theta)x},$$
$$F(x) = 1 - \left(1 + \frac{(\lambda + \theta)x}{\theta^2 + \lambda\theta + 1}\right)e^{-(\lambda+\theta)x}$$



Fig(3): The pdf of Weighted Shanker Distribution.

4. In (2022) Maryam Mohiuddin et al proposed new distribution are called the weighted Amarendra distribution, in this paper applied new distribution for two data set, and we derive some properties, the pdf for new distribution is:

$$f(x) = \frac{\theta^{c+4}}{\theta^3 c! + 9c + 1)!\,\theta^2 + (c+2)!\,\theta + (c+3)!} x^c (1 + x + x^2 + x^3) e^{-\theta x}; x > 0, \theta > 0, c > 0$$

And the CDF for same distribution is:

$$F(x) = \frac{\theta^3 \gamma(c+1;\theta x) + \left(\frac{1}{\theta}\right)\gamma(c+2;\theta x) + \left(\frac{1}{\theta^2}\right)\gamma(c+3;\theta x) + \left(\frac{1}{\theta^3}\right)\gamma(c+4;\theta x)}{\theta^3 c! + \theta^2 (c+1)! + (c+)!\,\theta + (c+3)!}$$

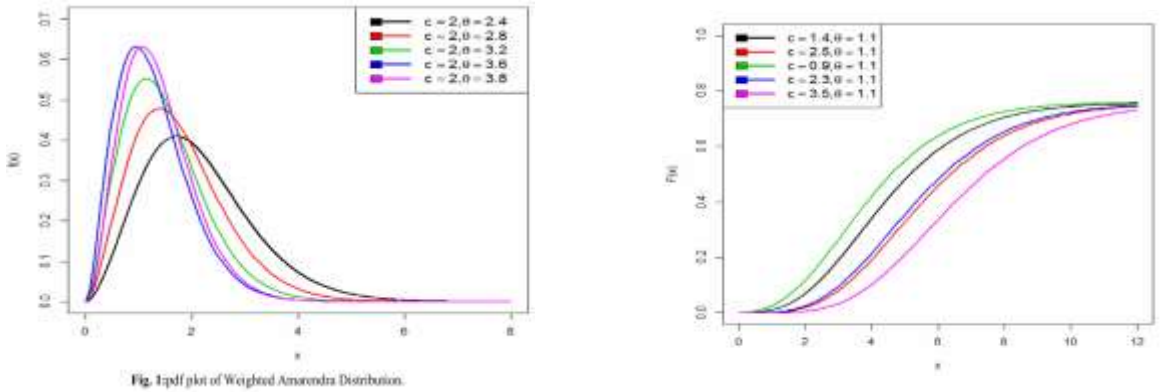


Fig. 1:pdf plot of Weighted Amarendra Distribution.

5. In (2019) Rama Shanker et al., included new distribution called weighted Quasi Lindley Distribution by added parameter for pdf become 4 parameters, the pdf for the new distribution is:

$$f(x) = \frac{\theta^{\beta}}{(\alpha + \beta)\Gamma(\beta)} (\alpha + \theta x) e^{-\theta x}; x > 0, \theta > 0, \alpha > -1, \beta > 0$$

The cdf for new distribution is:

$$F(x) = 1 - \frac{(\alpha + \beta)\Gamma(\beta, \theta x) + (\theta x)^{\beta} e^{-\theta x}}{(\alpha + \beta)\Gamma(\beta)}$$

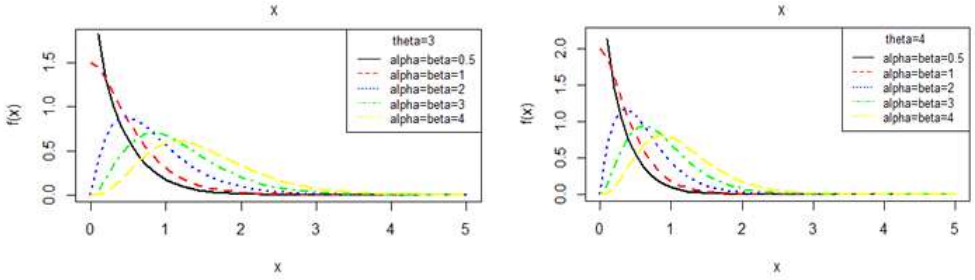


Figure 1. Behaviour of the pdf of WQLD for varying values of parameters

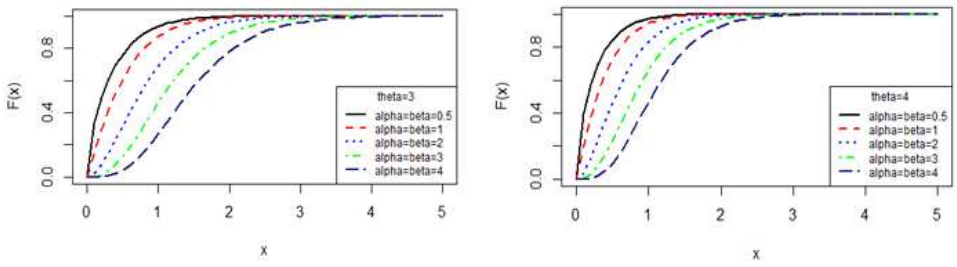


Figure 2. Behaviour pf the cdf of WQLD for varying values of parameters

6. In (2018) Aamir Saghir and G.G. Hamedani proposed A Generalized Weighted Maxwell Distribution (GWMD), we application of this distribution in real life phenomenon, the pdf for new distribution is:

$$f(x) = \frac{x^{m+2} e^{-\frac{x^2}{2\alpha^2}}}{2^{\frac{(m+2)}{2}} \alpha^{m+3} \Gamma\left(\frac{3+m}{2}\right)}, x > 0$$

And the cdf for new distribution is:

$$F(x) = \frac{\gamma\left(\frac{(m+3)}{2}, \frac{x^2}{2\alpha^2}\right)}{\Gamma\left(\frac{3+m}{2}\right)}, x \geq 0$$

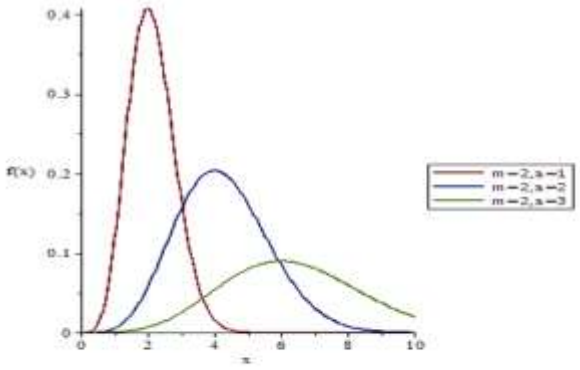


Figure 1: The Probability Density Function of GWMD for Different Values of α at $m = 2$.

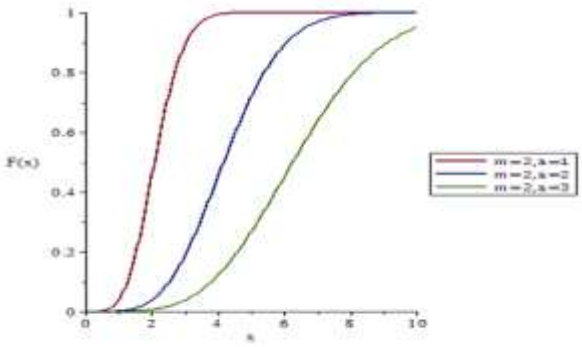


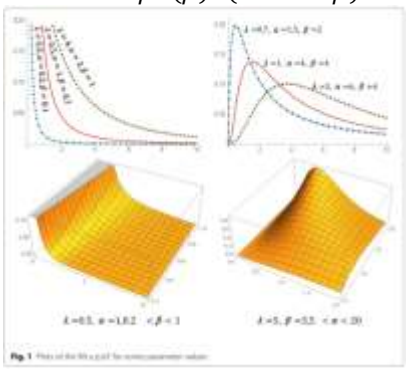
Figure 2: The Cumulative Distribution Function of GWMD for Different Values of α at $m = 2$.

7. In (2016) N.M.Kilany suggested the Weighted Lomax distribution (Pareto Type-II), the pdf of new distribution is:

$$f(x) = \frac{\Gamma(\alpha+1)\lambda^{1+\alpha-\beta}}{\Gamma(1+\alpha-\beta)\Gamma(\beta)}\left(\frac{x^{\beta-1}}{(x+\lambda)^{\alpha+1}}\right), x \geq 0, \alpha > 0, 0 < \beta < \alpha + 1$$

And the cdf for this distribution is:

$$F(x) = \frac{\Gamma(\alpha+1)\lambda^{-\beta}x^{\beta} \times 2F1\left(\alpha+1, \beta, +1; -\frac{x}{\lambda}\right)}{\beta\Gamma(\beta)\Gamma(1+\alpha-\beta)}$$



8. in (2024) P.Pandiyan and M. Sakthivel suggested the weighted Loai distribution by used form :

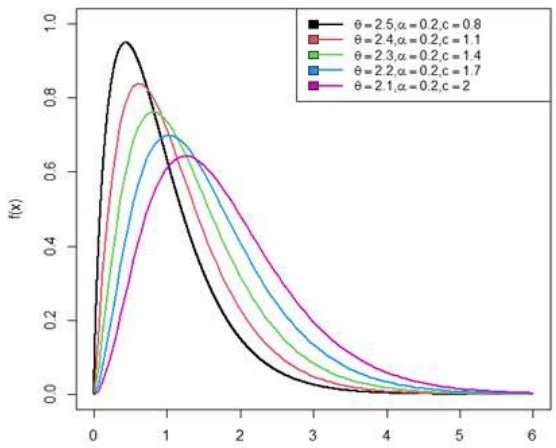
$$f(x) = \frac{w(x)f(x)}{w}, x > 0$$

The pdf of new distribution is:

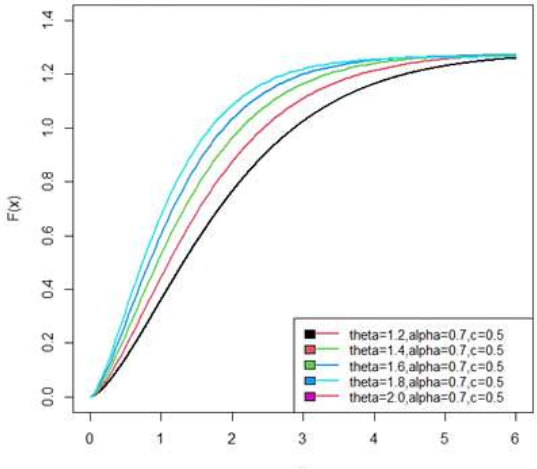
$$f(x) = \frac{\theta^{c+2}}{\left(\frac{\alpha(c+2)!}{2} + \frac{(\theta c1 + (c+1)!)}{\theta+1}\right)} x^c \left(\frac{1}{2}\alpha\theta x^2 + \frac{1}{\theta+1}(1+x)\right) e^{-\theta x}$$

The CDF for new distribution is :

$$F(x) = \frac{\left(\frac{\alpha}{2}\gamma(c+3,\theta x) + \frac{1}{\theta+1}(\theta\gamma(c+1,\theta x) + \gamma(c+2,\theta x)\right)}{\left(\frac{\alpha(c+2)!}{2} + \frac{(\theta c! + (c+1)!)}{\theta+1}\right)}$$



Figures.1:Pdf plot of Weighted Loai distribution



Figures.2 Cdf plot of Weighted Loai distribution

And there are reports that study weighted statistical distributions, which cannot all be covered in this report.

The important characteristics for weighted statistical distributions:

- Flexibility in Modeling.
- Improved Accuracy of Estimates.
- Handling Heterogeneous Data.
- Applications in Multiple Fields.
- Enhancing the power of Statistical tests.
- Customizing Relative Importance.

Conclusion

More accurate or representative data can be given more weight when using the right weights, which will improve the quality of statistical results.

References

1. Mahmood A. Shamran, (2020), "Modified Weighted Pareto Distribution Type I (MWPDTI)", Baghdad Science Journal, Vol. 17(3):869-877.

2. Brijesh P. Singh1 and Utpal Dhar Das, (2020), "Some Statistical Properties of a Weighted Distribution and Its Application to Waiting Time Data", Asian Journal of Probability and Statistics, Vol.9(1), 1-12.

3. Tamer S. Helal, (2022), "Statistical Properties of Weighted Shanker Distribution", Journal of Business and Environmental Sciences, Vol. 1 (1), 141-153.

4. Maryam M. et al., (2022), “Weighted Amarendra Distribution: Properties ana Applications to Model Real-Life Data”, Journal of Statistics Applications & Probability, No. 3, 1075-1093.

5. Rama S. et al., (2019), “Weighted Quasi Lindley Distribution with Properties and Applications”, International Journal of Statistics and Applications, 9(1): 8-20.

6. Aamir S. and G. G. Hamedani, (2018), “A GENERALIZED WEIGHTED MAXWELL DISTRIBUTION: PROPERTIES AND ESTIMATION”, Pak. J. Statist., Vol. 34(3), 195-209.

7. N.M.Kilany, (2016), “Weighted Lomax Distribution”, Kilany SpringerPlus, Vol.5: 1862.

8. P. Pandiyan, and M. Sakthivel, (2024), “A Stochastic Model for Weighted Loai Distribution with Properties and Its Applications”, International Journal of Mathematics and Statistics Invention (IJMSI), India, Vol. 12, Issue 1, 11-32.