

Review about the Chaos Theory

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Abstract: *Chaos theory redefines our definition of "randomness" and "predictability," revealing that the world is not simply a struggle between order and chaos, but rather a complex interaction between the two. One of the most important theories of the 20th century, it started out as a branch of mathematics but has since spread to almost every scientific discipline. In the field of chaos theory, nonlinear dynamical systems that appear random or irregular but are actually subject to precise mathematical laws are studied. Despite being deterministic—that is, their behavior can be theoretically predicted if their initial conditions are known—these systems are highly sensitive to those conditions, making them unpredictable in the long run. Another name for this sensitivity is the "butterfly effect".*

Keyword: randomness, predictability.

Introduction

Khaos, which means "gaping void" in Greek, is where the word chaos originates. According to mathematicians, while defining chaos is challenging, it is simple to "see it when you see it." To put it another way, chaos is a state in which a complex natural system behaves in a completely unpredictable or confused manner. According to the idea of chaos theory (Devaney 1989), One of the most recent theories in mathematics is chaos theory, which is not older than several decades, which deals with the non-linear motor system that shows a kind of chaotic behavior, and that this behavior is either through the inability to determine. The initial conditions or physical nature show us that the hidden system in this apparent chaos lays the foundations for studying weather forecasting and overpopulation systems. Chaos theory is the theory that the newest, however trivial and small, a tiny Change today can have a huge impact later (Morse 1967). Its central claim is that even small changes in initial conditions, such as rounding errors in numerical calculations, can produce radically different results for chaotic systems, rendering long-term predictions nearly impossible. This paper is intended to serve as a useful resource for anyone who is interested in learning more about this topic. In our day-to-day lives, chaos theory is among the most significant theories. It is the study of complex, nonlinear, dynamic systems. Mathematical study of systems that exhibit chaotic behaviors despite their apparent order (determinism). Additionally, it deals with systems that have an underlying order despite their seeming chaos. Alternatively put, these systems are unpredictable despite being deterministic (Robert 1976). Deterministic chaos, or simply chaos, is the term used to describe this behavior. Given her complexity, it is only natural for nature to be unpredictable. May was developing a model to address the relationship between insect birth rate and food supply in the early 1970s. He discovered that his equation took twice as long to return to its initial state at specific critical values, meaning that the period had doubled. Similar to how real insect populations are often unpredictable, his model became unpredictable after multiple period-doubling cycles. For many different systems, this period-doubling is a natural path to chaos, as mathematicians have discovered since May's discovery with insects.

1. Important concepts in chaos theory:-

1.1 Butterfly Effect:

The concept that a slight alteration in the initial circumstances (like a butterfly's wings flapping) may bring about significant, long-term changes. consequences (like a hurricane in another part of the world).

For instance, in weather models, a slight variation in the starting temperature or humidity can have a significant impact on the weather prediction for the following week.

1.2 Nonlinear Systems:

systems where there is no direct proportionality between inputs and outputs.

For instance, population growth or atmospheric movement in a resource-constrained setting (logistic model).

1.3 Dependency Sensitiveness:

The characteristic of chaotic systems that, in spite of their determinism, renders them unpredictable.

Example: Despite adhering to Newton's laws, the motion of a double pendulum appears random.

1.4 Chaos is not a random event:

There is no randomness in chaotic systems. Instead, they show intricate patterns that can be modeled by strange attractors, which are fractals—repeated geometric shapes—that draw the trajectories of the system in their direction.

2 The theoretical underpinnings of mathematics:

2.1 Equations with nonlinear differentials:

For instance, the Lorenz equation, which explains convection currents in the atmosphere:

$$\frac{dx}{dt} = \sigma(y - x)$$

$$\frac{dy}{dt} = x(\rho - z) - y$$

$$\frac{dz}{dt} = xy - \beta z$$

where σ, ρ and β are constants, and small changes in them lead to chaotic behaviour.

2.2. Logistic Maps:

A basic model that accounts for finite resources and describes population growth:

$$X_{n+1} = rX_n(1 - X_n)$$

When the value of r exceeds a certain threshold (≈ 3.57), the system enters a state of chaos.

2.3 Phase Space:

A dynamical system's state is represented geometrically, with each point in the space denoting the system's state at a specific point in time.

In chaotic systems, the paths of the system create an odd, fractal-structured attractor.

3. Examples of Chaos Theory in Practice

3.1 The climate and weather:

In the 1960s, Edward Lorenz noted that weather forecasts are sensitive to initial conditions, making it difficult to make accurate predictions after two weeks.

3.2 Physics and Engineering:

investigating bridge vibrations or disruptions in flow systems, such as blood flow in arteries.

Financial markets and economics;

examination of abrupt market swings that could be caused by nonlinear interactions between economic variables.

3.3Biology

simulating brain activity or cardiac dynamics, where chaos could account for elements like arrhythmia.

3.4 Cryptography and computer science:

using chaotic algorithms to produce random numbers that are secure in cryptography.

The Intersection of Chaos Philosophy

The Limits of Human Knowledge: According to the theory, some systems are intricate and sensitive, making them unpredictable even when their laws are understood.

The Right Balance Between Chaos and Order: A lot of natural systems, like ecosystems, are in a state of "ordered chaos" that permits adaptation and flexibility.

3.5 Obstacles and Remarks:

Difficulty of Practical Application: Very precise initial data is necessary for the analysis of chaotic systems, but this data is frequently unavailable.

The terms can be confused because some complex systems are not chaotic, and vice versa.

3.6 Chaos, Artificial Intelligence, and Machine Learning

Artificial neural networks are optimized using chaos concepts, which aid in simulating the nonlinear complexity of natural systems.

For example, optimization algorithms such as the modified butterfly effect are used to avoid local optimization during model training.

Data scientists study chaotic systems to uncover hidden patterns in vast amounts of data.

3.7 Chaos in the universe and astrophysics

Complex gravitational interactions can cause chaotic behavior in the motion of planets and other celestial bodies in multi-star systems.

As an illustration, consider the three-body problem in celestial mechanics, which is chaotic and lacks a general solution.

When galaxies collide with one another, they can experience chaotic disturbances as well.

3.8 The Gap Between Chaos and Quantum Theory

The relevance of chaos theory to quantum systems is debatable because the ideas of measurement and determinism are different from those of classical physics.

The study of quantum chaos examines the macroscopic chaotic behavior of quantum systems, such as the distribution of electrons in complex atoms.

3.9 Chaos in Art and Literature

Chaos theory has inspired works of art and literature that reflect the idea of complexity and uncertainty, such as:

Jackson Pollock's paintings, which depict organized chaos through the scattering of colors.

Jorge Luis Borges's novel "The Garden of Forking Paths," which explores the idea of non-linear time.

4.New Mathematical Tools Related to Chaos

4.1 The concept of fractal geometry

studies of shapes that are similar to themselves, like the Mandelbrot set.

Geographic terrain analysis, antenna design, and even X-ray tumor diagnosis are examples of applications.

4.2 The theory of bifurcation

examines how dynamical systems' paths change as parameters (like the logistic map's r parameter) change.

Conclusion

Chaos theory boils down to a fundamental idea: "The world is not completely random, and it cannot be reduced to simple, controllable equations." It forces us to reconsider our notions of determinism and infinity, revealing that even the simplest systems can conceal a complexity that can only be perceived through the interaction of their dynamic components.

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