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Applying Individual Growth Models to Analyze Regional Development Trends: A Multilevel Longitudinal Approach

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Abstract: Individual growth models *are increasingly applied in urban and regional planning as powerful statistical tools to analyze dynamic changes over time. This study aims to present and evaluate individual growth models in their various forms and estimation techniques, and to apply them in assessing spatial development trends across multiple years. The analysis is based on data collected from a random sample of 829 geographic units, where development indicators (such as infrastructure investment or population growth) were tracked over a four-year period (2010/2011 to 2013/2014). Additional contextual variables—such as population density and urban/rural classification—were also included. The dataset represents a two-level longitudinal structure: level one reflects within-unit variation over time, while level two captures differences between units (e.g., neighborhoods, districts, or regions). To explore these dynamics, four growth models were fitted and their parameters estimated: 1. The unconditional means model. 2. The unconditional linear growth model. 3. The conditional linear growth model. 4. The nonlinear growth model. Results from the unconditional means model showed that 28% of the variance in development indicators was attributable to differences between geographic units, highlighting the necessity of considering multilevel data structures in regional planning analysis. Moreover, incorporating time (years) alongside second-level predictors (such as population density and classification) in the conditional linear growth model enhanced model fit, explaining 30% of the variance in average development and 24% of the variance in growth/change rates over time. When comparing the quality of fit of the four growth models using the Model Deviance (D), BIC, and AIC measures, it became clear that the non-linear growth model had the lowest quality of fit, achieving the highest values for all three measures. The quality of fit of the growth model improves with the addition of explanatory variables, whether at the first or second level, that have a significant impact on the dependent variable.*

Keywords: Longitudinal data, individual growth models, multilevel modeling, urban and regional planning

Introduction:

Over the past few years, many statistical models have emerged that are used to measure student and school performance. These include value-added models, which provide a picture of a student's progress in academic achievement compared to the progress achieved by peers of the same ability level and background (McCaffrey et al., 2004).

Hierarchical models, multilevel models, and multilevel models address the multiple levels of educational data, where students represent the first level, classes represent the second level, schools represent the third level, and educational districts represent the fourth level, and so on. The items at any of these levels are considered nested relative to the items at the higher level. These models generally aim to explain change in the dependent variable (student academic performance) using multiple explanatory variables specific to each level of data, as well as partitioning the variance into a component specific to each level (Rasbash et al., 2009).

Individual growth models are considered one of the newly used statistical models in the educational field (Betebenner, 2009) and (Andrew, 2015). They are widely used to study the change that occurs to individuals over time by recording repeated measurements at successive points in time, or what is called longitudinal data, such as recording students' scores on a standardized test in a particular subject with the aim of studying the extent of improvement or decline in students' academic performance, repeated measurements of blood pressure and diabetes in patients with the aim of studying the extent of improvement in the patients' health as a result of treatment with a specific drug, and in weight measurements for some newborns over successive time periods to show the extent to which their weight is appropriate for their age, and in opinion polls on a specific topic at successive points in time in specific geographical areas, etc.

Considering the previous examples, we find that the measurements recorded for individuals through longitudinal data have a nested design within individuals, and we have a data structure with two levels: the first level represents the variation within each individual's measurements within the n, and the second level represents the variation between the individuals themselves i.

A two-level growth model includes several variables that are measured at the first or second levels, such as: gender,

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age, marital status of the individual, educational level of the parent, etc., and other variables that can contribute to increasing the explanatory power of the model.

As noted in longitudinal data, we may have a three- or four-level structure. For example, the results of a number of school students in tests of a single subject over a number of years could represent this situation, where the differences between students' scores represent the first level, the differences between the students themselves represent the second level, and the differences between the students' schools represent the third level. If the students' schools belong to different educational districts, these districts represent the fourth level. Thus, there are many examples of multi-level longitudinal data, the change of which can be studied over time by constructing a multi-level growth model.

Compared to traditional models used in longitudinal data analysis, such as repeated measures ANOVA (Analysis of Variance), individual growth models offer several advantages, including flexibility. They enable researchers to answer a variety of research questions, such as: Is there growth/change in the phenomenon under study? What is the form of this growth? Do individuals grow at different rates? What individual variables/characteristics can be used to predict change in growth parameters? Patterns of change within an individual's repeated periods are often referred to as time paths or growth curves.

The various characteristics that can vary from one individual to another, and these paths may be flat - that is, they do not show any change over time - or they may increase/decrease regularly over time, and they also determine the functional form of the phenomenon’s growth (linear or non-linear). (Bauer, 2007).

Growth models have been used in a variety of ways to study individual change/growth over time. In the educational field, Al-Yalei's study is considered a fundamental reference for explaining how to use individual growth software, multilevel models, and stepwise models. Regarding growth models, the study relied on the grades of a group of students in a single subject over four consecutive years. The steps involved using SAS to fit several conditional and unconditional growth models were demonstrated, the quality of CEP models was compared, and several statistical hypotheses were tested. Regarding other models, the study relied on two-level data: the student represented the first level and the school represented the second level. Several two-level models were fitted, and the effect of adding first- and second- level variables on model parameter estimation and the quality of fit was demonstrated.

In a study (Goldschmidt et al., 2012), the Gain Score Model was discussed. This model is considered a form of growth modeling, and does not address multiple levels of data. Each student's previous score on a test in a particular subject is subtracted from their current score on another test in the same subject to obtain a gain score. The tests must be standardized and the scores used are standard scores. The average gain for the school in that subject is calculated by adding the total gain scores and dividing the result by the number of students. The researchers also compared several different growth models, which become increasingly complex with multiple levels of data and the addition of multiple variables to each level.

In the field of physical health, Chen and Cohen (2006) used linear growth models to study the change in quality of life (QL) data over time. The change in initial health from adolescence to adulthood was analyzed as one aspect of quality of life. The study relied on longitudinal data taken from physical health measurements for 572 individuals born between 1965 and 1975. The sample members were interviewed three consecutive times: in the period from 1985 to 1986, from 1991 to 1994, and from 2004 to 2002. An unconditional and then a two-level conditional growth model was constructed. After that, several explanatory variables were added to the model. The researchers used the SAS PROC MIXED program to fit these models and discuss the results.

In the field of child psychology, Delucia and Pitts (2006) used individual growth models to analyze longitudinal data in several applications in this field. This study aimed to study the expected effects of Spina Bifida on child and family development. The study relied on data from a random sample of 135 children with an average age (at the beginning of the study) of 8.84 years. Sample members were interviewed three consecutive times, approximately two years apart. SAS PROC MIXED was used to reconcile the proposed growth models and compare the results of those models. The study concluded that the linear growth curves of emotional autonomy change as a function of the child's gender, the degree of Spina Bifida, and their interaction with other child characteristics.

Unconditional Linear Growth Model:

A growth model consists of fixed and random effects, which together best describe an individual's change trajectories over time. Fixed effects express the average change trajectories for all individuals within the sample under study. Random effects express the variance of an individual's trajectories around the sample means. A growth/change study usually begins with a two-level unconditional linear growth model. The first level is expressed through a linear growth model containing time as the sole explanatory variable for the first level. The second level model expresses the parameters of the growth model as random effects unrelated to the variables measured at the individual level. The first-level parameters will be expressed usingπ, and the second-level parameters will be expressed using

β Therefore, the first and second level models can be written as

$$Y_{ij} = \pi_{0j} + \pi_{1j}T_{ij} + e_{ij} \qquad , e_{ij} \sim (0, \sigma^2)$$

$$\pi_{0j} = \beta_{00} + u_{0j},$$

$\pi_{1j} = \beta_{01} + u_{1j},$

$$[u_{0j}\ u_1\ ] \sim N[(\frac{0}{0})\ (\tau_{00}\ \tau_{01}\ \tau_{10}\ \tau \qquad)]$$

-

By substituting (2) in (1), the linear growth model in its final form is as follows:

$$Y_{ij} = [\beta_{00} + \beta_{10}T_{ij}] + [u_{0j} + u_{1j}T_{ij} + e_{ij}]$$

Where: Zala - the dependent variable representing the response of individual A at time point Z, T_{ij}- a declarative variable representing the time points at which individual A's measurements were recorded. Model 3 consists of two parts:

Part 1: Contains three parameters/fixed effects: β_{00}, the fixed part is represented by the equation Intercept, β_{10} Explanatory variable coefficient B

Part Two: It contains three random effects, which are:

- Level 2 errors (including $u_1$andu_{oj}) represent the deviations of an individual's growth/change trajectories from the predicted mean.
- Level 1 random error (e_{ij}) represents the unexplained portion of the dependent variable for individual A at time point Z.

Model 3 differs from traditional regression models in that it considers the fixed part and slope as random effects that vary by level-2 units. It does not contain any explanatory variables measured at level-2, and the independence of the level-1 random error $e_{ij}$from the level-2 random errors $u_j$is assumed. Rewriting Model 3 without changing T_{ij}gives us the following:

(Hedeker and Gilbbons,2006)

(4)

$$Y_{ij}=\beta_{00} + u_{oj} + e_{ij}$$

Model 4 is called the One-Way ANOVA or Unconditional Means Model, which includes the Boom mean and random errors for the first and second levels, and does not contain any explanatory variables for either the first or second levels. The main objective of studying unconditional growth models - Models 3 and 4 - is to determine whether there is a systematic variation in the dependent variable that is worth studying or not. If there is a variation, where is it located? Within individual measurements or between individuals, and what is the magnitude of this variation in each case? These models are also considered the starting point for determining the extent of success in constructing other successive growth models by adding explanatory variables for the first and second levels to these models.

Model (4) is fitted to the data with the aim of estimating the fixed part β_{00}, which represents the average of the variable IY_{ij} in the population, and two types of random effects

τ_{00}, which represents the change in the averages of individuals (the second level), and σ^2, which represents the change in measurements within individuals (the first level). Based on the assumption of independence of random errors at the first and second levels, we find that:

(°)

Var (Yij) = $\tau_{00} + \sigma^2$

Thus, in the case of linear growth models (as in the case of multilevel models) that do not contain explanatory variables, the variance can be divided according to its source into two parts by calculating the within-group correlation coefficient.

Rasbash et al (2009) :Intraclass Correlation(θ)

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Section 1: Represents the percentage of variance due to the first-level units and is measured as follows:

$$\theta_1 = \sigma^2 / (\tau_{00} + \sigma^2)$$

Section Two: It represents the percentage of variance due to the second-level units and is measured as follows:

$$\theta_2 = \tau_{00} / (\tau_{00} + \sigma^2)$$

($\beta^{'}s$ s is easy, reduces the strong correlation between first- and second-level variables and the cross- interactions between them, and also reduces the possibility of numerical errors in some estimation methods (Raudenbush et al., 2011).

Non- Linear Growth Model

Determining the optimal functional form of a growth trajectory over time is a fundamental step in constructing a growth model and thus expressing repeated individual measurements as a function of time. If this is not done correctly, it leads to biased results (Raudenbush and Bryk, 2002). In many applied situations, whether in educational, medical, or behavioral fields, we find that the optimal functional form of growth is non-linear, especially of the second order (quadratic) or third order (cubic). The second-order growth model can be written in its general form as follows:

(Cudeck and Harring,2007) Level 1 Model:
(11)

$$Y_{ij} = \pi_{0j} + \pi_{1j} T_{ij} + \pi_{2j} T^2 ij + , \qquad e_{ij} \sim N(0, \sigma^2)$$

Level 2 Model:

$$\pi_{0j} = \beta_{00} + \sum_{q=1}^{m} \beta_{0q} X_{qj} + u_0 ,$$

$$\pi_{1j} = \beta_{10} + \sum_{q=1}^{m} \beta_{1q} X_{qj} + u_1 ,$$

$$\pi_{2i} = \beta_{20} + \sum_{q=1} {}_{2q} + u_{2j}$$

(12)

The first-level model (11) contains time as a first-order explanatory variable T_{ij}in addition to the square of this variable$T^2{}_{ij}$, while the second-level model (12) contains many second-level explanatory variables. The growth rate (GR) at any point in time can be obtained by taking the first derivative of the model (11) with respect to time T as follows:

(13)

$$GR = \pi_{1j} + \quad 2\pi_{2j} \; T_{ij}$$

In this section, we will be satisfied with the unconditional growth model. In this case, the first-level model (11) will remain unchanged, while the equations of the second-level model will not include any variables or explanations and will become as follows:

$$\pi_{0j} = \beta_{00} + u_{0j}$$

$$\pi_{1j} = \beta_{10} + u_{1j}$$

$$\pi_{2j} = \beta_{20} + u_{2j}$$

To fit the non-linear growth model, we need to estimate the fixed parameters, which include the regression coefficients, in addition to the random parameters, which include the error variances at different levels. The variance-covariance matrix takes the following form:

$$(\tau_{00}\;\tau_{01}\;\tau_{03}\;\tau_{10}\;\tau_{11}\;\tau_{12}\;\tau_{20}\;\tau_{21}\;\tau_{22})$$

Where the elements of the main diagonal represent the variances of the errors, and the remaining elements of the matrix represent the covariance between these errors.

As higher the value of the result in Equation (7), the greater the importance of taking the second level into account, and thus the greater the importance of multi-level analysis. The lower the result of Equation (7) or the variance of the second-level errors is not statistically significant, then there is homogeneity between the second-level units, and thus there is no need for multi-level analysis.

Conditional Linear Growth Model

It is noted in the growth model (3) that it contains one explanatory variable specific to the first level and does not contain any explanatory variables specific to the second level, but in many practical cases the researcher may wish to study longitudinal data through growth models that contain variables specific to the second level (individuals) such as gender, age, weight, etc.

Assuming that the data under study contain many explanatory variables of the second level, then Model (3) becomes, after adding these variables, as follows:

Level 1 Model:

$$Y_{ij} = \pi_{0j} + \pi_{1j}\, T_{ij} + \quad , \qquad e_{ij} \sim \; N(0, \sigma^2)$$

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(8)

Level Two Model:

(9)

$\pi_{pj} = \beta_{p0} + \sum_{q=1} \quad m \quad + u_{pj} \quad ,=0,1$

If we have one explanatory variable $X_j$for the second level, then model (9) becomes as follows:

$$\pi_{0j} = \beta_{00} + \beta_{01} X_j + u_0 ,$$

$$\pi_{1j} = \beta_{10} + \beta_{11} X_j + u_{1j}$$

$$[u_{0j}\ u1] \sim [(0\ 0), (\tau_{00}\ \tau_{01}\ \tau_{10}\ \tau_{11})]$$

Equations (8) and (9) are called the conditional linear growth model. Through this model, one can determine the extent to which the model parameters $\pi_{0j}$and π_{1j}change as a function of differences between individuals (level two). If the growth model contains some
continuous explanatory variables (e.g., X_ij), it is preferable for the researcher to center these
variables before fitting the data model by subtracting the mean of the variable from all values of this variable. In the two-level linear growth model, there are two options for this:

First: Centering on the general average (X), where the general average of the variable is subtracted from each value of this variable (X) - X).

Second: Centering on the group mean (individual measurements) ((X), where the group mean is subtracted from each value of this variable ((X -X_{ij}).

Hofmann and Gavin (1998) explained that the choice between the two previous options depends on the research question(s). The main interpretive benefit is to make the interpretations of the fixed effects in the model.

Growth Models Fitting

There are two basic approaches to fitting growth models: Multilevel Modeling (MLM) and Structural Equation Modeling (SEM). The two approaches are similar in many characteristics and give similar numerical results in many cases, but in some other cases there are differences between them. For example, in the case of multiple levels of data, it is preferable to use the MLM approach. On the other hand, it is preferable to use an approach if the explanatory and dependent variables are subject to measurement error (a"

Due to the widespread use of individual growth models in applied fields, whether educational, medical, or behavioral, many researchers have prepared and used some software to reconcile these models in addition to multi-level models and linear stepwise models.

SAS PROC MIXED is one of the most widely used programs for fitting individual growth models to longitudinal data based on maximum likelihood estimates. It allows treating growth parameters for each individual as random effects in the model. Many researchers have used it to fit multilevel and phylogenetic growth models in various fields.

• (McNeish, , Delucia and Pitts (2006) › (Singer and Willett ,2003), Singer (1998) lis

Some researchers have also used the R program to fit linear and nonlinear individual growth models, such as Bliese
Pinheiro and Bates (2000).

Given the multi-level data under study, the multi-level modeling approach (MLM) will be followed and the HLM7 program will be used to fit growth models to the data under study (Raudenbush et al., 2011). This program does not require writing many syntax commands, unlike other programs such as SAS. This program estimates the first-level coefficients that vary randomly using the empirical Bayes approach.

The second-level coefficients are calculated using generalized least squares, while the variance and covariance components are estimated using the maximum likelihood approach. Depending on the outputs of the HLM7 program, the quality of the growth model's fit can be measured using several measures such as: Model

, Bayesian Information criterion (BIC) , Deviance(D) Akaike Information

Pseudo R^2criterion (AIC) and the Deviance (D) statistic is defined as:

(O'Connell and McCoach, 2008)

(51)

D = -2 log(L)

Where L represents the likelihood value of obtaining maximum likelihood estimates of model parameters, and in general, models with a lower D value have a better goodness of fit than models with a higher D value.

The D statistic can be used to compare the goodness of fit of two nested models (i.e., one model can be derived by deleting some parameters or variables from the other model). Assuming that D1 and D2 represent the Deviance statistic for the two models, the difference between them (-D1 D2) follows an X2 distribution with degrees of freedom equal to the difference between the number of parameters estimated in the two models. Depending on the value of the D statistic, the remaining measures of goodness of fit of the model can be calculated as follows:

(51)

BIC = D + p log(N) (51)
AIC = D + 2p

Since N represents the sample size and m represents the number of parameters in the model, in stepwise models there is a different sample size at each level of the data hierarchy, Lukc (2005) suggested using the sample size at the first level when calculating the B1C statistic. The lower the BIC or AIC statistic, the better the model fit. Compared to other metrics, the BIC and AIC measures can be used to compare the fit of non-nested models. As for the pseudo R^2 statistic, it is based on the idea of measuring the proportion of variance in the dependent variable that is explained by the explanatory variables in the model. However, the problem with stepwise models is the presence of variance at each level of the data hierarchy which may lead - in some cases - to erroneous results such as negative pseudo R2 values. To solve this problem, more than one formula has been proposed to calculate the forced variance at each level of the data hierarchy. For more rich details on this topic, see Singer Snüjders (2005) , (Raudenbush and Bryk,2002),(1998).

Study Sample

To analyze repeated-measures data using growth models, the sample size selected must be at least 100 individuals. It is preferable that the number of measurements recorded for each individual be three or more, as this affects the estimation of model parameters and its statistical power. Hedeker and Gibbons (2006)

A random sample of 876 (20%) male and female students was selected from the total number of sixth-grade students in government schools in the State of Qatar (4379 students in the academic year 2010/2011, distributed across 48 schools). This was done using the personal ID number of each student and the SPSS program. The results of these students in the mathematics test (the final grade in this subject is 100) were recorded starting from the academic year 2010/2011. The results of these students in this subject were followed for the following three years, i.e. the years 2011/2012, 2012/2013, and 2013/2014. The cases of students who did not have a complete record of grades for the four years were also removed due to their leaving the country or dropping out of education. The number of these cases reached 47. Therefore, the research sample consists of the results of 829 students in mathematics for four consecutive years. The study sample has multiple variables, including:

• Students' results in mathematics are represented by the dependent variable Y_{ij}.

The years in which the students' results were recorded, represented by the explanatory variable
T_{ij}, which takes the values 0, 1, 2, and 3 to express the academic years from 2010/2011 to 2013/2014.

Gender: The number of students in the sample was 431, representing 52% of the total sample, and the number of female students was 398, representing 48% of the total sample. This variable was coded (male -1 and female -.)

Age in years: The values of this variable ranged between 11.4 and 15.7, with an average of 13.1 years.

Results of Growth Model Fitting and Discussion

In this section, various growth models will be fitted to the data under study, with the aim of analyzing students' academic performance in mathematics. Since each student in the sample has a complete score record over the four years under study, the study data are considered balanced, which has a positive impact on estimating the parameters of individual growth models. Singer (1998) explained that if the data under study are unbalanced, contain missing values, or have a high degree of correlation, this will negatively impact the fit of the growth model.

Unconditional Mean Model

We will begin by fitting the unconditional mean model, shown in Equation 4, with the aim of dividing the variation in students' mathematics scores (Y) into two components:

First: The variation among students themselves, regardless of the time variable, expressed by the parameter τ_00

Second: The variation within students' scores across time, expressed by the parameter σ^2

To fit Model 4, we need to estimate two types of parameters: the fixed parameter, represented by the regression coefficient β_00, and the random parameters, which include the first- and second-level error variances τ_(00 and σ^2)) (Table 1). Based on the outputs of the HLM7 program, Table 1 displays the results of fitting the unconditional average model. It is noted that: β_00 = 77.52, which represents the overall average of students' grades over the years of study, Tτ_00 = 16.55, and the variance between students σ^2 - 42.05, which represents the variance within students. The random parameters (σ^2 and τ_00) are statistically significant, meaning they differ significantly from zero (0). These estimates indicate that students differ in their average grades in mathematics, as there is a greater degree of variation within the class (the variance within students is greater than twice the variance between students). The variation in the dependent variable can also be studied by calculating The coefficient (θ) shown in Equation 7 is as follows:

= 16.55/(16.55+{2.05)= 0.282 θ

This indicates that approximately 28% of the variation in students' mathematics scores is due to differences between students (level 2), which underscores the importance of taking the property of gradualism (or multi-level) into account when analyzing such data. Here, the need to search for and add confounding variables specific to the first and second levels to the growth model is highlighted, in order to explain this variation within and between students' scores.

Unconditional Linear Growth Model

This model was discussed in Part 2, where students' scores represent level 1, while students represent level 2. The parameters of the level 1 model are expressed as a function of the change between level 2 units. This model contains the dependent variable $Y_{ij}$and a single explanatory variable ؟T_{ij}, measured at level 1 and representing the year y in which Student A's scores were shifted. To fit Model 3, we need to estimate the regression coefficients (β_{00} و β_{10}),

the first- and second-level error variances (σ^2 و τ_{00} و τ_{11}), and the covariance τ_{01}.. Table 2

In Table 2, we find the estimates of the fixed parameters and their significance as follows: β_{00} = 77.58, representing the average score of students over the years of study.
β_{10}Represents the average intercept (i.e., the value of the dependent variable Y at T -0).
Represents the average rate of change/growth (average slope). This means, on average, that a student who initially scored approximately 78 in mathematics (the 2010/2011 academic year) achieved an increase of approximately 4 points in each subsequent test. The standard errors of these parameter estimates are small, leading to a large T-ratio and, consequently, a small P- value.

As for the random parameters, we find that σ^2 = 34.17, representing the variance of the first-level errors. We note that their value decreases compared to the unconditional mean model by:

(42.05-34.17)/42.05 - 0.187

This means that approximately 19% of the variation within students' math scores was explained by adding the time variable in its linear form to the unconditional growth model. The variance-covariance matrix can be written as follows:

$$(\tau_{00}\ \tau_{01}\ \tau_{10}\ \tau_{11}) - (16.32\ -0.027\ -0.027\ 0.88)$$

The elements of the main diagonal of this matrix represent estimates of the variances

$\tau_{00}$and τ_{11}, where τ_{00}= 16.32 and represents the change in student mean scores, intercepts = 0.88 and represents the change in growth rates, and slopes τ_{01} = 0.027 and represents the covariance between student mean scores and growth rates. In addition to the model parameter estimates, Table 2 displays the standard errors of these parameter estimates, enabling us to test their statistical significance. Looking at the P-value column, we reject the hypothesis that the parameters of the growth model in the population are equal to zero (0). It is noted that the significance of the parameter estimate τ_{00}indicates that students differ in their grade levels in mathematics, and the significance of the parameter estimate τ_{11} indicates that students differ in the rate of change/growth of their grades in this subject. This model contains the dependent variable Y_{ij}, and a single explanatory variable Y_{ij}measured at the first level, representing the year in which Student A's grades declined. To fit Model 3, we need to estimate the coefficients of the conditional linear growth model.

After fitting the previous unconditional linear growth model, which showed differences in both mean scores and rate of change/growth among students, we will add some second-level variables to the model to demonstrate whether this difference is related to these variables or not. This is done using the conditional linear growth model discussed in Part 3. The first-level model remains unchanged, as in Equation 8, while the second-level model (students) is as follows:

= β_00 + β_01 x_1J + β_02 (x_(2J-x)+U_0J). π_0J

=π_1J= β_10 + β_11 x_1J + β_12 (x_2J -X) + U_1J

In Model 18, two explanatory variables were added: X1, representing gender (male, female), and X2, representing the student's age. Since X2 is a continuous variable, this variable is centered on its overall mean by subtracting the overall mean of the variable (13.1) from each of its values before fitting the model and estimating its parameters. Table 3

Table 3 shows the significance of all estimates of the coefficients of the explanatory variables and the interactions between them, with the exception of the gender variable (where the standard error is larger than the value of the coefficient of the variable itself). There is also a slight change in the values of the coefficients compared to their estimates in the unconditional growth model shown in Table 2. This is due to the centering of the second- level variable (age) around the year mean.

It is noted that the student's age variable has an effect on the rate of growth of his grades, as: $\beta_{02=0.46}$ indicates that a student whose age changes by one year leads to a change in the growth of his grades by 0.46 points. Thus, the estimated values of the interaction coefficients of the explanatory variables can be interpreted. Based on the variance estimates τ_{00} و τ_{11} and τ_{01} , the variance-covariance matrix can be written as follows:

(τ_00 τ_01 τ_10 τ_11)=(11.35 -0.021 -0.021 0.67)

Comparing these estimates with their counterparts in the unconditional growth model, the following is observed:

- A decrease in the variance of student grade averages by:

(16.31 - 11.35)/16.31 = 0.304

- A decrease in the variance of student growth rate by: (0.88 - 0.67)/0.88 = 0.24

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This means that adding the student age variable—in addition to the interactions between the explanatory variables—led to an improved fit of the data model, and that it is responsible for explaining 30% of the variance in student grade averages, as well as 24% of the variance in growth rate. Students' grades changed. As for the variance of first-order errors, σ^2, it remained almost unchanged.

Using Individual Growth Models to Study Student Achievement Performance: An Applied Study

Nonlinear Growth Model

As an extension of the analysis of the data under study, a second-order unconditional growth model will be fitted to demonstrate whether the change in students' grades (growth) in mathematics over the years of study (time) follows a linear or quadratic pattern, as the optimal form of growth takes the nonlinear form in many applied cases. To fit the quadratic growth model consisting of Equations (11) and (14), we need to estimate the second parameters, $\beta_{00}$and β_{10} andβ_{20}, in addition to the random parameters, which include the error variances at the first level (within students) and the second level (between students). Table 4

Table 4 displays the estimates of the fixed parameters in the quadratic growth model. Their significance is evident, except for the parameter β_{12}, which is specific to the square of the time variable T^2T2. This indicates that the quadratic formula is inappropriate for the growth rate in the data under study, and that the linear formula is more appropriate. Based on the output of the HLM7 program, the estimated variance-covariance matrix can be written as follows:

$$(18.03\ -2.058\ -1.771\ -2.058\ 0.91\ 0.455\ -1.771\ 0.455\ 0.41) =$$
$$(\tau_{00}\ \tau_{01}\ \tau_{02}\ \tau_{10}\ \tau_{11}\ \tau_{12}\ \tau_{20}\ \tau_{21}\ \tau_{22})$$

It is noted that there is an increase in the estimates of the error variance parameters τ_{00} and τ_{11}1 compared to their counterparts in the unconditional linear growth model shown in Table 2. It is also evident that the parameter τ_{22}, which represents the time-varying square slope variance, is not significant. Therefore, it can be concluded that the quadratic formula did not lead to an improvement in data fitting. However, the first-level error variance 2 showed a slight increase compared to the unconditional linear growth model. Comparing the Quality of Fit of Growth Models

In the previous sections, the data under study were studied by fitting four different growth models:

- The unconditional mean model (1), which does not contain any differential variables and is shown in Equation (4).

- The unconditional linear growth model (11), which contains the time variable as the sole explanatory variable measured at the first level and is shown in Equation (3).

- The conditional linear growth model (111), which contains explanatory variables measured at the second level and is shown in Equations (8) and (9).

- The second-order nonlinear growth model (IV), which is shown in Equations (11) and (14). Based on the outputs of the HLM7 program, the quality of fit of the four previous models will be compared using three measures previously discussed in Part (5). Table (5)

Table (5) shows that the nonlinear growth model has the lowest quality of fit. It achieved the highest values for the three measures, which is consistent with the results shown in Table 4, confirming that the quadratic formula does not fit the data under study, as the quality of model fit increases as the values of these three measures decrease. It is also noted that the quality of the growth model's fit is strengthened by adding explanatory variables at both the first and second levels. Model (11) fits better than Model (1) due to its inclusion of the time variable, while Growth Model (1II) is better than Models (A) and (11) due to its inclusion of explanatory variables specific to the first and second levels.

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Tables

Table (1): Results of fitting the unconditional average model

Fixed effects	estimote	Standard error	t- ratio	p-value
β_{00}	77.52	1.27	61.04	0.0001
Random effect	Standard deviation	Variance component	Chi-square	p-value
Intrecept, u_0	4.07	16.55	374.21	0.000
Level-1.e	6.48	42.05		

Table (2): Results of fitting the unconditional linear growth model

Fixed effects	estimote	Standard error	t- ratio	p-value
Intercept, β_{00}	77.58	1.23	63.07	0.0001
Time, β_{10}	3.62	0.15	24.13	0.0001
Random effect	Standard deviation	Variance component	Chi-square	p-value
Intercept, u_0	4.04	16.32	341.91	0.000
Time slope, u_1	0.94	0.88	385.66	0.000
Level-1.e	5.85	34.17		

Table (3): Results of fitting the conditional linear growth model

Fixed effects	Estimate	Standard error	t- ratio	p-value
INTERCEPT, β_{00}	77.59	1.72	61.09	0.0001
GENDER, β_{12}	0.13	0.19	0.68	0.5331
AGE, β_{02}	0.46	0.07	6.75	0.0022
Time, β_{10}	3.64	0.19	19.16	0.0001
GENDER*TIME, β_{11}	1.94	0.15	12.93	0.0085
AGE*TIME β_{12}	0.53	0.14	3.97	0.0041
Random effect	Standard deviation	Variance component	Chi-square	p-value
Intercept, u_0	3.37	11.35	344.27	0.000
Slope, u_1	.0.82	0.67	372.66	0.000
Level-1,e	5.84	34.14		

When analyzing such data, the importance of searching for and adding specific variables specific to the first and second levels to the growth model becomes clear, in order to explain this difference within and between students' scores.

Unconditional Linear Growth Model

This model was discussed in Part 2, where students' grades represent the first level, while students represent the second level. The parameters of the first-level model are expressed as a function of change between units of the second level. This model contains the dependent variable, R, and a single explanatory variable, Ti, measured at the first level, representing the year Z in which student A's grades were recorded. To fit Model 3, we need to estimate the regression coefficients (Boo and B10), the error variances of the first and second levels (62, T00, and T11), and the variance To1. Table 2

In Table 2, we find the estimates of the fixed parameters and their significance as follows: Boo - 77.58, representing the average grades of students over the years of study; average intercept (i.e., the value of the dependent variable Y at B1o -3.62); (T -0, representing the average rate of change/growth). This means, on average, that a student who initially scored approximately 78 in mathematics (the 2010/2011 academic year) achieved an increase. It is estimated at approximately 4 points each time a subsequent test is performed. It is noted that the standard errors of these parameter estimates are small, leading to a large T- ratio and, consequently, a small P-value.

As for the random parameters, we find that 0.02 - 34.17 represents the variance of the first-level errors. It is noted that their value decreases compared to the unconditional mean model by an amount of:

(42.05 - 34.17)/42.05 - 0.187

This means that approximately 19% of the variation within students' mathematics scores was explained by adding the time variable in its linear form to the unconditional growth model. The variance-covariance matrix can be written as follows:

(0.027)

(16.31 0.88

-0.021

T11/

(∞

The elements of the main diagonal of this matrix are estimates of the variances To0 and T11, where To0 is 16.32 and represents the change in the students' mean scores T1 - 0.88 intercepts and represents the change in the growth rate, and Tor - - 0.27, Slopes and represents the covariance between the students' mean scores and the growth rates. In addition to the model parameter estimates, Table 2 displays the standard errors of these parameter estimates, enabling us to test their statistical significance. Looking at the P-value column, we reject the hypothesis that the parameters of the growth model in the population are equal to zero (0). It is noted that the significance of the To0 parameter estimate indicates that students differ in their grade levels in mathematics, and the significance of the T11 parameter estimate indicates that students differ in the rate of change/growth of their grades in this subject.

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The elements of the main diagonal of this matrix represent estimates of the variances To0 and T11, where To0"16.32 represents the change in students' mean scores (T1 - 0.88 intercepts), which represents the change in the growth rate. Tor - - 0.27, Slopes, which represents the covariance between students' mean scores and growth rates. In addition to the model parameter estimates, Table 2 displays the standard errors of these parameter estimates, enabling us to test their statistical significance. Looking at the P-value column, we reject the hypothesis that the parameters of the growth model in the population are equal to zero (0). It is noted that the significance of the To0 parameter estimate indicates that students differ in their grade levels in mathematics, while the significance of the T11 parameter estimate indicates that students differ in the rate of change/growth of their grades in this subject.

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