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On a Pseudo Smarandache Ideals of BD-algebra

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Abstract**—***In this paper the notion of a pseudo Samarandache BD-algebra, a pseudo Samarandache ideal , a pseudo Smarandache closed ideal and a pseudo Samarandache completely closed ideal of a pseudo Samarandache BD-algebra are defined. There notion are stadied. The relationships among these types of Ideals are discussed.*

Keywords: BCK-algebra, *BD*-algebra, ideal of *BD*-algebra, a Smarandache of *BD*-algebra, ,a pseudo *BD*-algebra, a pseudo ideal of a pseudo BD-algebra, a pseudo closed ideal of a pseudo BD-A , a pseudo completely closed of a pseudo *BD*-algebra, a pseudo Smarandache ideal of BD – algebra, a pseudo Smarandache closed ideal of *BD*-algebra, a pseudo Smarandache completely closed ideal of BD –algebra.

Introduction

In 1966 by Y .Imai and K.Iseki introduceed the notion of BCKalgebra[12], In 1998, Y. B. Jun, E. H. Rogh and H. S. Kim introduceed the notion of a BH- algebra, and the notion of Id of a BH- algebra[12]. In 2022, T.Bantaojai and et. cl. introduceed the notion of a BD-A [1,12]. In this paper, we define the concepts of a pseudo Smarandache completely closed ideal and a pseudo Smarandache closed ideal of a pseudo Smarandache *BD*-algebra. We stated and proved some theorems which determine the relationships between these notions and some types of a pseudo Smarandache Ideals of a Smarandache *BD*-algebra.

1. Materials and Methods

In this section, some basic concepts about a BCK-algebra, a BD-algebra, a pseudo BD-algebra, pseudo Id and a pseudo closed ideal of a pseudo BD-algebra are given.

Definition 1.1[1,12].
A *BD*-algebra (BD-A) is a non-empty set ζ with a constant o and a binary " $\diamond$ " satisfying the following axioms hold $\forall \varepsilon,\eta \in \zeta$, if :
(1) $\varepsilon \diamond o = \varepsilon, \forall \varepsilon \in \zeta$,
(2) $(\varepsilon \diamond \eta) = o$ *and* $\eta \diamond \varepsilon = o$, *then* $\varepsilon = \eta$.

Definition 1.2[14].
A Smarandache (Sm) BD-A is defined to be a BD-A $(\zeta;\diamond,o)$ in which there exists a proper subset Q of ζ such that .
 i. $o \in Q$ and $|Q| \geq 2$
 ii. Q is a BD – algebra under the operation of ζ.

Definition 1.3[1,12].
Let I be a nonempty subset of a BD-A $(\zeta;\diamond,o)$ and $\lambda\ (\neq \emptyset) \subseteq \zeta$ .Then λ is called an ideal (Id)of ζ if it is satisfies:
i. $o \in \lambda$,
ii. $\varepsilon \diamond \eta \in \lambda$ *and* $\eta \in \lambda$ *imply* $\varepsilon \in \lambda$, $\forall \varepsilon,\eta \in \zeta$.

Definition 1.4[1,12].
A nonempty subset λ of a Sm BD – algebra $(\zeta;\diamond,o)$ is called a Sm Id of ζ if:
i. $o \in \lambda$,
ii. $\varepsilon \diamond \eta \in \lambda$ *and* $\eta \in \lambda$ *imply* $\varepsilon \in \lambda$, $\forall \varepsilon \in \zeta$.

Proposition 1.5. Every Id of a Sm BD-A $(\zeta;\diamond,o)$ is a Sm Id of ζ .

Definition 1.6.
An Id λ of a BD-A $(\zeta;\diamond,o)$ is called a CId Id of ζ if and only if $o * \varepsilon \in \lambda$ *for all* $\varepsilon \in \lambda$

Definition 1.7. A Sm Id λ of a Sm BD –A $(\zeta;\diamond,o)$ is called a Sm closed ideal (CId)of ζ if: *for all* $\varepsilon \in \lambda, o \diamond \varepsilon \in \lambda$

Proposition 1.8. Every CId Id of a Sm BHalgebra $(\zeta;\diamond,o)$is a Sm CId of ζ.

Definition 1.9. An Id λ of a BD-A $(\zeta;\diamond,o)$ is called a completely closed ideal CId of ζ if it is Satisfies: $\varepsilon \diamond \eta \in \lambda, \forall \varepsilon,\eta \in \lambda$.

Remark 1.10. Every a CCId of BD-A $(\zeta;\diamond,o)$ is CId of ζ.

Definition 1.11. A Sm Id λ of a Sm BD-A $(\zeta;\diamond,o)$ is called a Sm CCId of ζ if: $\varepsilon * \eta \in \lambda, \forall \varepsilon,\eta \in \lambda$.

Proposition 1.12. Every CCId of a BD-A $(\zeta;\diamond,o)$ is a Sm CCId of ζ.

Remarks 1.13. Every a Sm CCId of Sm BD-A $(\zeta;\diamond,o)$ is a Sm CId of ζ.

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Definition 1.14. A pseudo (Po) $BD\text{-}A$ $(\zeta;\diamond,o)$ is anon empty set with a constant 0 and two binary operations $\diamond$ and # satisfying the following, $\exists\ \varepsilon,\eta,\iota \in \zeta$

(1) $\varepsilon \diamond o = \varepsilon \# o = \varepsilon$

(2) $(\varepsilon \diamond \eta) = o\ ,(\varepsilon \# \eta) = o\ and\ (\eta \diamond \varepsilon) = 0\ ,(\eta\ \#\ \varepsilon) = 0$ implies $\varepsilon = \eta$.

Definition 1.15.

A nonempty subset λ of a Ps $BD\text{-}A$ $(\zeta,\diamond,\#,o)$ is called a Pseudo Ps Id of ζ if:

i. $o \in \lambda$,

ii. $\varepsilon \diamond \eta\ ,\varepsilon \# \eta \in \lambda\ and\ \eta \in \lambda\ imply\ \ \varepsilon \in \lambda,\ \ \forall\ \varepsilon,\eta \in \zeta.$

Definition 1.16.

A Po Id λ of a Ps BD-algebra $(\zeta,\diamond,\#,o)$ is called a Po CId of ζ, if for every $\varepsilon \in \lambda$,we have $o \diamond \varepsilon, o \# \varepsilon \in \lambda$.

Definition 1.17.

A Po Id λ of a Ps BH -algebra $(\zeta,\diamond,\#,o)$ is called a Po CCId of ζ, if satisfies: $\varepsilon \diamond \eta, \varepsilon \# \eta \in \lambda, for\ all\ \varepsilon,\eta \in \lambda$.

Remarks 1.18.

Every a Ps CCId of a Ps BD-A $(\zeta,\diamond,\#,o)$ is a Ps CId of ζ.

2. Main Results

In this section, the concepts a Ps Sm BD-A, a pseudo Sm Id, a Ps Sm CIds and a pseudo Sm compeletly CIds of a Ps Sm BD-A are given.

Definition 2.1.

A Ps Sm BD-A $(\zeta,\diamond,\#,o)$ is defined to be a Ps BD-A in which there exists a proper subset Q of ζ such that

i.$o \in Q\ and\ |Q| \geq 2$

ii. Q is BD – A under the operations "$\diamond$" and "#" of ζ.

Example 2.2.

The a pseudo BH- algebra ζ = {0,1 ,2 ,3 ,4 } with constant 0 and binary operations" $\diamond$" and" #" defined the following tables

*	0	1	2	3	4
0	0	0	0	0	4
1	1	0	0	2	4
2	2	1	0	2	4
3	3	2	0	0	4
4	4	2	1	0	0

#	0	1	2	3	4
0	0	0	0	2	4
1	1	0	2	3	1
2	2	2	0	1	0
3	3	0	0	0	2
4	4	1	1	1	0

and Q={0,1,2} is a Ps Sm BD-A .

Definition 2.3.

Let $(\zeta,\diamond,\#,o)$ be a Ps Sm BH-algebra An on empty subset λ of ζ is called a Ps Sm Id of ζ related to Q (or briefly , a Ps Sm Id of ζ if.

1- $o \in \lambda$,
2- $\forall\,\eta \in \lambda,\ \ \varepsilon \diamond \eta\ ,\ \ \varepsilon \# \eta \in \lambda\, imply\ \ \varepsilon \in \lambda, \forall\,\varepsilon \in Q$

Example 2.4.

Consider the Ps Sm BH- algebra ζ={0,1,2,3,4} with the binary Operations "$\diamond$" and " # " defined by the tables.

*	0	1	2	3	4
0	0	0	0	3	4
1	1	0	0	2	3
2	2	1	0	2	4
3	3	3	2	0	4
4	4	2	1	0	0

#	0	1	2	3	4
0	0	0	0	3	4
1	1	0	0	2	1
2	2	2	0	1	0
3	3	3	2	0	2
4	4	1	2	1	0

And Q={0,1,2} the subset λ={0,1,3} is a Ps Sm Id of ζ .

Proposition 2.5.

Let $(\zeta,\diamond,\#,o)$ be a pseudo Sm – BD-A .Then every a Ps Id of ζ is a Ps Sm Id of ζ .

Proof: It is clear

Remark 2.6.

The following example shows that convers of proposition is not correct in general.

Example 2.7.

Consider the a Ps Sm BD- A ζ={0,1,2,3} with binary operations "$\diamond$" and "#" defined by the following tables.

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*	0	1	2	3
0	0	0	2	3
1	1	0	1	2
2	2	2	0	1
3	3	3	2	0

#	0	1	2	3
0	0	0	2	3
1	1	0	0	2
2	2	2	0	1
3	3	3	2	0

And Q ={0,1}. The subset λ={0,2} is a Ps Sm Id of ζ but it is not a Ps Id of ζ , since $3 \diamond 2 = 2 \in \lambda$ *and* $3\#2 = 2 \in \lambda$ *but* $3 \notin \lambda$ **Theorem (2.8)**

Let $(\zeta,\diamond,\#,o)$ be a Ps Sm BD- algebra and λ be a pseudo Sm Id such that $\varepsilon \diamond \eta, \varepsilon \# \eta \notin \lambda$, *for all* $\varepsilon \notin \lambda$ *and* $\eta \in \lambda$, *then* λ *is a* Ps Id *of* ζ.

Proof:

Let λ be a Ps Sm Id of $(\zeta,\diamond,\#,o)$, $\varepsilon \in \zeta$, *and* $\eta \in \lambda$,

1- $o \in \lambda$,
2- *let* $\varepsilon * \eta, \varepsilon \# \eta \in \lambda$ *and* $\eta \in \lambda$.

Then we have two cases. Case 1 if $\varepsilon \in Q \Longrightarrow \varepsilon \in \lambda$

Case 2 if $\varepsilon \notin Q$, *either* $\varepsilon \in \lambda$, *or* $\varepsilon \notin \lambda$. If $\varepsilon \in \lambda \Longrightarrow \lambda$ *is a pseudo ideal*if $\varepsilon \notin \lambda, \Longrightarrow \varepsilon \diamond \eta, \varepsilon \# \eta \notin \lambda$

And this contradiction since $\varepsilon \diamond \eta$, $\varepsilon \# \eta \in \lambda$. Therefore , λ is a Ps Id .

Definition 2.9.

A Ps Sm Id λ of a Ps Sm BD-A $(\zeta,\diamond,\#,o)$ is called a Ps Sm CId of ζ if $o \diamond \varepsilon$, $o \# \varepsilon \in \lambda$, $\forall \varepsilon \in \lambda$.

Example 2.10.

The a Ps Sm Id λ={0,1,3} of ζ in example (2.4) is a Ps Sm CId of ζ.

Definition 2.11.

A Ps Sm Id λ of a Ps Sm BD-A $(\zeta,\diamond,\#,o)$ is called a Ps Sm CCId of ζ if $\varepsilon \diamond \eta$ *and* $\varepsilon\#\eta \in \lambda$, $\forall \varepsilon , \eta \in \lambda$.

Example 2.12.

the a Ps Sm Id λ={0,4} of ζ in example (2.4) is a Ps Sm CCId of ζ.

Proposition 2.13.

Let $(\zeta,\diamond,\#,o)$ is a Ps Sm BD –A .Then every a pseudo Sm CCId λ of ζ is a Ps Sm CId of ζ.

Proof: It is clear

Remark 2.14. The following example shows that convers of proposition is not correct inganeral.

Example 2.15. Consider the a pseudo Sm BD- algebra ζ={0,1,2,3} with binary operation"$\diamond$" "#" defined by the following tables.

*	0	1	2	3
0	0	0	1	3
1	1	0	3	1
2	2	3	0	2
3	3	2	1	0

#	0	1	2	3
0	0	0	2	3
1	1	0	3	1
2	2	3	0	3
3	3	3	1	0

And Q={0,1} , Then ζ a Ps Sm BD- A where the pseudo Sm Id λ ={0,1,2} is a Ps Sm CId of ζ.

But is not a Ps Sm CCId λ of ζ. Since $1 \diamond 2 = 3 \notin \lambda, 1\#2 = 3 \notin \lambda$, *and* $1,2 \in \lambda$.

Remark 2.16.

Let $(\zeta,\diamond,\#,o)$ a Ps Sm BD -A and λ be a Ps CCId of ζ then λ is a Ps Sm CCId of ζ .

Proposition 2.17.

Let $(\zeta,\diamond,\#,o)$ be a Ps Sm BD-A and λ be Ps Sm CId such that $\varepsilon \diamond \eta$, $\varepsilon \# \eta \notin \lambda$ for all $\varepsilon \notin \lambda$ *and* $\eta \in \lambda$, then λ is a Ps CId of ζ.

Proof: Let λ be a Ps Sm CId of $\zeta \Longrightarrow$I is a Ps Sm Id of ζ. By theorem (2.8) , λ is a Ps Sm CId of ζ implies that $o \diamond \varepsilon, o\#\varepsilon \in \lambda$. Therefore, I is a Ps CId of ζ.

Proposition 2.18.

Let $(\zeta,\diamond,\#,o)$ be a Ps Sm BD-A and λ be a Ps Sm CCId such that $\varepsilon \diamond \eta$, $\varepsilon \# \eta \notin \lambda$ for all $\varepsilon \notin \lambda$ *and* $\eta \in \lambda$, then λ is Ps CCId of ζ.

Proof: Let λ be a Ps Sm CCId of ζ. This yield λ is a Ps Sm Id of ζ. by theorem (2.8) we have λ is a Ps Id of ζ [since λ a Ps Sm CCId of ζ] . It followos $\varepsilon \diamond \eta$ *and* $\varepsilon \# \eta \in \lambda$, *for all* $\varepsilon, \eta \in \lambda$. Hence, λ is a Ps CCId of ζ.

3. Conclusion

In this paper, the notions of a Ps Sm BD-A , a Ps Sm Id of BD-A , a Ps Sm CId of BD-A , a Ps Sm CCId of a BD-A are introduced. Furthermore, the results are examined in terms of the relationships between a Ps Sm CId of BD-A , a Ps Sm CCId of BD-A

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