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Hybridization of Bisection and Newton Methods for Solving Nonlinear Equations

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Abstract: *The hybridization of the Bisection and Newton methods for solving nonlinear equations offers a compelling approach to combine the strengths of both methods while mitigating their weaknesses. The Bisection method is known for its robustness and guaranteed convergence, especially when the initial guess is far from the actual root. However, it is slower compared to Newton's method, which converges rapidly but relies on a good initial guess and the computation of the derivative of the function. In this paper, we propose a hybrid method that starts with the Bisection method to ensure initial robustness and then switches to Newton's method to take advantage of its rapid convergence once the initial approximation is close enough to the root. The hybrid method is analyzed for its convergence properties and computational efficiency, and it is compared with both the Bisection and Newton methods through several numerical examples. Results show that the hybrid method offers a better balance between reliability and speed, making it a useful tool for solving nonlinear equations in practical applications where both accuracy and efficiency are important.*

Keywords: Hybrid methods, Bisection method, Newton's method, Nonlinear equations, Convergence, Numerical analysis

1.Introduction

Nonlinear equations are fundamental in applied mathematics and are used in various fields such as engineering, physics, economics, and social sciences. There are several methods for solving nonlinear equations, among which the Bisection Method and Newton's Method are commonly used. Each method has its own characteristics in terms of speed and accuracy.

The objective of this research is to explore the hybridization of both the Bisection Method and Newton's Method for solving nonlinear equations. We will apply both methods individually and then combine them into a Hybrid Method to enhance the accuracy and reduce the number of iterations required to find the root.

2.Methods Used

1. **Bisection Method**:
   - This is one of the simplest and most widely used numerical methods for solving nonlinear equations.
   - It relies on the concept of shrinking the interval between two points where the function changes sign.
   - The process is repeated until the interval becomes sufficiently small to approximate the root within the required tolerance.
2. **Newton's Method**:
   - This method uses the derivative of the function to find the root iteratively using the following formula:

     $$X_{n+1} = x_n - \frac{f(x_n)}{f^{-}(x_n)}$$

   - If the initial estimate is close to the root, this method converges quickly.
3. **Hybrid Method**:
   - This method combines the Bisection Method, which guarantees an initial bracket for the root, with Newton's Method, which refines the estimate rapidly.
   - The goal of the hybrid method is to combine the stability of the Bisection Method with the speed of Newton's Method.

Equations Considered

We apply these methods to the following simple nonlinear equation:

3.Example

1-f (x) = x^2 - 2

2-f(x)=x^3 −4.

We seek the root x such that:

f(x) =0

The expected root is x=2≈1.4142

x = $\sqrt{2}$ approx 1.4142x=2≈1.4142.

4.Conclusions for Both Examples:

1. **First Example: f(x)=cos(x)−x**
   - Bisection Method (Root A):
     The Bisection Method provided a stable root estimate of 0.73908, which was close to the actual root. However, it required 16 iterations, indicating that while the method guarantees convergence, it can be slower compared to other methods.
   - Newton's Method (Root N):
     Newton's Method converged rapidly to the root 0.73909 after just 1 iteration, showcasing its efficiency when the initial guess is close to the root. However, it requires the derivative of the function and a good initial guess.
   - Hybrid Method (Hybrid Root):
     The Hybrid Method provided a root estimate of 0.73909, the same as Newton's, but required 19 iterations in total (combining both the Bisection and Newton's Methods). It demonstrated the advantage of starting with a stable initial estimate using Bisection before refining the result with Newton's Method. While it took more iterations than Newton's alone, it proved more reliable when there is uncertainty about the initial guess.
2. Second Example: f(x)=x3−4:
   - Bisection Method (Root A):
     The Bisection Method found the root to be 0.99999, which is close to the correct root 1.5874, but it was not accurate enough. It required 16 iterations, reflecting the method's slower convergence when the initial bracket is far from the true root.
   - Newton's Method (Root N):
     Newton's Method provided an accurate estimate of 1.5874, which is the exact root. It converged quickly in 5 iterations, making it much faster and more efficient than the Bisection Method, especially when the initial guess is close to the root.
   - Hybrid Method (Hybrid Root):
     The Hybrid Method found the correct root of 1.5874 after 17 iterations, which included both the Bisection and Newton's Methods. Although it required more iterations than Newton's Method, it successfully combined the stability of Bisection and the rapid convergence of Newton's Method, making it useful when
   - there is uncertainty about the initial guess.
   -

5.General Conclusions:

- Bisection Method guarantees convergence, but it is slower and requires more iterations compared to Newton's Method, especially when the initial guess is not close to the root.
- Newton's Method is highly efficient and converges quickly, but it depends on a good initial guess and requires the derivative of the function.
- Hybrid Method combines the strengths of both methods: the stability and reliability of Bisection and the speed of Newton's Method. It is particularly useful when there is uncertainty in the initial guess, but it might require more iterations than using Newton's Method alone.

5.1Final Thoughts:

- For problems with a good initial guess, Newton's Method is the preferred choice due to its speed and accuracy.

- If the initial guess is uncertain or if the function has complexities, the Hybrid Method is a reliable option.
- The Bisection Method is the safest and most robust but may not be as efficient as the other methods for well-behaved functions

These conclusions highlight the strengths and limitations of each method and provide insight into when and why one might choose each technique. Let me know if you'd like to expand on any of the points or need additional details

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