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Review of Mathematical Side for Constructing an Activation Function for Fuzzy Wavenets by Deriving Composite Functions

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Abstract*— From this study we try to present new idea in way to more future wonderful ideas(as researches), exploiting mathematical side of specific ,through several ways employing very wide resources of mathematical functions and their properties on variables that with natural in improve features of activation functions, from these studies this study which construct an activation function through compose two(and more) functions and then drive it .That functions with properties to be continuous and drivable in general that will be features of produced function(s) later. We can drive membership function under fuzzy logic by this as composite function on wavelet function or vice versa ,and for many different functions(as future works) that to use it in practical applications such for NNs. Also we compose wavelets from families such SLOG mother wavelets with RASP and POLYWOG wavelets, that specifically arises from NNs representation and classification problems ,which also drivable .Each one from these functions have derivative which also can be composed to get an other activation function .*

Keywords— Fuzzy Wavenets, Rasp's Family, Polywog's Family, Slog's family.

1. INTRODUCTION

Most if not all studies and works on NNs and WNNs in this moment try to improve their works and options ,from these studies what award to improve structure of the important part in network which is the activation function. In prior study there is idea to construct activation function as summation between two sigmoid functions(which odd function) or three functions as in reference[1]. Other ways award to exploit wavelet to combine with NNs and through use wavelets as activation function in what named WNNs. A new techniques that hybrid many ways such NNs and wavelets analysis with fuzzy logic and fuzzy learning rules in scaling function as in references[2][3],or exploiting logarithmic wavelet modeling as occurred in reference[4],and more studies which that not acquainted on it. The idea to Design and development from the electronic circuits of mathematical models in order to generate activation functions employed in the Artificial Neural Networks(ANN)in reference[5],which present six models for activation functions. The work in this paper has been based on casting an activation function in head principle through composite the functions, that aim to make activation function in form of composite to get on a best features for function and then for data that process by this function. Several ways exploit this will show in more accuracy. A way depend on composite two mother wavelets(and daughters) from different wavelets families, which compose SLOG by RASP wavelet families and with POLYWAG wavelet family. While the other way by compose POLYWAG by SLOG family . Also for attempt to improve the computations these composite functions have been derived with first derivative ,and to promote the work a derivative extended to parameters wavelet scale and shift. The properties of chosen wavelet functions were helpful to forming the function which not requisite be in composed function .

Some Families of Wavelets

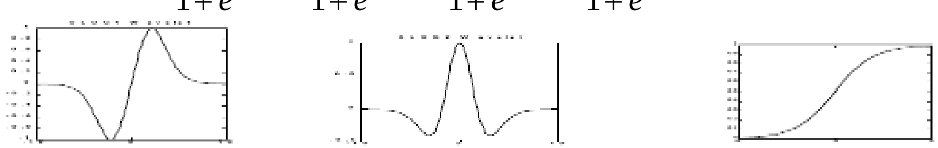
2.1 SLOG Wavelets :

This family of mother wavelets result from finite term sums of weighted and delayed logistic functions[6].A logistic function which is a type of monotonically increasing, smooth, asymptotic sigmoid (*S*-shaped) function which usually represents the threshold function at the neuron output of the neural networks model .

Sigmoid function which centered at zero $\int_{-\infty}^{+\infty} \frac{dx}{e^{x}+1} = 0$ (1)

The first SLOG mother wavelet exhibiting the following Superposition LOGistic sigmoids;

$$h_{s\log_1}(x) = \frac{1}{1+e^{-x+1}} - \frac{1}{1+e^{-x+3}} - \frac{1}{1+e^{-x-3}} + \frac{1}{1+e^{-x-1}} \quad (2)$$



(a) *(b)* *(c)*

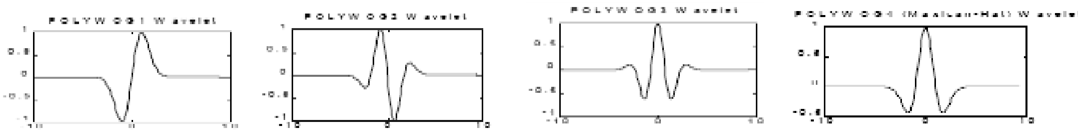
Figure(1): **(a)** $SLOG_1$ wavelet **(b)** A $SLOG_2$ wavelet **(c)** Sigmoid function .

Which represent a minimum-oscillation wavelet(down –up-down),its plot were normalized to have unity amplitudes .

2.2 POLWOG Wavelets :

These wavelets arise from POLYnomials WindOwed with Gaussians type of functions ,that represent all derivatives of Gaussian function $= e^{(-\frac{x^2}{2})}$,In this work a second derivative was applied which with form[6];

$$h_{polywog_2}(x) = k\,(x^3 - 3x).e^{-x^2/2} \quad , \; k = 0.7246 \qquad (3)$$



Figure(2): Some POLYWOG mother wavelets .

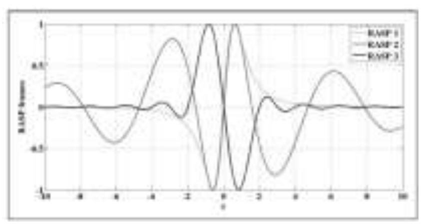
2.3 RASP Wavelets :

These Rational functions with Second-order Poles wavelets are real, odd functions with zero mean .The distinction among these mother wavelets is their rational form of functions being strictly proper and having simple/double poles of second order.

$RASP_1$ Wavelet with form[6].

$$h_{Rasp_1}(x) = \frac{x}{(x^2+1)^2} \qquad (4)$$

$$h_{Rasp_2}(x) = \frac{x\cos(x)}{x^2+1} \qquad (5)$$



Figure(3):First, second and third RASP Wavelets.

The figure shows the RASP1,RASP2 and RASP3 as RASP mother wavelets in [7]. That are real, odd functions with zero mean wavelets, arises in combination with the residue theorem of complex variables C. The distinction among these wavelets is their rational form of strictly proper functions and having simple/double poles of second order. According to the admissibility condition, the families of RASP mother wavelets have zero mean values. This through the residue theorem for evaluation of integrals.

3. Compose the Activation Function :

❶The compose of two functions in form :

$$g \circ f(x) = g[f(x)]$$

for two functions f and g on x which mean circling gof .

We can compose (or in other word circling) more than two functions. This work circling functions in (2) and (3) as putting ;

$$f(x) = h_{s\log_1}(x) \quad \text{and} \quad g(x) = h_{polywog_2}(x) \; , \; k = 0.7246$$

Compose f on g as; $g \circ f(x)$:

$$g \circ f(x) = (h_{polywog_2} \circ h_{s\log_1})(x)$$

$$= h_{polywog_2}(h_{s\log_1}(x)) = g(f(x))$$

Represented by variable x as in form (2):

$$g(f(x)) = k[(f(x))^3 - 3f(x)].e^{[-(f(x))^2/2]} \quad , \quad k = 0.7246 \qquad (6)$$

Represented by variable x as in form (1):

$$g(f(x)) = k[[\frac{1}{1+e^{-x+1}} - \frac{1}{1+e^{-x+3}} - \frac{1}{1+e^{-x-3}} + \frac{1}{1+e^{-x-1}}]^3 - 3[\frac{1}{1+e^{-x+1}} - \frac{1}{1+e^{-x+3}} - \frac{1}{1+e^{-x-3}} + \frac{1}{1+e^{-x-1}}]].e^{[-[\frac{1}{1+e^{-x+1}} - \frac{1}{1+e^{-x+3}} - \frac{1}{1+e^{-x-3}} + \frac{1}{1+e^{-x-1}}]^2/2]} \quad (7)$$

If we simplify $f(x)$ apart as in form:

$$f(x) = \frac{1+u}{e^{-2x}+u} - \frac{1+v}{e^{-2x}+v} = \frac{1+u}{w+u} - \frac{1+v}{w+v} \quad (8)$$

where $u = 1+e^{-x-1}+e^{-x+1}$ (9)

, $v = 1+e^{-x-3}+e^{-x+3}$ (10)

and $w = e^{-2x}$ (11)

Or use other form ;

$$f(x) = 2 - \frac{e^{-x-3}+u}{e^{-x-1}+u} - \frac{e^{-x+3}+v}{e^{-x+1}+v} \quad (12)$$

where $u = e^{-x-3}+e^{-2x-4}+1$ (13)

and $v = e^{-x+3}+e^{-2x+4}+1$ (14)

So the composite will be by use (9), (10) and (11) as:

$$\tilde{\lambda}(x) = g(f(x)) = k.[[\frac{1+u}{w+u} - \frac{1+v}{w+v}]^3 - 3[\frac{1+u}{w+u} - \frac{1+v}{w+v}].e^{[-[\frac{1+u}{w+u} - \frac{1+v}{w+v}]^2/2]} \quad (15)$$

for $k = 0.7246$,and $\tilde{\lambda}(x)$ the implicated function.

4.Some Properties of Composted Activation Function

Properties of functions like even $\vartheta(x) = \vartheta(-x)$,and odd function $-\vartheta(x) = \vartheta(-x)$.

For (18)and(19),function;

$$u(-x) = -(x^3 - 3x) = -u(x) \quad \text{,and for } v(x) = e^{-x^2/2} = v(-x)$$

The features of these functions apart on the composite function, in time u is odd function, v is not ,but v is even function and u is not even function.

Composite function;

$$\vartheta(-x) = \frac{1}{1+e^{-[k((-x)^3-3(-x))e^{-(-x)^2/2}]+1}} - \frac{1}{1+e^{-[k((-x)^3-3(-x))e^{-(-x)^2/2}]+3}} - \frac{1}{1+e^{-[k((-x)^3-3(-x))e^{-(-x)^2/2}]-3}} + \frac{1}{1+e^{-[k((-x)^3-3(-x))e^{-(-x)^2/2}]-1}}$$

$$\vartheta(-x) = \frac{1}{1+e^{kuv+1}} - \frac{1}{1+e^{kuv+3}} - \frac{1}{1+e^{kuv-3}} + \frac{1}{1+e^{kuv-1}} \quad (16)$$

Or $\vartheta(-x) = \frac{e^{-kuv-1}}{e^{-kuv-1}+1} - \frac{e^{-kuv-3}}{e^{-kuv-3}+1} - \frac{e^{-kuv+3}}{e^{-kuv+3}+1} + \frac{e^{-kuv+1}}{e^{-kuv+1}+1}$

When communicate the terms of function to coincide with essential function in form(17);

$$= \frac{1}{e^{-kuv+1}+1}(e^{-kuv+1}) - \frac{1}{e^{-kuv+3}+1}(e^{-kuv+3}) - \frac{1}{e^{-kuv-3}+1}(e^{-kuv-3}) + \frac{1}{e^{-kuv-1}+1}(e^{-kuv-1})$$

It is clear that this is equivalent to essential, but with set from additional parentheses are $(e^{-kuv+1}),(e^{-kuv+3}),(e^{-kuv-3})$ and (e^{-kuv-1}) ,respectively. Which can represented as polynomial with power of exponential.

$$= (e^{-kuv+1})(1st\ term.e^0 + 2nd\ term.e^2 + 3rd\ term.e^{-4} + 4th\ term.e^{-2})$$

$$= (e^{-kuv+1})(\frac{1}{e^{-kuv+1}+1} - \frac{1}{e^{-kuv+3}+1}.e^2 - \frac{1}{e^{-kuv-3}+1}.e^{-4} + \frac{1}{e^{-kuv-1}+1}.e^{-2})$$

The feature of function to be centered at origin (with mean zero);

$$\int_{\infty}^{\infty} \vartheta(x)dx = \int_{\infty}^{\infty} f(g(x))dx$$

$$= \ln(1+e^{kuv-1}) - \ln(1+e^{kuv-3}) - \ln(1+e^{kuv+3}) + \ln(1+e^{kuv+1})$$

$$= \ln(1) - \ln(1) - \ln(1) + \ln(1) = 0$$

Integration here is through depend on k,u,v as constants with respect to values of x.

The value of function at zero is zero ;

$$\vartheta(x) = \frac{1}{1+e^{1}} - \frac{1}{1+e^{3}} - \frac{1}{1+e^{-3}} + \frac{1}{1+e^{-1}} = \frac{e^{-1}}{1+e^{-1}} - \frac{e^{-3}}{1+e^{-3}} - \frac{1}{1+e^{-3}} + \frac{1}{1+e^{-1}}$$

$$= \frac{e^{-1}+1}{1+e^{-1}} - \frac{e^{-3}+1}{1+e^{-3}} = 0$$

at $u(0) = 0, v(0) = 1$

At zero $\lambda(x)$ be also zero, at $w(0) = 1, u(0) = 1 + e^{-3} + e^{-4}, v(0) = 1 + e^{3} + e^{4}$.

$$\lambda(0) = g(f(0)) = k.[[\frac{1+u}{1+u} - \frac{1+v}{1+v}]^3 - 3[\frac{1+u}{1+u} - \frac{1+v}{1+v}].e^{[-[\frac{1+u}{1+u} - \frac{1+v}{1+v}]^2/2]}$$

$$= k.[0].e^{0} = 0$$

5.Derive $f(g(x))$ Composite Activation Function:

Here derive form (15) with respect to x through chain rule by functions u,v ;

$$\frac{\partial f(g(x))}{\partial x} = \frac{\partial f(g(x))}{\partial u}.\frac{\partial u}{\partial x} + \frac{\partial f(g(x))}{\partial v}.\frac{\partial v}{\partial x} \quad (17)$$

$$\frac{\partial f(g(x))}{\partial u}.\frac{\partial u}{\partial x} = \frac{kve^{-kuv+1}}{(1+e^{-kuv+1})^2} - \frac{kve^{-kuv+3}}{(1+e^{-kuv+3})^2} - \frac{kve^{-kuv-3}}{(1+e^{-kuv-3})^2} + \frac{kve^{-kuv-1}}{(1+e^{-kuv-1})^2}$$

Here we suppose σ as , $\sigma = kuv$,and then substitute it as;

$$\frac{\partial f(g(x))}{\partial u}.\frac{\partial u}{\partial x} = kv[\frac{e^{-\sigma+1}}{(1+e^{-\sigma+1})^2} - \frac{e^{-\sigma+3}}{(1+e^{-\sigma+3})^2} - \frac{e^{-\sigma-3}}{(1+e^{-\sigma-3})^2} + \frac{e^{-\sigma-1}}{(1+e^{-\sigma-1})^2}].\frac{\partial u}{\partial x} \quad (18)$$

,with $\frac{\partial u}{\partial x} = 3(x^2 - 1)$

,and ;

$$\frac{\partial f(g(x))}{\partial v}.\frac{\partial v}{\partial x} = ku[\frac{e^{-\sigma+1}}{(1+e^{-\sigma+1})^2} - \frac{e^{-\sigma+3}}{(1+e^{-\sigma+3})^2} - \frac{e^{-\sigma-3}}{(1+e^{-\sigma-3})^2} + \frac{e^{-\sigma-1}}{(1+e^{-\sigma-1})^2}].\frac{\partial v}{\partial x} \quad (19)$$

,with $\frac{\partial v}{\partial x} = -xe^{-x^2/2} = -xv$, for $k = 0.7246$

By compile terms in (26)it will be;

$$\frac{\partial f(g(x))}{\partial x} = kv[3(x^2-1).[\frac{e^{-\sigma+1}}{(1+e^{-\sigma+1})^2} - \frac{e^{-\sigma+3}}{(1+e^{-\sigma+3})^2} - \frac{e^{-\sigma-3}}{(1+e^{-\sigma-3})^2} + \frac{e^{-\sigma-1}}{(1+e^{-\sigma-1})^2}]$$
$$- xu.[\frac{e^{-\sigma+1}}{(1+e^{-\sigma+1})^2} - \frac{e^{-\sigma+3}}{(1+e^{-\sigma+3})^2} - \frac{e^{-\sigma-3}}{(1+e^{-\sigma-3})^2} + \frac{e^{-\sigma-1}}{(1+e^{-\sigma-1})^2}]] \quad (20)$$

It is cleared here the activation function constructed from a polynomial of sigmoids and exponential functions .

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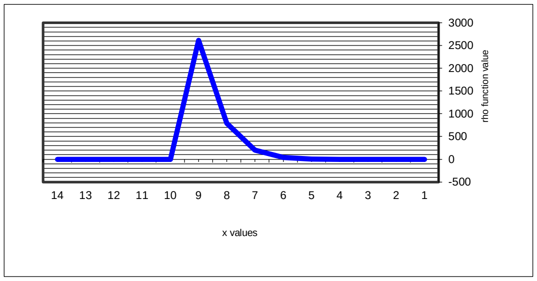
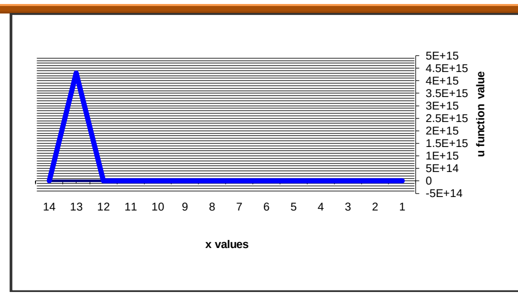
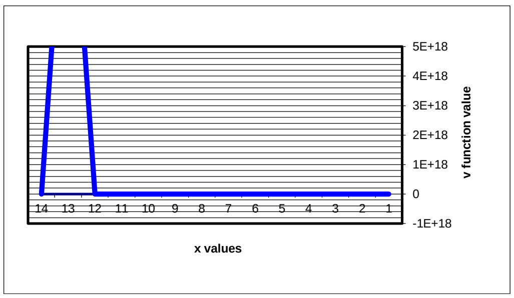


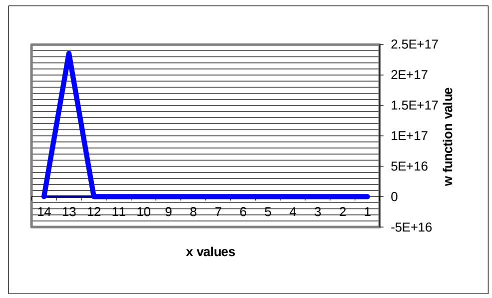
(a) A form ρ .



(b):A function u



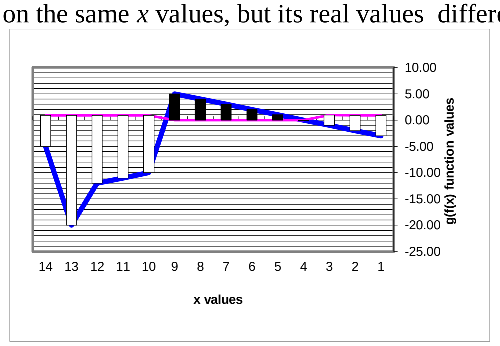
(c): A function v .



(d) :A function w

Figure(4): The functions from the composite form $g \circ f$

The functions in figures as shown defined on the same *x* values, but its real values different with change between *x*=12 into *x*=14 .



Figure(5):Graphing of $g \circ f$ function .

2. ACKNOWLEDGMENT (HEADING 5)

The preferred spelling of the word "acknowledgment" in America is without an "e" after the "g." Avoid the stilted expression "one of us (R. B. G.) thanks .". Instead, try "R. B. G. thanks.". Put sponsor acknowledgments in the unnumbered footnote on the first page.

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[1] Eason, G., Noble, B., & Sneddon, I. N. (1995). On certain integrals of Lipschitz-Hankel type involving products of Bessel functions, Phil. Trans. Roy. Soc. London, vol. A247, pp. 529-551.

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