

Review on inverse Side of the High order Composes Wavelets Families by Gaussian Fuzzy Number

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Abstract: A second direction for the fourth compose for wavelet function(s) as a second order compose for two wavelets from RASP's, SLOG's, POLYWOG'S wavelet families with a Gaussian fuzzy numbers to generate a new membership function MF. The features of the used wavelets add good features, so the generated wavelet have standard features. That all will be composed in MF to be in closed interval [0,1]. The new MF will employed in a fuzzy-models, or in control systems with standard features. The data on the functions implemented numerically and discussed to verify the activity of the new membership function.

Keywords: Wavelets, RASP wavelet, POLYWOG wavelet, SLOG wavelet, Gaussian MF.

1. Introduction;

In NNs and WNNs we want to improve works and options for taking decisions, from works that improve structure of the essential point in network is the activation function(s). In last study there is idea to construct activation function as composition between sigmoid functions for fourth order for four functions[1]. We exploit a new techniques that hybrid many ways such NNs and wavelets analysis with fuzzy logic and fuzzy learning rules in scaling function [2][3]. The idea to design present many models for activation functions from different families. The work in this paper has been based on casting many ways for activation function(s) through composite the functions, that aim to make activation function in form of composite to get more features. The data that process by these functions will process through the composition[4][5]. We suppose several ways in exploiting formulas. A way depend on composite two wavelets from families POLYWAG and SLOG by RASP wavelet family with Gaussian fuzzy number[6][7]. While the other way by compose POLYWAG by RASP family. We also attempt for improving the computations to derive resulted functions with first derivative. For promoting the work a derivative of function extended to parameters wavelet scaling and shifting. The resulted properties of composed wavelet functions were better than forming the wavelet functions .

2. Suggestion for Fourth Order Composes;

The compose among wavelets functions from RASP family with other one from SLOG family and MF through fourth order composes as by taking the second side for the functions in reference [1][3] as first suggestion;

$$f(x) = F_{RASP_1}(x) = \frac{x}{(x^2 + 1)^2} \quad \dots (1)$$

$$g(x) = F_{RASP_2}(x) = \frac{x \cdot \cos x}{x^2 + 1} \quad \dots (2)$$

$$h(x) = F_{SLOG_1}(x) = \frac{1}{1 + e^{-x+1}} - \frac{1}{1 + e^{-x+3}} - \frac{1}{1 + e^{-x-3}} + \frac{1}{1 + e^{-x-1}} \quad \dots (3)$$

$$\text{and } l(x) = e^{-x^2/2} \quad \dots (4)$$

The compose of the functions will be as;

If we reordering the compose of the functions as;

$$\begin{aligned}
 g \circ f(x) &= \frac{\left(\frac{x}{(x^2+1)^2}\right) \cdot \cos\left(\frac{x}{(x^2+1)^2}\right)}{\left(\frac{x}{(x^2+1)^2}\right)^2 + 1} = \frac{\left(\frac{x}{(x^2+1)^2}\right) \cdot \cos\left(\frac{x}{(x^2+1)^2}\right)}{\frac{x^2 + (x^2+1)^4}{(x^2+1)^4}} \\
 &= \frac{x}{(x^2+1)^2} \cdot \cos\left(\frac{x}{(x^2+1)^2}\right) \cdot \frac{(x^2+1)^4}{x^2 + (x^2+1)^4} \\
 g(f(x)) &= x \cdot \cos\left(\frac{x}{(x^2+1)^2}\right) \cdot \frac{(x^2+1)^2}{x^2 + (x^2+1)^4} \quad \dots(8)
 \end{aligned}$$

So here compute compose $h \circ l$ for (8), get;

$$\begin{aligned}
 h(l(x)) &= h\left(e^{\left[-x \cdot \cos\left(\frac{x}{(x^2+1)^2}\right) \cdot \frac{(x^2+1)^2}{x^2 + (x^2+1)^4}\right]^{1/2}}\right) \quad \dots(9) \\
 &= \frac{1}{1 + e^{-\left(e^{\left[-x \cdot \cos\left(\frac{x}{(x^2+1)^2}\right) \cdot \frac{(x^2+1)^2}{x^2 + (x^2+1)^4}\right]^{1/2}}\right) + 1}} - \frac{1}{1 + e^{-\left(e^{\left[-x \cdot \cos\left(\frac{x}{(x^2+1)^2}\right) \cdot \frac{(x^2+1)^2}{x^2 + (x^2+1)^4}\right]^{1/2}}\right) + 3}} \\
 &\quad - \frac{1}{1 + e^{-\left(e^{\left[-x \cdot \cos\left(\frac{x}{(x^2+1)^2}\right) \cdot \frac{(x^2+1)^2}{x^2 + (x^2+1)^4}\right]^{1/2}}\right) - 3}} + \frac{1}{1 + e^{-\left(e^{\left[-x \cdot \cos\left(\frac{x}{(x^2+1)^2}\right) \cdot \frac{(x^2+1)^2}{x^2 + (x^2+1)^4}\right]^{1/2}}\right) - 1}}
 \end{aligned}$$

Putting $u = (x^2 + 1)^2$, $v = x$, functions respect to x , then the form (9) will be as;

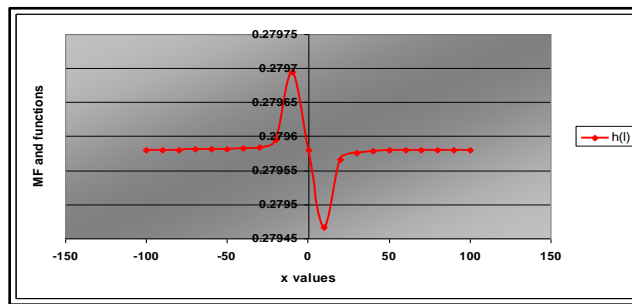
$$\begin{aligned}
 h(l(x)) &= h\left(e^{\left[-\frac{u \cdot v}{v^2 + u^2} \cdot \cos\left(\frac{v}{u}\right)\right]^{1/2}}\right) \\
 &= \frac{1}{1 + e^{-\left(e^{\left[-\frac{u \cdot v}{v^2 + u^2} \cdot \cos\left(\frac{v}{u}\right)\right]^{1/2}}\right) + 1}} - \frac{1}{1 + e^{-\left(e^{\left[-\frac{u \cdot v}{v^2 + u^2} \cdot \cos\left(\frac{v}{u}\right)\right]^{1/2}}\right) + 3}} \\
 &\quad - \frac{1}{1 + e^{-\left(e^{\left[-\frac{u \cdot v}{v^2 + u^2} \cdot \cos\left(\frac{v}{u}\right)\right]^{1/2}}\right) - 3}} + \frac{1}{1 + e^{-\left(e^{\left[-\frac{u \cdot v}{v^2 + u^2} \cdot \cos\left(\frac{v}{u}\right)\right]^{1/2}}\right) - 1}} \quad \dots(10)
 \end{aligned}$$

3. **First treatment** :Applying the values in equation (10) for variable x in the interval $[-100,100]$ with step 10, The computed values for the resulting function are positive and all in the interval $[0,1]$.as in the table (2) ,the graphic of function through many charts(pointer ,surface and radar) shown in figure(2)

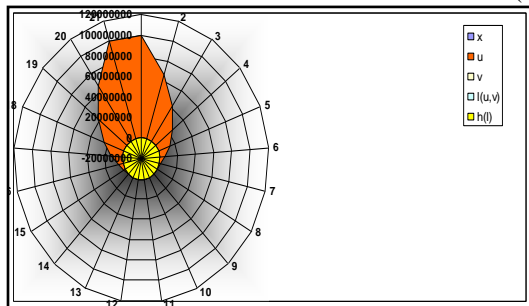
Table (1):Resulted Data for first treatment

$x = v$	u	$l(u, v)$	$h(l)$
-100	100020001	1.00000005	0.279580482
-90	65626201	1.000000686	0.279580525
-80	40972801	1.000000976	0.279580593
-70	24019801	1.000001457	0.279580704
-60	12967201	1.000002314	0.279580903
-50	6255001	1.000003997	0.279581295
-40	2563201	1.000007803	0.279582179
-30	811801	1.000018478	0.279584659
-20	160801	1.000062191	0.279594815
-10	10201	1.000490267	0.279694256
0	1	1.000000000	0.279580366

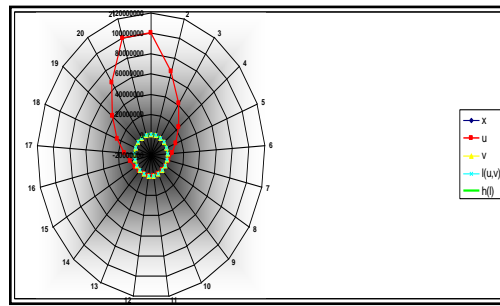
10	10201	0.999509973	0.279466497
20	160801	0.999937813	0.279565917
30	811801	0.999981523	0.279576073
40	2563201	0.999992197	0.279578553
50	6255001	0.999996003	0.279579437
60	12967201	0.999997686	0.279579828
70	24019801	0.999998543	0.279580027
80	40972801	0.999999024	0.279580139
90	65626201	0.999999314	0.279580207
100	100020001	0.9999995	0.27958025



(a)



(b)



(c)

Figure (1): Drawing of Membership Function for values in table(1)

4. Second treatment: Third Suggestion for Fourth Order Composes

The compose among wavelets functions from Polywog family with other one from SLOG family and MF through fourth order composes as by taking the functions in (1) and (4), the other suggested functions in the compose are ;

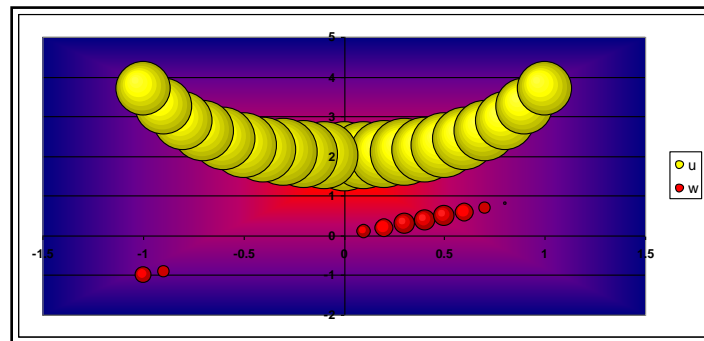
$$h_{polywog_2}(x) = k(x^3 - 3x) \cdot e^{-x^2/2}, \text{ with } k = 0.7246 \quad \dots(11)$$

$$g(x) = h_{SLOG_2}(x) = \frac{3}{1 + e^{-x-1}} - \frac{3}{1 + e^{-x+1}} - \frac{1}{1 + e^{-x-3}} + \frac{1}{1 + e^{-x+3}} \quad \dots(12)$$

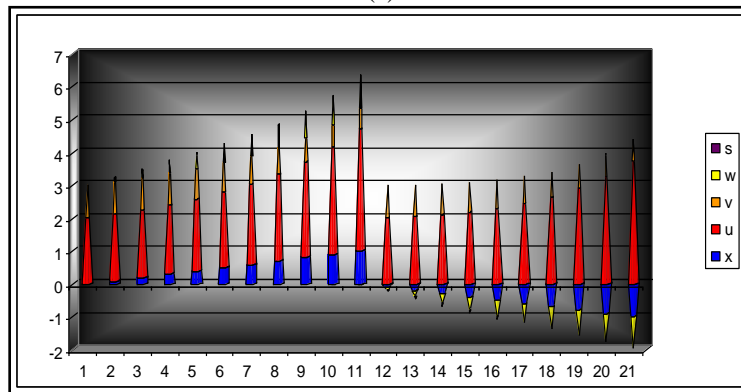
Table (2): Results from Data for second treatment

$x = w$	$u = e^{x^2} + 1$	$v = e^{-x^2/2}$	$s = (wv/(u^2))(u(\sin(v) - \cos(v)) + (2v^2 \cos(v)))$
0	2	1	0
0.1	2.010050167	0.995012479	0.040911776
0.2	2.040810774	0.980198673	0.075082894
0.3	2.094174284	0.955997482	0.096905774
0.4	2.173510871	0.923116346	0.102936944
0.5	2.284025417	0.882496903	0.092643024
0.6	2.433329415	0.835270211	0.068608089

0.7	2.63231622	0.782704538	0.036002846
0.8	2.896480879	0.726149037	0.001337193
0.9	3.247907987	0.666976811	-0.029176643
1	3.718281828	0.60653066	-0.051108278
-0.1	2.010050167	0.995012479	-0.040911776
-0.2	2.040810774	0.980198673	-0.075082894
-0.3	2.094174284	0.955997482	-0.096905774
-0.4	2.173510871	0.923116346	-0.102936944
-0.5	2.284025417	0.882496903	-0.092643024
-0.6	2.433329415	0.835270211	-0.068608089
-0.7	2.63231622	0.782704538	-0.036002846
-0.8	2.896480879	0.726149037	-0.001337193
-0.9	3.247907987	0.666976811	0.029176643
-1	3.718281828	0.60653066	0.051108278



(a)



(b)

Figure (2): Drawing of Membership Function for values with 3-D in table(2)

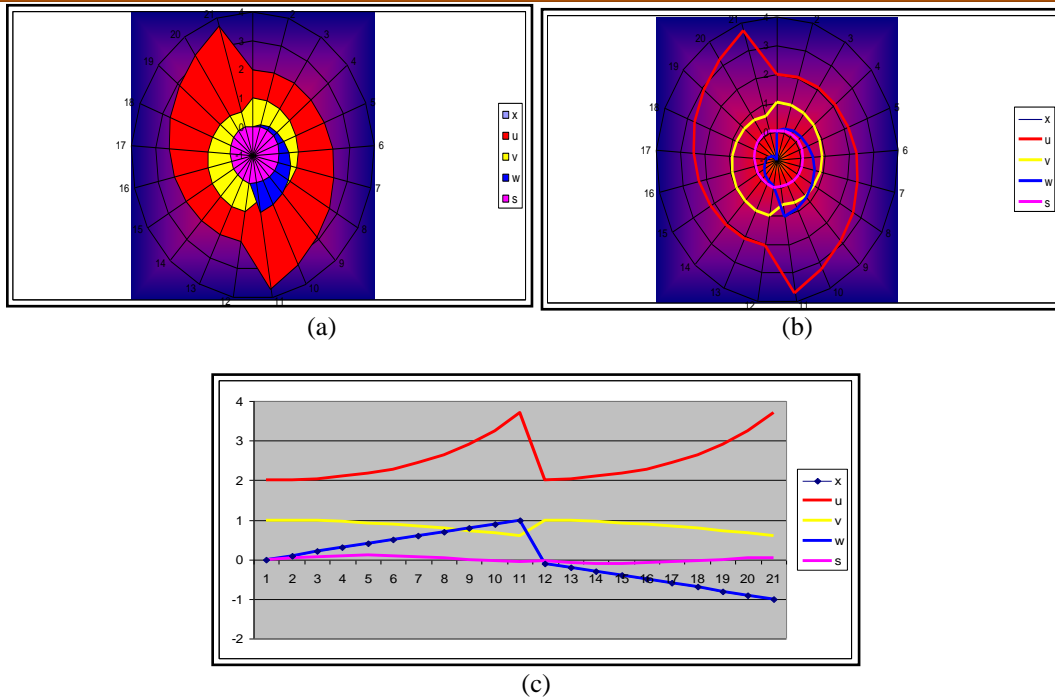


Figure (3): Drawing of Membership Functions for values in table(2) with surface figuring

Table (3): Results from Data for second treatment with $g(f(x))$

x	$g(x) = e^{-x^2/2}$	$f(x) = (x \cos(x)) / ((x^2 + 1))$	$g(f(x))$
0	1	0	0.5
0.1	0.995012	0.098515	0.504876
0.2	0.980199	0.188474	0.518081
0.3	0.955997	0.262937	0.535791
0.4	0.923116	0.317607	0.553069
0.5	0.882497	0.351033	0.565569
0.6	0.83527	0.364119	0.570887
0.7	0.782705	0.359322	0.568909
0.8	0.726149	0.339857	0.561219
0.9	0.666977	0.309088	0.550124
1	0.606531	0.270151	0.537855
-0.1	0.995012	-0.09852	0.504876
-0.2	0.980199	-0.18847	0.518081
-0.3	0.955997	-0.26294	0.535791
-0.4	0.923116	-0.31761	0.553069
-0.5	0.882497	-0.35103	0.565569
-0.6	0.83527	-0.36412	0.570887
-0.7	0.782705	-0.35932	0.568909
-0.8	0.726149	-0.33986	0.561219
-0.9	0.666977	-0.30909	0.550124
-1	0.606531	-0.27015	0.537855

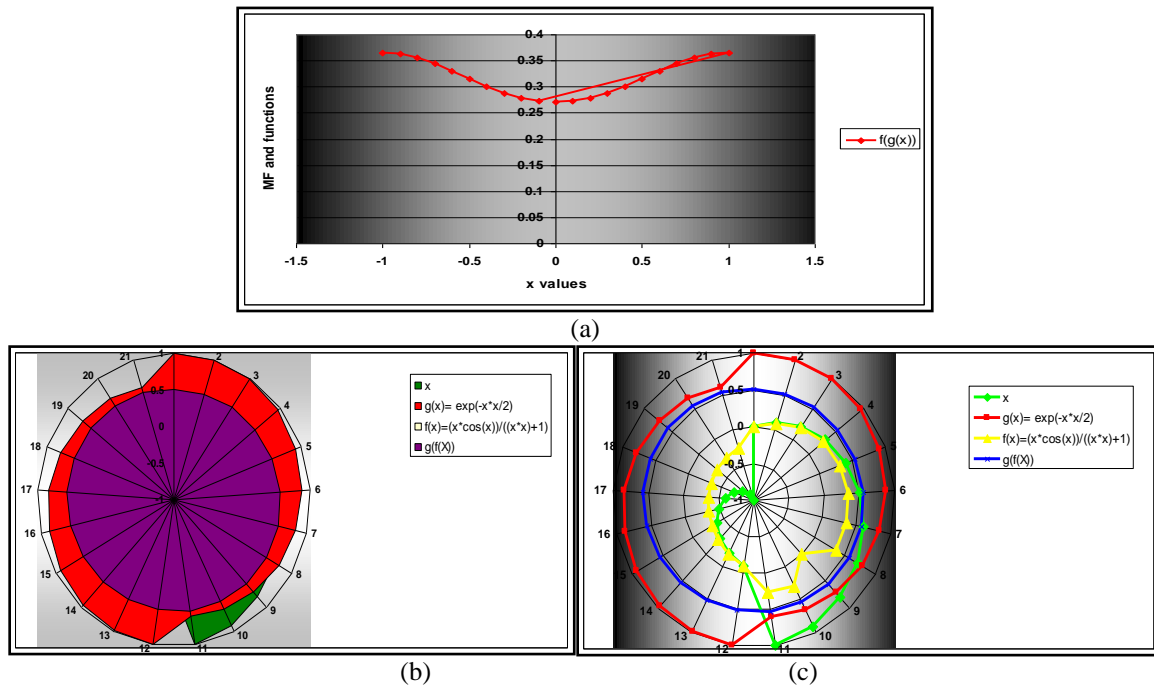


Figure (4): Drawing of Membership Functions for values in table(3) with surface and linear figuring

Take positive and negative values the results are all positive in interval [0,1]

Table (4): Results from Data for second treatment with $f(g(x))$

x	$g(x) = e(-x^2/2)$	$f(x) = (xcos(x)/x^2 + 1)$	$f(g(x))$
0	1	0	0.270151153
0.1	0.995012479	0.098515264	0.272242808
0.2	0.980198673	0.188474342	0.278373093
0.3	0.955997482	0.262936648	0.28810535
0.4	0.923116346	0.317607239	0.300706151
0.5	0.882496903	0.351033025	0.315147056
0.6	0.835270211	0.364118654	0.330126176
0.7	0.782704538	0.359321833	0.344124641
0.8	0.726149037	0.339856931	0.355510487
0.9	0.666976811	0.30908783	0.362693506
1	0.60653066	0.270151153	0.364318745
-0.1	0.995012479	-0.098515264	0.272242808
-0.2	0.980198673	-0.188474342	0.278373093
-0.3	0.955997482	-0.262936648	0.28810535
-0.4	0.923116346	-0.317607239	0.300706151
-0.5	0.882496903	-0.351033025	0.315147056
-0.6	0.835270211	-0.364118654	0.330126176
-0.7	0.782704538	-0.359321833	0.344124641
-0.8	0.726149037	-0.339856931	0.355510487
-0.9	0.666976811	-0.30908783	0.362693506
-1	0.60653066	-0.270151153	0.364318745

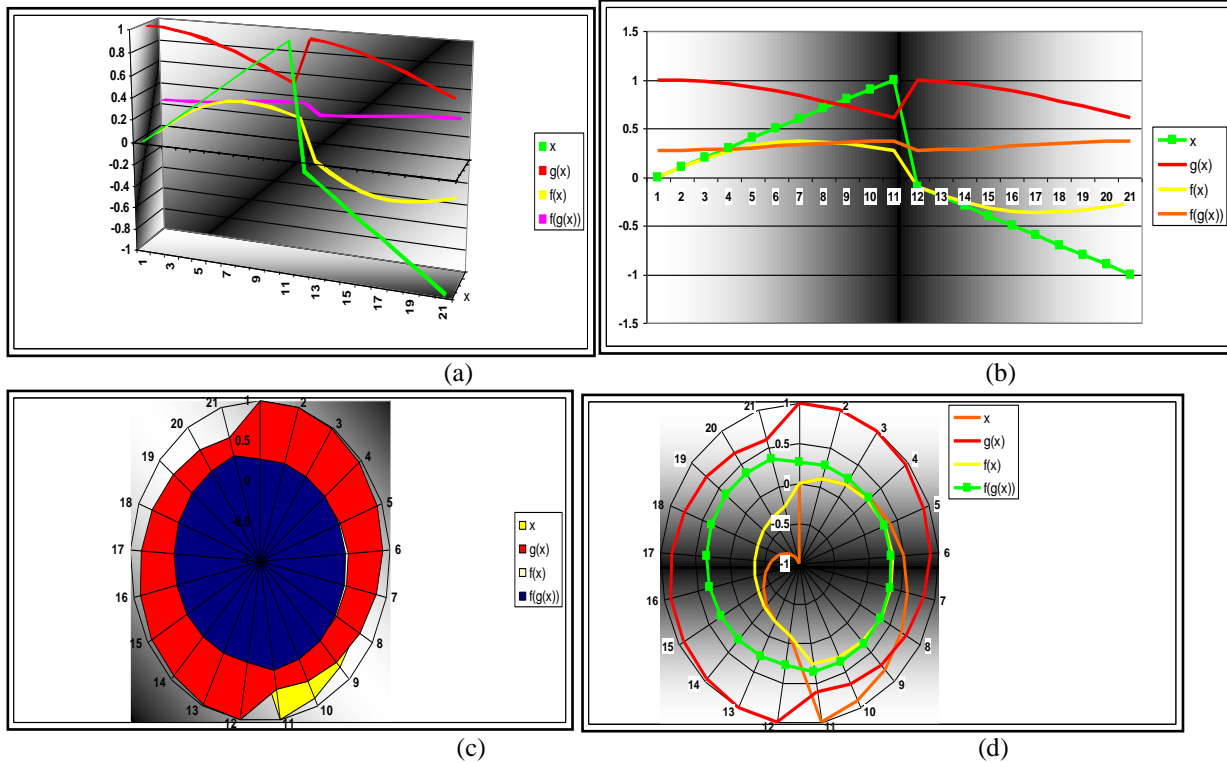


Figure (5): Drawing of Membership Functions for values in table(4) with surface figuring

Table (5): Results from Data for second treatment with $f(g(x)) = k(g(x)^3 - 3g(x)).e^{-g(x)^2/2}$

x	$g(x) = e^{-x^2/2}$	$f(g(x)) = k(g(x)^3 - 3g(x)).e^{-g(x)^2/2}$ with $k = 0.7246$
0	1	-0.878984232
0.1	0.995012479	-0.883335241
0.2	0.980198673	-0.89586328
0.3	0.955997482	-0.915010491
0.4	0.923116346	-0.938251576
0.5	0.882496903	-0.962242622
0.6	0.835270211	-0.983089852
0.7	0.782704538	-0.996750446
0.8	0.726149037	-0.999532132
0.9	0.666976811	-0.988608619
1	0.60653066	-0.962438218
-0.1	0.995012479	-0.883335241
-0.2	0.980198673	-0.89586328
-0.3	0.955997482	-0.915010491
-0.4	0.923116346	-0.938251576
-0.5	0.882496903	-0.962242622
-0.6	0.835270211	-0.983089852
-0.7	0.782704538	-0.996750446
-0.8	0.726149037	-0.999532132
-0.9	0.666976811	-0.988608619
-1	0.60653066	-0.962438218

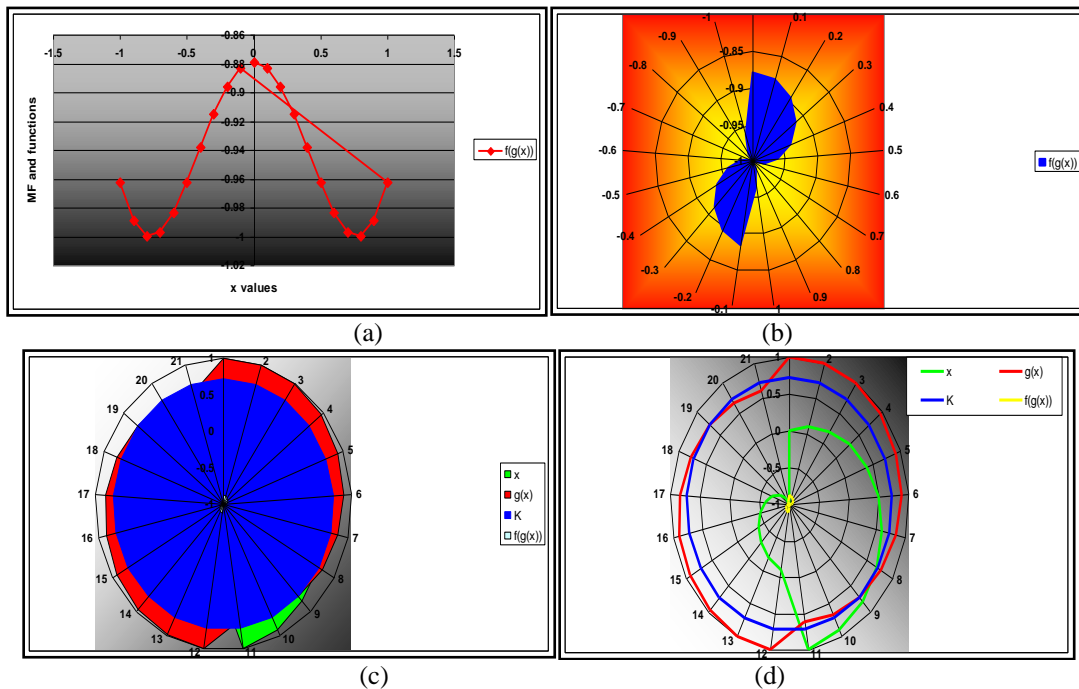


Figure (6): Drawing of Membership Functions for values in table(5) with surface figuring

5. Third treatment :

In this treatment we apply the values in equation (10) for variable x in the interval $[-100,100]$ with step 10, The computed values for the resulting function are positive and all in the interval $[0,1]$.as in the table (2) ,the graphic of function through many charts(pointer ,surface and radar) shown in figure(7)

Table (6): Results for third treatment with $I(x^2) = e^{x^2/2}$

x	$f(x) = x^2$	$I(x^2) = e^{x^2/2}$	$I(f(x))$
5.75	33.0625	16.53125	6.6156E-08
5.5	30.25	15.125	2.69958E-07
5.25	27.5625	13.78125	1.03485E-06
5	25	12.5	3.72665E-06
4.75	22.5625	11.28125	1.26071E-05
4.5	20.25	10.125	4.00653E-05
4.25	18.0625	9.03125	0.000119613
4	16	8	0.000335463
3.75	14.0625	7.03125	0.000883826
3.5	12.25	6.125	0.002187491
3.25	10.5625	5.28125	0.005086069
3	9	4.5	0.011108997
2.75	7.5625	3.78125	0.022794181
2.5	6.25	3.125	0.043936934
2.25	5.0625	2.53125	0.079559509
2	4	2	0.135335283
1.5	2.25	1.125	0.324652467
1	1	0.5	0.60653066
0	0	0	1
-5.75	33.0625	16.53125	6.6156E-08
-5.5	30.25	15.125	2.69958E-07

-5.25	27.5625	13.78125	1.03485E-06
-5	25	12.5	3.72665E-06
-4.75	22.5625	11.28125	1.26071E-05
-4.5	20.25	10.125	4.00653E-05
-4.25	18.0625	9.03125	0.000119613
-4	16	8	0.000335463
-3.75	14.0625	7.03125	0.000883826
-3.5	12.25	6.125	0.002187491
-3.25	10.5625	5.28125	0.005086069
-3	9	4.5	0.011108997
-2.75	7.5625	3.78125	0.022794181
-2.5	6.25	3.125	0.043936934
-2.25	5.0625	2.53125	0.079559509
-2	4	2	0.135335283
-1.5	2.25	1.125	0.324652467
-1	1	0.5	0.60653066

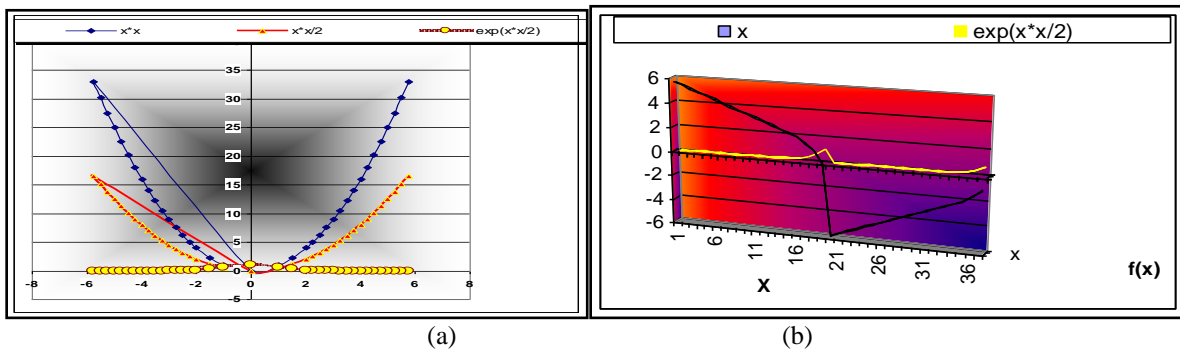


Figure (7): Drawing of Membership Functions for values in table(6) with surface figuring

Conclusion :

The work aims to proceed a new way form which try to restrict in computations by the form of activation function which changing from the composite functions properties. The success of computations in this work through experimental calculations on NNs or WNNs in spite the complex forms. The computations reflect on the features values through NNs (WNNs) in two directions. First direction that drive $RASP_1$, $RASP_2$ and $SLOG_1$ mother wavelets and then by scaling and shifting on a variable values through a composed functions to getting on an activation function with special properties. The compose of $RASP_2$ mother wavelet with mean zero by a $SLOG_2$ mother wavelet in once, which is close to Fourier transform in series of cosine functions variable(s). The graphing of composed function is changed from negative to positive values, which mean effective of properties of functions through data. Some computations be are complex. The use of activation function as communication of many functions like ; exponential, trigonometric, linear ,...,etc) in sequential serial summation or/and subtraction may eliminated or null the computations. The computational intelligence for work will drive a composite function by wavelet function or membership function. This feature performed by association property for composite function. The range of results values is accurate and important by properties of functions .

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