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An investigation of channel estimation schemes for millimeter wave massive MIMO systems with lens antenna array

[1] *Eze Gerald C.

[1] Department of Electronic/Electrical Engineering, Federal Polytechnic, Oko, Anambra State, Nigeria.
eze.gerald@federalpolyoko.edu.ng

Abstract: *This work studies the beamspace channel estimation (CE) schemes for millimeter-wave (mm-wave) massive multiple input multiple output (MIMO) systems with lens antenna array. Lens antenna array is assumed as an effective beam selection mechanism in mm-wave massive MIMO systems. Firstly, the selecting network for mm-wave beamspace systems is used to formulate the CE problem and reduce the number of required radio-frequency (RF) chains. Efficient CE schemes are needed to use the advantage of the beam selection paradigm. Previously, compressive sensing (CS)-based schemes are used to utilize existing sparsity for CE in these mm-wave beamspace systems. Among them, least squares (LS), support detection (SD) and orthogonal matching pursuit (OMP) are the most popular ones. Then, by utilization of the architectural features of mm-wave beamspace channel, we propose an improved sparsity mask detection (SMD) based CE scheme with moderate number of instants (i.e., pilot overhead). The performance analysis shows the effectiveness of the improved SMD scheme can estimate the support of sparse beamspace channel with comparable accuracy than traditional schemes. Numerical results verify that the improved SMD scheme outperforms SD scheme and other classical schemes such as both LS and OMP variants.*

INDEX TERMS Lens antenna array, compressed sensing, channel estimation, millimeter-wave system, massive MIMO.

I. INTRODUCTION

The merger of millimeter wave (mmWave) beamspace network and massive multiple-input multiple-output (MIMO), that is mmWave massive MIMO, has been mainly considered as a promising technique for the future mobile networks [1]–[5]. On the one hand, the large bandwidths available in mmWave bands (i.e., ranging from 30 GHz to 300 GHz) can substantially boost the spectral efficiency (SE) and energy efficiency (EE) of mobile networks [6], [7]. Moreover, realizing mm-wave massive MIMO in practice is challenging. The main key challenging task is that each antenna in massive MIMO usually needs one dedicated radio-frequency (RF) chain [8].The output results produced, shows high energy consumption in mm-wave massive MIMO systems, as the number of antennas plus RF chains increases [9]. Particularly, each RF chain includes mixers, high-resolution digital-to-analog converters, etc. Such structure leads to a huge hardware cost in mmWave massive MIMO systems.

Recently, the concept of beamspace MIMO with lens antenna array was proposed and considered as an effective method to significantly decrease the number of needed RF chains in mmWave massive MIMO [6].

By the employment of lens antenna array, that is an electromagnetic lens with energy focusing capability plus a matching antenna array with elements located on the focal surface of the lens, the spatial channel can be transformed into beamspace channel by focusing the signals from different directions (beams) on different antennas [10]–[11]. Thus, the mm-wave beamspace channel is sparse [6], and we can select a small number of dominant beams according to the sparse beamspace channel to significantly decrease the dimension of MIMO system and the number of required RF chains [12]–[14]. As a result, mm-wave massive MIMO with lens antenna array can be considered as a promising solution to relieve the bottleneck of huge energy consumption [6].

However, to have the EE and SE-approaching performance, beam selection needs the base station (BS) to have the information of beamspace channel of huge size, which is challenging to realize, especially when the number of RF chains is limited. To solve this challenging problem, some advanced methods based on compressive sensing (CS) have been proposed and considered in [14]–[18]. The main idea of these methods is to efficiently utilize the sparsity of mm-wave channel in the angle domain. Moreover, these methods are designed for hybrid beamforming systems [19]–[21], where the phase shifter network is realized by high-resolution phase shifters.

Related works

In [22], the authors addressed this beam selection problem by reducing the dimension of beamspace channel, and estimate the dimension-reduced channel by classical algorithms, such as least squares (LS). The LS scheme can efficiently estimate the beamspace channel with quite low computational complexity, and its pilot overhead is also low. In [23], the authors investigated the use of sparsity mask detection (SMD) scheme to estimate the beamspace channel in mm-wave massive MIMO systems with lens antenna array. The main idea is to first determine which beams with large power should be used (i.e., sparsity mask [22]-[23]) by a beam training procedure between the BS and users. The SMD scheme is often used to first select beams with higher power using a beam training process between the base station and users, and the advantage of this scheme is to lessen the training pilots. Moreover, the number of training pilots needed to check all beams is directly proportional to the number of base station antennas, which is a huge number (for e.g., 256 antennas). In [24], the authors studied the channel estimation (CE) problem for mm-wave massive MIMO systems with lens antenna array and proposed support detection (SD)-based CE scheme with low pilot overhead and reliable performance. The performance analysis of the proposed SD-based CE scheme can detect and support the sparse beamspace channel with higher accuracy than the classical CS schemes, such as orthogonal matching pursuit (OMP), compressive sampling orthogonal matching pursuit (CS-OMP) [25] and LS. The number of pilots is significantly lessened based on the SD-based CE scheme [24]. In [33], the authors proposed the use of OMP and SD together for better CE. The numerical results validated that the proposed scheme enhances the CE quality over the traditional schemes.

In the OMP scheme, firstly all channel components are considered for estimation, resulting in increased number of instants and decreased accuracy. This decrement in accuracy is majorly because of the presence of numerous channel components with negligible values. In estimation, these components generate errors, as their values are made to be nonzero according to the minimum square error (MSE) metric. Thus, it is unnecessary for the estimation of all components, and a subset of channels is sufficient for accurate estimation. While the SD scheme significantly lessens the support set, but it lacks responsiveness to different channel conditions and paths. It depends on a fixed series of indices, having prior knowledge of the number of transmitted paths.

In this work, we propose an improved SMD-based CE scheme and compare it with stage-wise orthogonal least square (SOLS), CS-OMP, LS, SMD, and SD schemes [22].

The contribution of this work is that the proposed improved SMD-based CE scheme can estimate the support of sparse beamspace channel components more accurately than the classical CS schemes.

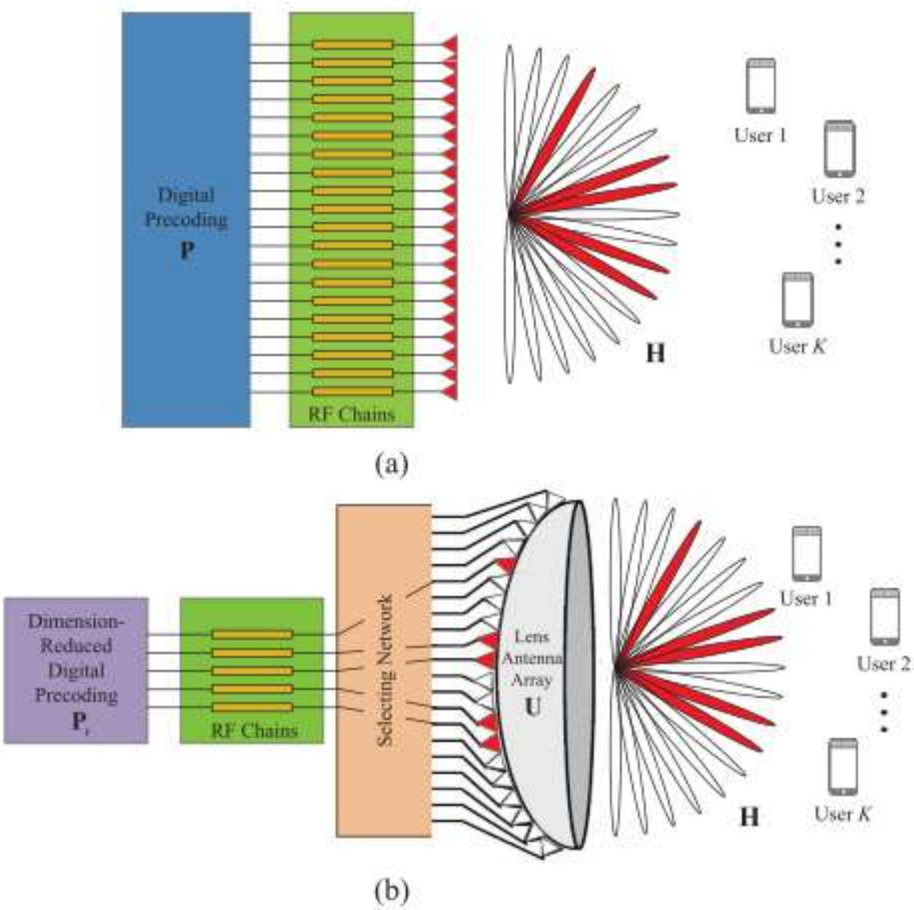

Figure 1: Comparison of system Architectures: (a) traditional mm-wave massive MIMO; (b) mm-wave massive MIMO with lens antenna array [24].

II. SYSTEM MODEL AND PROBLEM FORMULATION

Here, we assume a time division duplexing (TDD) mm-wave massive MIMO system, where the BS is having number of transmitted antennas N_T and RF transmitting chains N_T^{RF} simultaneously communicates with K single-antenna users [10]–[12].

A. Traditional mm-wave Massive MIMO

Fig. 1 (a) depicts the traditional mm-wave massive MIMO. For a narrowband system, y^{UL} is the received signal vector for all K users in the uplink can be presented by

$$y^{UL} = \sqrt{\alpha}Hs + n_k, \quad (1)$$

where α represents transmit signal to noise ratio (SNR), s is the K-dimensional transmitted signal vector with the power constraint, H is the N_T x K antenna domain channel matrix,
$H = [h_1, h_2, \ldots, h_k]$ is the uplink channel matrix, h_k is the channel vector of the k-th user, and n_k is complex additive white Gaussian noise.

B. Channel Model

The Saleh-Valenzuela channel model is adopted and it represents the sparse scattering environment of mmWave channels [13].

$$\sqrt{\frac{N_T}{L_k+1}}\sum_j^{L_k} \beta_k^{(j)}\, a\left(\psi_k^{(j)}\right) = \sqrt{\frac{N_T}{L_k+1}}\sum_j^{L_k} c_{k,j,}\, , \quad (2)$$

where $\beta_k^{(j)}$ denotes the complex gain and $\psi_k^{(j)}$ denotes the spatial direction. The $c_{k,0,}$denotes the line-of-sight (LoS) component of h_k with $\beta_k^{(0)}$ and $\psi_k^{(0)}$, $c_{k,j,}$denotes the j-th non-line-of-sight (NLoS) component of h_k and L_k represents the number of NLoS components, which can be usually achieved by channel measurement [26], $a(\psi)$ is the $N_T \times 1$ array steering vector.

C. Beamspace massive MIMO with Lens Antenna Array

The traditional channel (2) in the spatial domain can be transformed to the beamspace channel by employing a carefully designed lens antenna array [6] as shown in Fig. 1 (b). Essentially, such lens antenna array plays the role of a spatial DFT matrix U of size $N_T \times N_T$, which contains the array steering vectors of N_T orthogonal directions (beams) covering the entire space as

$$\boldsymbol{U} = \left[a(\dot{\psi}_1), a(\dot{\psi}_2), \ldots, a(\dot{\psi}_{N_T})\right]^{\boldsymbol{H}}, \quad (3)$$

where $\dot{\psi}_n$represents the spatial directions pre-defined by lens antenna array. Then, according to Fig. 1 (b), the system model of beamspace massive MIMO with lens antenna array can be denoted by

$$y^{UL} = \sqrt{\alpha}\, UHs + n_k, \quad (4)$$

where UH is the beamspace channel matrix between the BS and the kth user. The beamspace channel UH has a sparse structure [6], because of the limited number of dominant scatterers in the mm-wave prorogation environments [15]. We assume $N_T^{RF} = K$ .To obtain the EE and SE-approaching performance, beam selection needs the information of beamspace channel.

III. BEAMSPACE CHANNEL ESTIMATION

We introduce a pilot transmission strategy based on the specific structure of mm-wave massive MIMO with lens antenna array. Then, design of an adaptive selecting network is used to achieve the efficient measurements of beamspace channel for channel estimation. After that, an improved SMD-based CE scheme is proposed to estimate the beamspace channel with low pilot overhead and reliable performance. Lastly, the complexity and performance analyses are given to present the advantages of our scheme.

A. Pilot Transmission

For uplink TDD CE of the beamspace channel, all users require to transmit the pilot sequences to the BS over Q instants that is each user transmits one pilot symbol in each instant, and we consider that the beamspace channel remains unchanged within such channel coherence time e.g., Q instants) [27].The channel coherence time impact on the pilot overhead are presented in [22]. It is very pertinent to note that the channel coherence time is very small because of the high carrier frequency; it still has a huge number of symbols due to the large mm-wave bandwidth [22]. In this work, we assume the pilot transmission strategy, where Q instants are divided into M blocks and each block has instants $Q = MK^2$, the pilot transmission strategy proposed and considered in [22] and [24] is applied here. For the m-th block, we define ϑ_m of size $K \times K$ as the pilot matrix, which has K mutually orthogonal pilot sequences transmitted by K users over K instants [27]-[28]. Then, according to Fig. 1 (b) and the channel reciprocity [17] in TDD systems, the received uplink signal matrix Y_m^{UL} of size $N_T \times K$ at the BS in the m-th block can be shown as

$$Y_m^{UL} = UH\vartheta_m + \vartheta_m n_k,\ m = 1, 2, \ldots, M, \quad (5)$$

where, $\vartheta_m n_k$ is the is noise matrix in the m-th block and $\vartheta_m n_k = N_C(0, \sigma^2)$ has circularly symmetric complex Gaussian distribution having circular symmetry N_C with zero mean and variance σ^2 is the uplink noise power. As the uplink pilot power is normalized to 1, $1/\sigma^2$ is taken as the uplink SNR.

B. Selecting Network

The selecting network is a one-bit phase shifter that replaces the selecting network block as shown in figure 1(b), the selecting network is used to reduce number of RF chains at the BS, the combiner W_m of size $N_T \times K$ to combine the received uplink signal matrix Y_m^{UL} (5) and also designed to minimize cross-correlation [24]. Then, we can achieve R_m of size $K \times K$ in the baseband sampled by $N_T^{RF} = K$ RF chains as

$$R_m = W_m^H Y_m^{UL} = W_m^H UH\vartheta_m + W_m^H \vartheta_m n_k, \quad (6)$$

After that, BS correlates R_m with ϑ_{jk}^* to estimate $\dot{H}$

$$Z_m = R_m \vartheta_m^* = W_m^H \dot{H} + N_k, \quad (7a)$$

where $\vartheta_m \vartheta_m^* = I_K$ and $\vartheta_m^* \vartheta_m = I_K$. By multiplying the pilot matrix ϑ_m on the right side of (6), the $K \times K$ measurement matrix Z_m of the estimated beamspace channel matrix $\dot{H}$ is obtained and N_k is the effective noise matrix. The m matrix of the $\dot{H}$ channel measurement block is equal to:

$$Z_K = W\dot{h_k} + n_k, \quad (7b)$$

Thus, we focus on estimating the beamspace channel $\dot{h_k}$ of the k-th user without loss of generality, and the similar approach can be directly applied to other users to obtain the complete beamspace channel. The beamspace channel $\dot{H}$ estimate is $\dot{H} = [\dot{h_1}, \dot{h_2}, \dots, \dot{h_k}]$. Fortunately, the beamspace CE does not suffer from the serious SNR loss because of the beamforming gains always exist in mm-wave massive MIMO systems with lens antenna array. Moreover, estimating the beamspace channel with low pilot overhead is tasking due to the number of RF chains is limited while the size of beamspace channel is huge [29].

C. Improved SMD scheme

After W has been designed by the selecting network, (7) can be solved by traditional CS schemes. Moreover, during uplink transmission mode, users' transmit power to the BS is always low, which is the typical case in mmWave massive MIMO systems because of the lack of beamforming gain and the low transmit SNR of users, will be corrupted by noise. To recover the original signals at the base station by traditional CS schemes is usually inaccurate, leading to the degraded performance. By utilization of the architectural characteristics of mmWave beamspace channel, we propose an improved SMD-based CE, which can detect the support more accurately and obtain better performance than traditional CS schemes, especially in the high and low SNR regimes. The pseudo-code of the proposed improved SMD-based CE is summarized in Algorithm 1, which can be explained as follows.

Algorithm 1: *Improved SMD scheme*

```
1 Input: measurement vector (Z_m) in (7), combining matrix(W) in (7), L, V and Order
2 CE using OMP Algorithm
3 h_0 = OMP (z,W)
4 n = length (h_0); Z = z
5 for l = 1: L do
6       order = max |h_0|
7       sel_matrix (k, :) = order - V/2: 1: order + V/2
8       for i = 1: length (sel_matrix (l, :))
9             if sel_matrix (l, i) > n
10                  sel_matrix (l, i) = sel_matrix (l, i) - n
11            else if sel_matrix (l, i) < 1
12                  (l, i) = sel (l, i) + n
13            end
14      end
15      W2 = W(:,sel_matrix (l, :))
16      [M, ~] = size (W_2)
17      h_2 = W_2 (W_2 * W_2^H + I_M)^-1 Z   %MMSE
18      temp = zeros (n,l)
19      temp ( sel_matrix (l, :)) = h_2
20      if l ≥ 1
21            Z = Z - W * temp;
22            CE using SD Algorithm
23      h_0 = SD (Z, W)
24      end
25 end
26 sel_final = unique (reshape (sel_matrix, L× V, l));
27 W_final = W (:, sel_final);
28 [MN, ~] = size (W_final);
29 h_k = W_final (W_final * W_final^H + I_MN)^-1 z   %MMSE
Output: Estimated beamspace channel h_k
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The improved SMD-based CE is integrated with minimum mean square error (MMSE) algorithm at steps 17 and 29, and combination of OMP and SD schemes at steps 3 and 23. The nonzero elements of W are estimated by MMSE algorithm instead of LS algorithm used in SMD and SD
schemes as explained in [22]-[24].

D. Channel Estimation Performance

In assessing the performance of the Beamspace mmWave MIMO CE schemes with Lens Antenna Array, we numerically evaluate the Normalized Mean-Square-Error (NMSE) criterion [30]-[31], which is defined as:

$$NMSE = \mathcal{E}\left\{10\log_{10}\left\|\dot{H} - UH\right\|_F^2 / \|UH\|_F^2\right\} \quad (8)$$

Representing $\dot{H}$ by estimation for the true beamspace channel matrix UH with any of the considered schemes. The assumed benchmark CE techniques are based on compressive sensing and matrix completion tools [30]-[31].

1) Performance measures

Two performance measures considered are the EE and the SE. The EE is measured in (Gbits /J) while the SE is measured in (bits/s/Hz). The achieved SE for NMSE is given by [30]-[31]

$$SE = \mathcal{E}\left\{\log_2 det\left|I_{N_T} + N_T(\sigma^2 + NMSE)\right|^{-1} UHH^H U^H\right\} \quad (9)$$

EE is defined as ratio between system SE and power consumption

$$EE = \frac{W \times SE}{P_T + P_{BB} + KP_{RF} + KP_{SW}} \quad (10)$$

where W is the system bandwidth, P_{BB} is the amount of power consumed by the baseband signal processing, P_{RF} is the amount of power consumed by the RF chain, P_T is the transmit power, and P_{SW} is the amount of power consumed by the switch. The total power consumption P can thus be expressed as $P = P_T + P_{BB} + KP_{RF} + KP_{SW}$.

IV. SIMULATIONAND EXPERIMENTAL RESULTS

In the numerical evaluation, we assume a typical mm-wave massive MIMO system, where the BS equips a lens antenna array with $N_T = 256$ antennas and $N_T^{RF} = 16$ RF chains to serve $K = 16$ users. We consider a Saleh Valenzuela channel model with one LoS component, $L_k = 2$ NLoS components, $\psi_k^{(0)}$ and $\psi_k^{(j)}$ have the i.i.d. uniform distribution within [-1, 1], $\beta_k^{(0)} \sim N_C(0,1)$, and $\beta_k^{(j)} \sim N_C(0, 10^{-5})$. Lastly, the uplink SNR is $1/\sigma^2$. The results presented come from averaging 100 independent Monte Carlo realizations.

For improved SMD based channel estimation, we retain $V = 19$ strongest elements as analyzed above for each channel component. For Traditional OMP based CE [32], we consider that the sparsity level of the beamspace channel is equal to 24, and we also add small modification on the designed sensing matrix to enable it to be applied in mm-wave massive MIMO systems with lens antenna array. In this work, we consider that CE schemes employ $Q = 96$ instants (i.e., $M = 6$ blocks), and the SMD-based CE scheme employ $Q = N = 256$ instants for pilot transmission.

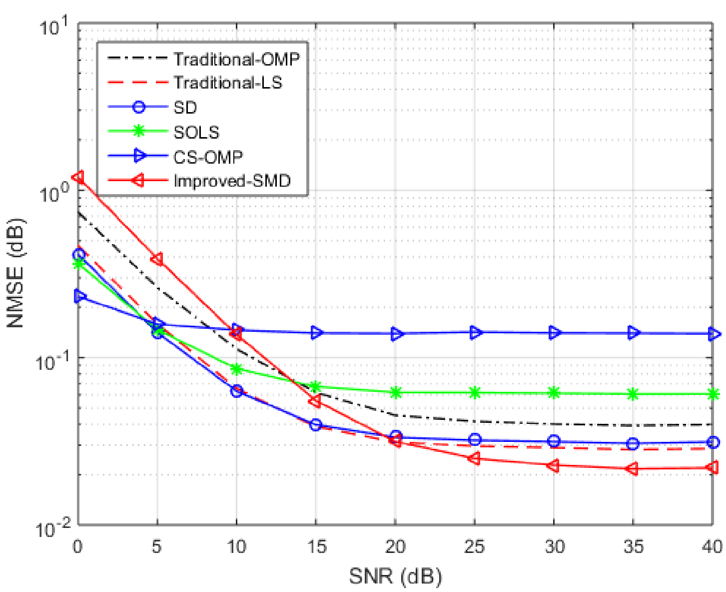

Fig. 2: NMSE performance comparison versus SNR among Traditional -OMP, Traditional -LS, SD, SOLS, CS-OMP and improved SMD-based channel estimation.

Figure 2 presents the impact of normalized mean square error (NMSE) performance comparison among the Traditional-OMP, Traditional-LS, SD, SOLS, CS-OMP and improved SMD-based CE schemes, where the performance of NMSE of different CE schemes referred to (8). We can see that the improved-SMD based CE scheme has best NMSE performance from SNR range of 20dB to 40dB with $Q = 96$ instants. The NMSE performance of both traditional-LS and SD are close to each other. Additionally, we can see that traditional-LS and SD based CE schemes enjoy higher accuracy than improved-SMD, Traditional–OMP, SOLS, CS–OMP based CE schemes when the SNR is low (i.e., less than 20 dB).We can deduct the fact that when SNR is high, the total number of instants Q is huge enough, Traditional–OMP and SOLS based CE schemes can perfectly estimate the support of the beamspace channel. Thus, we can conclude that the improved-SMD-based CE scheme is good at high SNR due to the NMSE performance of all considered CE schemes are steady. This is the reason that although the nonzero elements of beamspace channel can be estimated perfectly with sufficiently high SNR, the error generated by regarding the elements with small power as zero does not disappear.

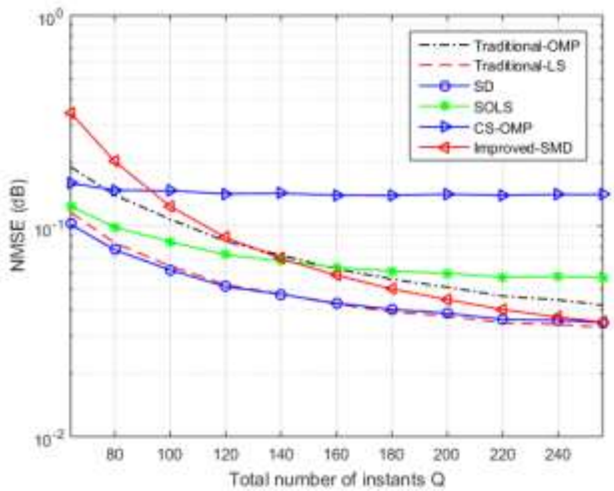

Fig. 3: NMSE performance comparison versus the total number of instants Q for pilot transmission.

Figure 3 depicts the impact of normalized mean square error (NMSE) performance comparison against the total number of instants Q for pilot transmission. We can see that to obtain the same NMSE value, the total number of instants Q needed by both traditional-LS and SD based CE is much lower than improved-SMD, Traditional–OMP, SOLS, CS–OMP based CE schemes. For instance, to obtain the NMSE of 1×10^{-1}, the total number of instants needed by both traditional-LS and SD based CE is Q = 60, while the improved-SMD based CE scheme only needs Q = 120 instants. Thus, as value of NMSE decreases, the number of instants Q (i.e., pilot overhead) needed in improved-SMD based CE is the third best among the CE schemes.

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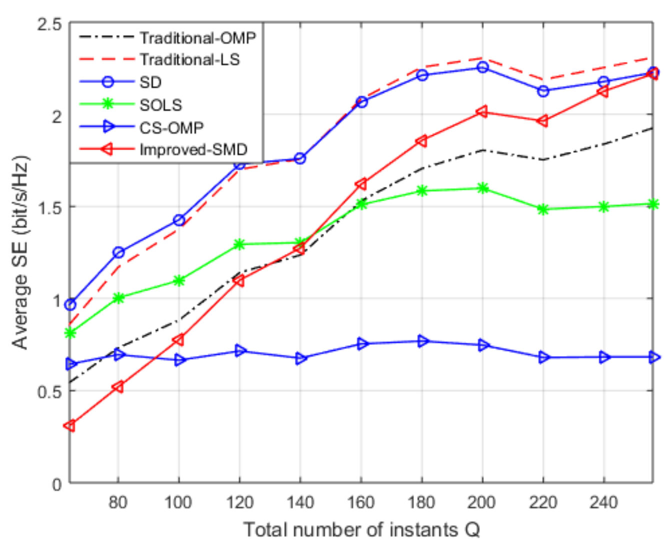

Fig. 4: Average SE performance comparison versus the total number of instants Q for pilot transmission.

Figure 4 depicts the impact of SE performance comparison against the total number of instants Q for pilot transmission. We can observe that both the traditional-LS and SD based CE schemes have highest SE values ranging from 1bit/s/Hz to 2.25bit/s/Hz for

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the number of instants Q ranging from 60 to 256.The SE performance gap between is traditional-LS and SD based CE schemes is very small. The improved-SMD based CE has the third best performance for the number of instants Q ranging from 160 to 256 followed by Traditional–OMP and SOLS, while CS–OMP based CE has the worst performance because of huge pilot overhead.

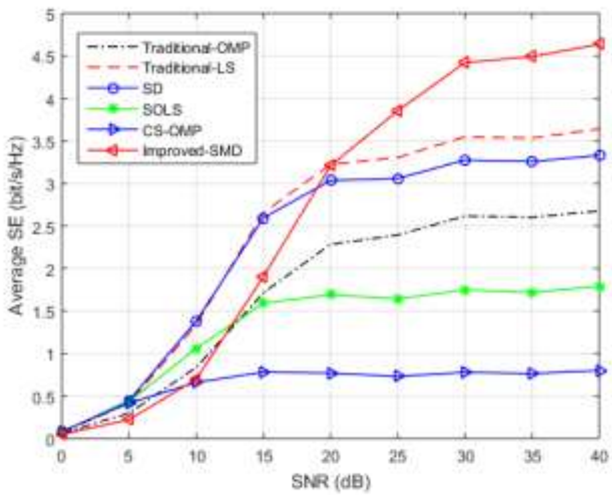

Fig. 5: Average SE comparison against SNR for the sparse scattering environment.

We also present the SE comparison results plotted against the SNR in Fig. 5, where the total number of instants $Q = 96$ and the performance of Average SE of different CE schemes referred to (9). One can observe that the improved-SMD based CE has the highest average SE among other schemes such as Traditional-OMP, Traditional-LS, SD, SOLS, and CS-OMP for SNR ranging from 20dB to 40dB. The Traditional-LS and SD based CE schemes have second and third best SE performance ranging from 15dB to 40dB, respectively. Performance gap between the Traditional-LS and SD based CE schemes is very tight, when SNR is less than 15dB. The Traditional-OMP and SOLS based CE schemes have fourth and fifth best SE performance, respectively. The CS-OMP CE scheme has the poorest SE performance among schemes. The SE values of improved-SMD, Traditional-LS, SD, Traditional-OMP, SOLS, and CS-OMP schemes at SNR = 40 dB are 4.62bit/s/Hz, 3.64bit/s/Hz, 3.45bit/s/Hz, 2.70bit/s/Hz, 1.64bit/s/Hz and 0.83bit/s/Hz, respectively.

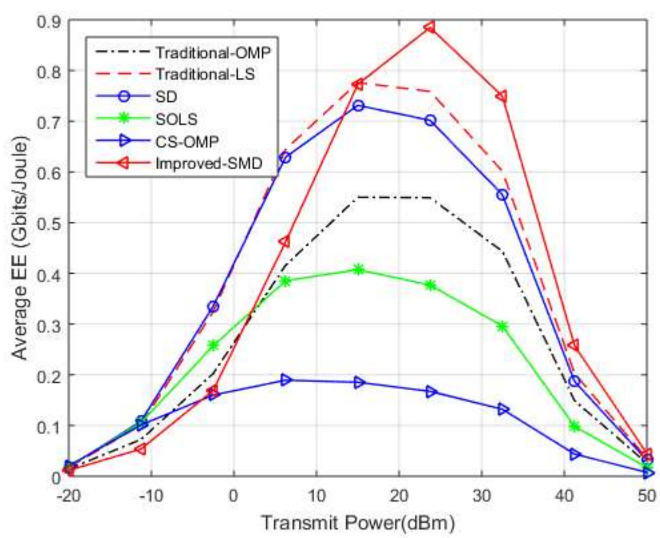

Fig. 6: EE comparison versus transmit power for the sparse scattering environment.

Figure 6 presents the impact of EE performance comparison against the transmit power, where the total number of instants $Q = 96$ and the performance of Average EE of different CE schemes referred to (10). One can observe that the reduction of the power consumption in the CE schemes produces an increase in terms of EE. The improved-SMD based CE scheme outperforms other schemes such as Traditional-OMP, Traditional-LS, SD, SOLS, and CS-OMP, when the transmit power range is from 16.7dBm W to 50dBm. The EE performance gap between the traditional-LS and SD based CE schemes is negligible, when the transmit power range is from -20dBm W to 4dBm. The Traditional-LS and SOLS schemes are the fourth and fifth best EE performance CE schemes.

It can be found that CS–OMP based CE has severely degrading EE performance and the worst among all considered schemes. The reason is that the RF chains increase the power consumption. Beside, when the transmit power is increased sufficiently more than power consumed by the RF chain, the baseband signal processing, and the switch in the power consumption model, increasing transmit power will decrease EE instead. The EE increases with the transmit power at least in the considered range of values, the EE maximum values of improved-SMD, traditional-LS, SD, Traditional-OMP, SOLS, and CS–OMP are 0.88Gbits/Joule, 0.78 Gbits/Joule, 0.72 Gbits/Joule,0.55Gbits/Joule,0.40 and 0.1.99 Gbits/Joule, respectively.

V. CONCLUSIONS

This work is based on improving SMD scheme that will solve CE problem for mm-wave massive MIMO systems with lens antenna array. Lens antenna array is considered as an effective beam selection mechanism in mm-wave massive MIMO systems. The selecting network is used to formulate the beamspace CE problem. Then, we propose an improved SMD based CE scheme with moderate number of instants (i.e., pilot overhead).The performance analysis verifies that the improved SMD based CE scheme can support sparse signal recovery with low complexity, when compared with that of Traditional-LS scheme. These CE schemes have their own advantages and disadvantages. Numerical results show the effectiveness of using the improved SMD scheme to obtain better NMSE performance, SE and EE performance than Traditional-LS, SD, Traditional-OMP, SOLS, and CS-OMP schemes.

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