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Review of the Gamma function and Its Applications

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Abstract **:** *In this research, there is a review of the use of the Gamma function to evaluate definite integrals involving eplas and instantaneous transformations using simple techniques. Research shows that the Gamma function is not only a formula and a proof, but it is a performance basis for applications in the evaluation of integrals and the simplification of the proofs of some important identities and theorems*

Keywords:- Gamma function, Laplace transform, Fourier transform

1-Introduction

The importance of integral transforms is that they provide powerful operational methods for solving initial value problems and initial value boundary value problems for equations differential and linear integrals. the transformations are so effective that they can convert systems of equations differential and integral equations into algebraic equations [1]

integral transformations are widely used and therefore there are many works on the theory and application of integral transformations such as , Mellin , Sumudu , Temimi ,Yang, Novel and Shehu ,to name a few [5,6,7,8,9,10].

Recently, Tariq Elzaki translated integral transformations again, called Elzaki transformation and more applied is the solution of ordinary partial differential equations and the system of partial differential equations [11]. Introduced by the Swiss mathematician Leonhard Euler in the 18th century, the Gamma function is the extension of the factorial function to the real numbers. Both the Beta and Gamma functions are very important in calculus because complex integrals can be moderated to simpler functions using the Beta and Gamma functions[2].

2**.** Definitions and Basic Properties of the Gamma Function

Definition:[2]
The Gamma function T(σ) is defined by

T(σ) = $\int_0^\infty t^{\sigma-1}\ e^{-\delta} d\delta$ where $\sigma > 0$

Important results

1)T(1)=1

2)T(ε +1)= ε T(ε) = ε!

3)T(σ) T(1- σ)= $\frac{\pi}{\sin \pi\sigma}$

4)T(2 σ)= $\frac{2^{2\sigma-1}}{\sqrt{\pi}} T(\sigma) T(\sigma + \frac{1}{2})$

5)T(σ)= $\int_0^\infty t^{\sigma-1} e^{-\delta} d\delta$

6) $\int_0^{\frac{\pi}{2}} \cos^{2\sigma-1}\theta \sin^{2Y-1}\theta\, d\theta = \frac{\mathrm{T}(\sigma)\ \mathrm{T}(\aleph)}{2\mathrm{T}(\sigma\aleph)}$

Definition(2) :[12]

Laplace transforms of a functionγ:[0,∞) → R denoted by L[γ](δ) is defined as

L[γ](δ) =$\int_0^\infty \gamma(\varepsilon)e^{-\delta\varepsilon}$ dε

γ (δ)= $\lim\limits_{\rho\to\infty} \int_0^\infty \gamma(\varepsilon)e^{-\delta\varepsilon}$ dε

where $\delta \in R$ is a parameter of the transform.

Definition(3) :[4]

Let f:R→ β the fourier transform of f∈ $L(R)$, , is defined by

F(γ)(ε)=$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \gamma(\mu)e^{-(i\mu\varepsilon)}$ dμ

For $\varepsilon \in R$ for which the integral exists.

3.1 Laplace transform [12]

The Laplace transform is a powerful mathematical tool with diverse applications in different field , such as engineering , physic, economics, and control theory, It is a mathematical technique that converts the time function in to an s function , allowing the function to be analyzed in the frequency domain.

Laplace transforms are used in solving initial value problems some techniques for solving systems of ordinary differential equations [ED] including solving with the Laplace transform. We will find the Laplace transforms of some functions that are very useful for solving problems in science and engineering with the Gamma function. We will demonstrate the application for three different functions, and the idea can be used for many other cases:

Proposition1[3]

Let f be a continuous function in R. here are the Laplace transforms given functions

.

$(i)\gamma(\delta) = \delta^n e^{a\delta}, l\{\gamma(\delta)\} = \dfrac{\varepsilon!}{(\omega-\alpha)^{\varepsilon+1}}$, $\omega > \alpha$

Using the Gamma function, that is : γ (ω)= $\frac{\varepsilon!}{(\omega-\alpha)^{\varepsilon+1}}$, $\omega > 0$

$(ii)\gamma(\delta) = e^{a\delta}\sin w\delta\,, l\{\gamma(\delta)\} = \dfrac{w}{(\omega-\alpha)^2+w^2}$, $\omega > \alpha$

By taking the Gamma function to both sides, yields: γ (ω)= $\frac{w}{(\omega-\alpha)^2+w^2}$, $\omega > 0$

$(iii)\gamma(\delta) = e^{a\delta}\cos w\delta\,, l\{\gamma(\delta)\} = \dfrac{\omega-\alpha}{(\omega-\alpha)^2+w^2}$, $\omega > \alpha$

We can apply the Gamma function, that is : γ (ω)= $\frac{\omega-\alpha}{(\omega-\alpha)^2+w^2}$, $\omega > 0$

3.2 **Fourier transform[4]**

This is a mathematical process used to transform a mathematical function with a real variable and complex values into another function. The Fourier transform plays an important role in the theory of signals and systems.

Let's start with the popular one-sided descending exponential which is defined as

F(δ)=[$\begin{matrix} 0 & \delta < 0 \\ e^{-\delta} & \delta \geq 0 \end{matrix}$

Proposition2 [3]

The Fourier transform of the one- sided decaying exponential is

F[γ](ϑ) = $\frac{1}{(i\vartheta+1)\sqrt{2\pi}}$

We applied Gamma function F[γ](ϑ) = $\frac{1}{\sqrt{2\pi}}\{\frac{1}{(i\vartheta+1)}\}$

The next to consider is the one- sided growing exponential which defined as

F(δ)=[$\begin{matrix} 0 & \delta < 0 \\ e^{\delta} & \delta \geq 0 \end{matrix}$

Proposition3 [3]

The Fourier transform of the one- sided growing exponential is

F[γ](ϑ) = $\frac{1}{(i\vartheta-1)\sqrt{2\pi}}$

We applied Gamma function F[γ](μ) = $\frac{1}{\sqrt{2\pi}}\{\frac{1}{(i\vartheta-1)}\}$

The next to consideration is the double – sided exponential which is defined as

F(δ) $= e^{-\delta}$,$\forall$ δ

Proposition4 [3]

The Fourier transform of the double – sided exponential is

F[γ](ϑ) = $\frac{1}{1+\vartheta^2}\sqrt{\frac{2}{\pi}}\delta$

We applied Gamma function F[γ](ϑ) =$\frac{1}{1+x^2}\sqrt{\frac{2}{\pi}}$

4-Conclusion

Applying the Gamma function to the Laplace and Fourier transform of some functions this makes the Gamma function a powerful tool in solving some mathematical problems. We will also apply the Gamma function to other transformation in the future.

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