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Some Results of Intuitionistic Fuzzy Subalgebras in BD-algebra

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Abstract: This study introduces new forms of intuitionistic fuzzy structures within BD -algebras, focusing on multiplication-based fuzzy subalgebras. It examines their interconnections with extension-based counterparts and explores how these fuzzy constructs behave under algebraic homomorphisms.

Key words: fuzzy subalgebra, intuitionistic fuzzy subalgebra, multiplication intuitionistic fuzzy subalgebra, image (pre-image) of subalgebra.

1- *Introduction:*

In 1966 by Y .Imai and K.Iseki introducee d the notion of BCKalgebra[24], In 1998, Y. Ɓ. Jun, E. H. Rogh and H. S. Kím introducee d the notion of a BH- ɑ lge brɑ , and the notion of ideal of a BH- ɑ lge brɑ [12]. In 2022, T.Bantaojai and et. cl. introducee d the notion of a BD-algebra [1-3]. In this paper, we define the concepts of intuitionistic fuzzy subalgebra in BD-algebra and multiplication intuitionistic fuzzy subalgebra in BD-algebra. Also, we give some theorems image (pre-image) of subalgebra of them are debated.

Def. 2.1. ([1-3]).
A *BD*-algebra (BD-A) is a non-empty set ζ with a constant o and a binary " $\diamond$ " satisfying the following axioms hold $\forall \varepsilon, \eta \in \zeta$, if :
(1) $\varepsilon \diamond o = \varepsilon, \forall \varepsilon \in \zeta$,
(2)$(\varepsilon \diamond \eta) = o$ and $\eta \diamond \varepsilon = o$, $then$ $\varepsilon = \eta$.
***Remark* 2.2[1-3].**
A *BD-A* can be (partially) order by $\varepsilon \leq \eta$ if and only if $(\varepsilon \diamond \eta) = 0$, $\forall \varepsilon, \eta \in \zeta$.
***Prop.* 2.3[1-3].**
In any *BD-A* $(\zeta; \diamond, o)$, the following hold: $\forall \varepsilon, \eta, \iota, \kappa \in \zeta$
(1) $\varepsilon \diamond \varepsilon = o$,
(2) $\varepsilon \diamond o = \varepsilon$,
(3)$(\varepsilon \diamond \eta) \diamond \iota = (\varepsilon \diamond \iota) \diamond \eta$,
(4)$(\varepsilon \diamond \eta) \diamond (\iota \diamond u) = (\varepsilon \diamond \iota) \diamond (\eta \diamond \kappa)$,
(5) $(\varepsilon \diamond (\varepsilon \diamond \eta)) \diamond \eta = o$,
(6) $(\varepsilon \diamond (\varepsilon \diamond \iota) \diamond \eta) \diamond \eta = o$,
***Remark* 2.4[1-3].**
Let $(\zeta; \diamond, o)$ be a *BD-A*, then
(1) If $\varepsilon \leq o, \forall \varepsilon \in \zeta$, then ζ contains only .
(2) If $\varepsilon \leq \eta$, then $\varepsilon * (\varepsilon * (\varepsilon * \eta)) = o, \forall \varepsilon, \eta \in \zeta$.
(3) If $\varepsilon \leq \eta$ such that $\varepsilon * \iota \leq \eta$, then $o \leq \iota, \forall \varepsilon, \eta, \iota \in \zeta$.
Def. 2.5. ([1-3]).
A subset S of a *BD*-A $(\zeta; \diamond, \epsilon)$ is Name **subalgebra of ζ (SA)** if $\varepsilon \diamond \iota \in S$ whenever $\varepsilon, \iota \in S$.
Prop. 2.7. ([1-3]).
Let { $_i$ | i∈Λ} be a family of **SA**s of *BD*-A $(\zeta; \diamond, \epsilon)$. The intersection of them of *SA*s of ζ is an *SA* of ζ.
Def. 2.8. ([25]).
Let $(\zeta; \diamond, \epsilon)$ be a nonempty set, a map. $\xi: \zeta \rightarrow [0,1]$.is name a fuzzy set ξ of ζ .
Def. 2.9. ([25]).
Let ξ be a fuzzy subset (FS) of a set . For t ∈ [0, 1], the set
$\xi t = U(\xi, t) = \{\varepsilon \in \zeta \mid \xi(\varepsilon) \geq t\}$, is name **upper level of ξ** and the set $L(\xi, t) = \{\varepsilon \in \zeta \mid \xi(\varepsilon) \leq t\}$ is name **lower level of ξ**.
Def. 2.10. ([25]).

Let $f:(\zeta;\diamond,\epsilon)\ \rightarrow (\zeta';\diamond',\epsilon')$ be a map. Non-empty sets ζ and ζ' resp.
If ξ is FS of ζ, then FS β of ζ' by:

$$f(\xi)(\iota)=\begin{cases} sup\{\xi(\varepsilon):\varepsilon\in f^{-1}(\iota)\} & if \quad f^{-1}(\iota)=\{\varepsilon\in\zeta, f(\varepsilon)=\iota\}\neq\emptyset \\ 0 & otherwise\end{cases}$$

is name the image of ξ under f.

Simil., if β is FS of ζ', then FS $\xi=(\beta\ o\ f)$ of ζ (i.e FS by $\xi(\varepsilon)=\beta(f(\varepsilon))\ \forall\varepsilon\in\zeta$) is name the pre-image of β under f.

Def. 2.11. ([24]).
A FS ξ of set ζ **has sup property** if for set T of ζ, $\exists t_0\in T\ \ni \xi(t_0)=sup\{\xi(t)|\ t\in T\}$.

Def. 2.12. ([1-3]).
Let $(\zeta;\diamond,\epsilon)$ be AB-A, a FS ξ of ζ is Name a **fuzzy subalgebra of ζ (F-SA)** if $\forall\varepsilon,\iota\in\zeta$, $\xi(\varepsilon\diamond\iota)\ \geq\ min\{\xi(\varepsilon),\xi(\iota)\}$.

Prop. 2.13. ([1-3]).
Let ξ be a FS of AB-A $(\zeta;\diamond,\epsilon)$. If ξ is a F-SA of , then for any t ∈ [0,1], ξ_t is a SA of ζ .

Prop. 2.14. ([1-3]).
Let $f:(\zeta;\diamond,\epsilon)\ \rightarrow(\zeta';\diamond',\epsilon')$ be a homo. from AB-As ζ to ζ' resp.
1- If F-SA β of ζ', then $f^{-1}(\beta)$ is a F-SA of ζ.
2- If F-SA ξ of ζ , then $f(\xi)$ is a F-SA of ζ'.

Def. 2.15.([24]).
An intuitionistic fuzzy subset A (IFS) in a non-empty set ζ is form A $=\{(\varepsilon,\zeta_A(\varepsilon),\eta_A(\varepsilon))\mid\varepsilon\in\zeta\}$ which $\xi_A:\zeta\ \rightarrow[0,1]$ and $\eta_A:\zeta\ \rightarrow[0,1]$ and
$0\ \leq\ \xi_A(\varepsilon)\ +\ \eta_A(\varepsilon)\ \leq\ 1\ \forall\varepsilon\in\zeta$.

Remark 2.18.([24]).
If IFS A of a non-empty set , then $\xi_A(\varepsilon)+\ \nu_A(\varepsilon)\ =\ 1$, i.e., $\nu_A(\varepsilon)\ =\ 1\ -\ \xi_A(\varepsilon)=\xi_A^c(\varepsilon)\ \forall\varepsilon\in\zeta$. Now ξ_A is fuzzy set while $\eta_A=\ \xi_A^c$ is complement of ξ_A.

Def. 2.16.
Let A $=\{(\varepsilon,\xi_A(\varepsilon),\eta_A(\varepsilon))\mid\varepsilon\in\zeta\}$be an IFS of AB-A $(\zeta;\diamond,\epsilon)$. A is named **intuitionistic fuzzy subalgebra of ζ(IF-SA)** if $\forall\varepsilon,\iota\in\zeta$
(IFS₁) $\xi_A(\varepsilon\diamond\iota)\ \geq\ \min\{\xi_A(\varepsilon),\xi_A(\iota)\}$.
(IFS₂) $\eta_A(\varepsilon\diamond\iota)\ \leq\ \max\{\eta_A(\varepsilon),\eta_A(\iota)\}$.
That mean ξ_A is a F-SA and η_A is a doubt F-SA.

Prop. 2.17.
If IFS A $=\{(\varepsilon,\xi_A(\varepsilon),\eta_A(\varepsilon))\mid\varepsilon\in\zeta\}$ of AB-A $(\zeta;\diamond,\epsilon)$ satisfies the inequalities $\xi_A(\epsilon)\ \geq\ \xi_A(\varepsilon)$ and $\eta_A(\epsilon)\ \leq\ \eta_A(\varepsilon),\forall\varepsilon\in\zeta$.

Theorem 2.18.
An IFS A $=\{(\varepsilon,\xi_A(\varepsilon),\eta_A(\varepsilon))\mid\varepsilon\in\zeta\}$ is an IF-SA of AB-A $(\zeta;\diamond,\epsilon)\Leftrightarrow\forall$ t ∈ [0,1], $U(\xi_A,t)$ & $L(\eta_A,s)$ are SAs in ζ.

3. *M*ultiplication Intuitionistic of Fuzzy Subalgebra.

Def. 3.1.
Let ξ be a FS of a set ζ and $\beta\in(0,1)$. A multiplication of ξ, by ξ_β^M is define map. $\xi_\beta^M:\zeta\rightarrow[0,1]$ by $\xi_\beta^M(\varepsilon)=\ \beta\,.\,\xi(\varepsilon)$, $\forall\varepsilon\in\zeta$.

Def. 3.2.
Let $A\ =\{(\varepsilon,\xi_A(\varepsilon),\eta_A(\varepsilon))\mid\varepsilon\in\zeta\}$ be an IFS of BD-A $(\zeta;\diamond,\epsilon)$ and let $\beta\ \in(0,1)$ an object having the form $A_\beta^M\ =\{(\varepsilon,(\xi_A)_\beta^M,(\eta_A)_\beta^M)|\ \varepsilon\in\zeta\}$ is Name **a β-multiplication intuitionistic of fuzzy subset of A (MI-FS)** if $(\xi_A)_\beta^M(\varepsilon)=\ \beta\,.\,\xi_A(\varepsilon)$ and $(\eta_A)_\beta^M(\varepsilon)=\ \beta\,.\,\eta_A(\varepsilon)$, $\forall\varepsilon\in\zeta$.

Def. 3.3.
Let $A_\beta^M\ =\{(\varepsilon,(\xi_A)_\beta^M,(\eta_A)_\beta^M)|\ \varepsilon\in\zeta\}$ be a (MI-FS) of A and $\beta\in(0,1)$, A_β^M is said to be **F-SA of ζ** if $\forall\ \varepsilon,\iota\ \in\zeta$
(IFS₁) $\beta\,.\,\xi_A(\varepsilon\diamond\iota)\ \geq\ \min\{\beta\,.\,\xi_A(\varepsilon),\ \beta\,.\,\xi_A(\iota)\}$,
(IFS₂) $\beta\,.\,\eta_A(\varepsilon\diamond\iota)\leq\ \max\{\beta\,.\,\eta_A(\varepsilon),\ \beta\,.\,\eta_A(\iota)\}$.
ξ_A is a FS in ζ & η_A is a doubt FS in ζ.

Theorem 3.4.

If $A = \{(\varepsilon, \xi_A(\varepsilon), \eta_A(\varepsilon)) \mid \varepsilon \in \zeta\}$ is an IF-SA of *BD*-A $(\zeta;\diamond,\epsilon)$, then the MI $A_\beta^M = \{(\varepsilon, (\xi_A)_\beta^M, (\eta_A)_\beta^M) \mid \varepsilon \in \zeta\}$ of A is a F-SA of ζ, $\forall \beta \in (0,1)$.

Proof:

Let $A = \{(\varepsilon, \xi_A(\varepsilon), \eta_A(\varepsilon)) \mid \varepsilon \in \zeta\}$ be an IF-SA of ζ and $\beta \in (0,1)$, then $\forall \varepsilon, \iota \in \zeta$,

$$(\xi_A)_\beta^M(\varepsilon \diamond \iota) = \beta . \xi_A(\varepsilon \diamond \iota) \geq \beta . \min\{\xi_A(\varepsilon), \xi_A(\iota)\}$$
$$= \min\{\beta . \xi_A(\varepsilon), \beta . \xi_A(\iota)\}$$
$$= \min\{(\xi_A)_\beta^M(\varepsilon), (\xi_A)_\beta^M(\iota)\}$$

and $(\eta_A)_\beta^M(\varepsilon \diamond \iota) = \beta . \eta_A(\varepsilon \diamond \iota) \leq \beta . \max\{\eta_A(\varepsilon), \eta_A(\iota)\}$

$$= \max\{\beta . \eta_A(\varepsilon), \beta . \eta_A(\iota)\}$$
$$= \max\{(\eta_A)_\beta^M(\varepsilon), (\eta_A)_\beta^M(\iota)\}.$$

Hence, the MI of A is a F-SA of ζ. ■

Prop. 3.5.

Every MI $A_\beta^M = \{(\varepsilon, (\xi_A)_\beta^M, (\eta_A)_\beta^M) \mid \varepsilon \in \zeta\}$ of A is F-SA of *BD*-A $(\zeta;\diamond,\epsilon)$ satisfies the inequalities $\beta . \xi_A(\epsilon) \geq \beta . \xi_A(\xi)$ and $\beta . \eta_A(\epsilon) \leq \beta . \eta_A(x)$, $\forall \varepsilon, \iota \in \zeta$.

Proof.

$$\beta . \xi_A(\epsilon) = \beta . \xi_A(\varepsilon \diamond \varepsilon) \geq \beta . \min\{\xi_A(x), \xi_A(\varepsilon)\}$$
$$= \min\{\beta . \xi_A(\varepsilon), \beta . \xi_A(\varepsilon)\}$$
$$= \beta . \xi_A(\varepsilon) \text{ and}$$
$$\beta . \eta_A(\epsilon) = \beta . \eta_A(\varepsilon \diamond \varepsilon) \leq \beta . \max\{\eta_A(\varepsilon), \eta_A(\varepsilon)\}$$
$$= \max\{\beta . \eta_A(\varepsilon), \beta . \eta_A(\varepsilon)\}$$
$$= \beta . \eta_A(\varepsilon).$$ ■

Theorem 3.6.

Let $A = \{(\varepsilon, \xi_A(\varepsilon), \eta_A(\varepsilon)) \mid \varepsilon \in \zeta\}$ be an IFS of *BD*-A $(\zeta;\diamond,\epsilon)$ $\ni$the MI $A_\beta^M = \{(\varepsilon, (\xi_A)_\beta^M, (\eta_A)_\beta^M) \mid \varepsilon \in \zeta\}$ of A is a F-SA of ζ, for some $\beta \in (0,1)$, then $A = \{(\varepsilon, \xi_A(\varepsilon), \eta_A(\varepsilon)) \mid \varepsilon \in \zeta\}$ is an IF-SA of ζ.

Proof:

Assume that $A_\beta^M = \{(\varepsilon, (\xi_A)_\beta^M, (\eta_A)_\beta^M) \mid \varepsilon \in \zeta\}$ of A is an MI of F-SA of ζ, for some $\beta \in (0,1)$. Let $\varepsilon, \iota \in \zeta$,

$$\beta . \xi_A(\varepsilon \diamond \iota) = (\xi_A)_\beta^M(\varepsilon \diamond \iota) \geq \min\{(\xi_A)_\beta^M(\varepsilon), (\xi_A)_\beta^M(\iota)\}$$
$$= \min\{\beta . \xi_A(\varepsilon), \beta . \xi_A(\iota)\}$$
$$= \beta . \min\{\xi_A(\varepsilon), \xi_A(\iota)\} \text{ and}$$
$$\beta . \eta_A(\varepsilon \diamond \iota) = (\eta_A)_\beta^M(\varepsilon \diamond \iota) \leq \max\{(\eta_A)_\beta^M(\varepsilon), (\eta_A)_\beta^M(\iota)\}$$
$$= \max\{\beta . \eta_A(\varepsilon), \beta . \eta_A(\iota)\}$$
$$= \beta . \max\{\eta_A(\varepsilon), \eta_A(\iota)\}$$

which implies that $\xi_A(\varepsilon \diamond \iota) \geq \min\{\xi_A(\varepsilon), \xi_A(\iota)\}$and

$\eta_A(\varepsilon \diamond \iota) \leq \max\{\eta_A(\varepsilon), \eta_A(\iota)\}$

Hence, $A = \{(\varepsilon, \xi_A(\varepsilon), \eta_A(\varepsilon)) \mid \varepsilon \in \zeta\}$ is IF-SA of ζ. ■

Def. 3.7. [8].

Let $t \in [0,1]$, $\beta \in (0,1)$ & FS ξ in a non-empty set ζ, $U_\beta(\xi, t) = \{\varepsilon \in \zeta \mid \beta . \xi(\varepsilon) \geq t\}$ is **Name multiplication of upper level of ξ**, and $L_\beta(\xi, t) = \{\varepsilon \in \zeta \mid \beta . \xi(\varepsilon) \leq t\}$ is **Name multiplication of lower level of ξ.**

Theorem 3.8.

If a MI $A_\beta^M = \{(\varepsilon, (\xi_A)_\beta^M, (\eta_A)_\beta^M) \mid \varepsilon \in \zeta\}$ of A is F-SA in *BD*-A $(\zeta;\diamond,\epsilon)$, then for $t, s \in [0,1]$, $U_\beta(\xi_A, t)$& $L_\beta(\eta_A, s)$ are SAs in ζ.

Proof.

Let $A_\beta^M = \{(\varepsilon, (\xi_A)_\beta^M, (\eta_A)_\beta^M) \mid \varepsilon \in \zeta\}$ be F-SA of ζ and $U_\beta(\xi_A, t) \neq \emptyset \neq L_\beta(\eta_A, s)$ and follow for every $\varepsilon, \iota \in \zeta$such as $\varepsilon \in U_\beta(\xi_A, t)$, $\iota \in U_\beta(\xi_A, t)$, then $\beta . \xi_A(\varepsilon) \geq t$ and $\beta . \xi_A(\iota) \geq t$, so therefore

$\beta . \xi_A(\varepsilon \diamond \iota) \geq \min\{\beta . \xi_A(\varepsilon), \beta . \xi_A(\iota)\} \geq t$, so as so

$(\varepsilon \diamond \iota) \in U_\beta(\xi_A, t)$. thus $U_\beta(\xi_A, t)$ is SA in ζ.

In a similar way, $L_\beta(\eta_A, s)$ is SA in ζ. ■

Theorem 3.9.

A MI $A_{\beta}^{M} = \{(\varepsilon, (\xi_A)_{\beta}^{M}, (\eta_A)_{\beta}^{M}) | \varepsilon \in \zeta\}$ of A of BD-A $(\zeta;\diamond,\epsilon)$. If $\forall$ t, s $\in$ [0,1], the set U_{β} $(\xi_A, t)$and L_{β} (η_A, s) are SAs of ζ, then A_{β}^{M} is F-SA of ζ.

Proof.

Assume that for each , t, s $\in$ [0,1] , U_{β} (ξ_A, t) & L_{β} (η_A, s) are SAs in ζ.

I found $\varepsilon', \iota' \in \zeta$ be such that

$\beta . \xi_A (\varepsilon' \diamond \iota') < \min\{ \beta . \xi_A (\varepsilon') , \beta . \xi_A (\iota')\}$, then by taking

$t_0 = \frac{1}{2} \{ \beta .(\xi_A (\varepsilon' \diamond \iota')) + \min\{ \beta . \xi_A (\varepsilon') , \beta . \xi_A(\iota')\}\}$, we get

$\beta . \xi_A (\varepsilon' \diamond \iota') < t_0 < \min\{ \beta . \xi_A (\varepsilon') , \beta . \xi_A(\iota')\}$ and hence

$(\varepsilon' \diamond \iota') \in U_{\beta}$ (ξ_A, t_0), $x' \in U_{\beta}$ (ξ_A, t_0), $\iota' \in U_{\beta}$ (ξ_A, t_0),

i.e., U_{β} (ξ_A, t_0), is not a SA of ζ, which make a C!.

Hence U_{β} $(\xi_A, t_0)$is a SA of ζ.

Finally, assume $\beta . \eta_A (\varepsilon' \diamond \iota') > \max\{ \beta . \eta_A (\varepsilon'), \beta . \eta_A (\iota')\}$. then by taking

$s_0 = \frac{1}{2} \{(\beta . \eta_A (\varepsilon' \diamond \iota')) + \max\{ \beta . \eta_A (\varepsilon'), \beta . \eta_A (\iota')\}\}$, we get

$\max\{ \beta . \eta_A (\varepsilon'), \beta . \eta_A (\iota')\} > s_0 > \beta .(\eta_A (\varepsilon' \diamond \iota'))$ and hence

$(\varepsilon' \diamond \iota') \in L_{\beta} (\eta_A, s_0)$, $\varepsilon' \in L_{\beta} (\eta_A, s_0)$, $\iota' \in L_{\beta} (\eta_A, s_0)$,

i.e., $L_{\beta} (\eta_A, s_0)$, is not a doute SA of ζ, which make a C!.

Therefore, $L_{\beta} (\eta_A, s_0)$ is a doute SA of ζ.

Hence $A_{\beta}^{M} = \{(\varepsilon, (\xi_A)_{\beta}^{M}, (\eta_A)_{\beta}^{M}) | \varepsilon \in \zeta\}$ is F-SA of ζ. ■

Theorem 3.10.

Let MI $A_{\beta}^{M} = \{(\varepsilon, (\xi_A)_{\beta}^{M}, (\eta_A)_{\beta}^{M}) | \varepsilon \in \zeta\}$ of A. If there exists as sequence $\{x_n\}$ in $\zeta \ni \beta . \xi_A(x_n) = 1$ and $\beta . \eta_A(x) = 0$, then $\beta . \xi_A(\epsilon)$=1 and $\beta . \eta_A(\epsilon)$=0.

Proof.

By Prop. (3.6), $\xi_A(\epsilon) \geq \xi_A(x)$ $\forall x \in \zeta$, therefore, $\beta . \xi_A(\epsilon) \geq \beta . \xi_A(x_n)$, for n $\in N$.

It is considered, $1 \geq \beta . \xi_A(x_n) = 1 \Rightarrow \beta . \xi_A(\epsilon) = 1$.

By Prop. (3.6), $\eta_A(\epsilon) \leq \eta_A(x)$, thus $x \in \zeta$, thus

$\beta . \eta_A(\epsilon) \leq \beta . \eta_A(x_n)$, for n $\in N$.

$\Rightarrow 0 \leq \beta . \eta_A(\epsilon) \leq \beta . \eta_A(x_n) = 0 \Rightarrow \beta . \eta_A(\epsilon) = 0$. ■

Prop. 3.11.

If the MI $A_{\beta}^{M} = \{(\varepsilon, (\xi_A)_{\beta}^{M}, (\eta_A)_{\beta}^{M}) | \varepsilon \in \zeta\}$ of A is F-SA, then $\forall \varepsilon \in \zeta$,

$\beta . \xi_A(\epsilon \diamond \varepsilon) \geq \beta . \xi_A(\varepsilon)$ and $\beta . \eta_A(\epsilon \diamond \varepsilon) \leq \beta . \eta_A(\varepsilon)$.

Proof.

$\forall \varepsilon \in \zeta$,

$$
\begin{aligned}
\beta . \xi_A(\epsilon \diamond \varepsilon) &\geq \min\{\beta . \xi_A(\epsilon), \beta . \xi_A(\varepsilon)\} \\
&= \min\{\beta . \xi_A(\varepsilon \diamond \varepsilon), \beta . \xi_A(\varepsilon)\} \\
&\geq \min\{\min\{\beta . \xi_A(\varepsilon), \beta . \xi_A(\varepsilon), \beta . \xi_A(\varepsilon)\} \\
&= \beta . \xi_A(\varepsilon) \text{ and}
\end{aligned}
$$

$$
\begin{aligned}
\beta . \eta_A(\epsilon \diamond \varepsilon) &\leq \max\{\beta . \eta_A(\epsilon), \beta . \eta_A(\varepsilon)\} \\
&= \max\{\beta . \eta_A(\varepsilon \diamond \varepsilon), \beta . \eta_A(\varepsilon)\} \\
&\leq \max\{\max\{\beta . \eta_A(\varepsilon), \beta . \eta_A(\varepsilon), \beta . \eta_A(\varepsilon)\} \\
&= \beta . \eta_A(\varepsilon).
\end{aligned}
$$
■

Def. 3.12.

Let $A_{\beta}^{M} = \{(\varepsilon, (\xi_A)_{\beta}^{M}, (\eta_A)_{\beta}^{M}) | \varepsilon \in \zeta\}$ $of A$ and $B_{\beta}^{M} = \{(\varepsilon, (\xi_B)_{\beta}^{M}, (\eta_B)_{\beta}^{M}) | \varepsilon \in \zeta\}$ $of B$ Be two MI of FSs of AB-A $(\zeta;\diamond,\epsilon)$, then **the intersection of A_{β}^{M} & B_{β}^{M}** by $A_{\beta}^{M} \cap B_{\beta}^{M}$ as

$A_{\beta}^{M} \cap B_{\beta}^{M} = \{min\{(\xi_A)_{\beta}^{M}, (\xi_B)_{\beta}^{M}\}, \max\{(\eta_A)_{\beta}^{M}, (\eta_B)_{\beta}^{M}\}\}$.

The **complement of A** by $\bar{A}_{\beta}^{M}$ by $\bar{A}_{\beta}^{M} = \{(x, (\eta_A)_{\beta}^{M}, (\xi_A)_{\beta}^{M}) | \varepsilon \in \zeta\}$.

Theorem 3.13.

Let A and B be two MSs of F-SAs of BD-A $(\zeta;\diamond,\epsilon)$, then $A \cap$ B is MI of F-SA of ζ.

Proof.

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Let $\varepsilon,\ \iota \in A \cap \mathrm{B}$, then $\varepsilon, \iota\ \in \mathrm{A}$ and B, then

$(\xi_{\mathrm{B}})_{\beta}^{\mathrm{M}}(\varepsilon \diamond \iota) = \min\{(\xi_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\varepsilon \diamond \iota), (\xi_{\mathrm{B}})_{\beta}^{\mathrm{M}}(\varepsilon \diamond \iota)\}$
$\geq \min\{\min\{(\xi_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\varepsilon), (\xi_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\iota)\}, \min\{(\xi_{\mathrm{B}})_{\beta}^{\mathrm{M}}(\varepsilon), (\xi_{\mathrm{B}})_{\beta}^{\mathrm{M}}(\iota)\}\}$
$= \min\{\min\{(\xi_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\varepsilon), (\xi_{\mathrm{B}})_{\beta}^{\mathrm{M}}(\iota)\}, \min\{(\xi_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\varepsilon), (\xi_{\mathrm{B}})_{\beta}^{\mathrm{M}}(\iota)\}\}$
$= \min\{(\xi_{A\cap\mathrm{B}})_{\beta}^{\mathrm{M}}(\varepsilon), (\xi_{A\cap\mathrm{B}})_{\beta}^{\mathrm{M}}(\iota)\}$ and
$(\eta_{A\cap\mathrm{B}})_{\beta}^{\mathrm{M}}(\varepsilon \diamond \iota) = \max\{(\eta_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\varepsilon \diamond \iota), (\eta_{\mathrm{B}})_{\beta}^{\mathrm{M}}(\varepsilon \diamond \iota)\}$
$\leq \max\{\max\{(\eta_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\varepsilon), (\eta_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\iota)\}, \max\{(\eta_{\mathrm{B}})_{\beta}^{\mathrm{M}}(\varepsilon), (\eta_{\mathrm{B}})_{\beta}^{\mathrm{M}}(\iota)\}\}$
$= \max\{\max\{(\eta_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\varepsilon), (\eta_{\mathrm{B}})_{\beta}^{\mathrm{M}}(\iota)\}, \max\{(\eta_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\varepsilon), (\eta_{\mathrm{B}})_{\beta}^{\mathrm{M}}(\iota)\}\}$
$= \max\{(\eta_{A\cap\mathrm{B}})_{\beta}^{\mathrm{M}}(\varepsilon), (\eta_{A\cap\mathrm{B}})_{\beta}^{\mathrm{M}}(\iota)\}$

Hence $A \cap \mathrm{B}$ isMI of F-SA of ζ. ■

Theorem 3.14.
The MI $A_{\beta}^{M} = \{(\varepsilon, (\xi_A)_{\beta}^{M}, (\eta_A)_{\beta}^{M}) \mid \varepsilon \in \zeta\}$ is MI of F-SA of BD-A $(\zeta;\diamond,\epsilon) \Leftrightarrow$ the FSs $(\xi_{\mathrm{A}})_{\beta}^{\mathrm{M}}$ and $(\bar{\eta}_{\mathrm{A}})_{\beta}^{\mathrm{M}}$ are F-SAs of ζ.

Proof.
Let $A_{\beta}^{M} = \{(\varepsilon, (\xi_A)_{\beta}^{M}, (\eta_A)_{\beta}^{M}) \mid \varepsilon \in \zeta\}$ be MI of F-SA of ζ.
Cleary$(\xi_{\mathrm{A}})_{\beta}^{\mathrm{M}}$ is a F-SA of ζ, for every$\varepsilon, \iota \in \zeta$, we have

$(\bar{\eta}_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\varepsilon \diamond \iota) = 1 - (\eta_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\varepsilon \diamond \iota)$
$\geq 1 - \max\{(\eta_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\varepsilon), (\eta_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\iota)\}$
$= \min\{1 - (\eta_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\varepsilon), 1 - (\eta_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\iota)\}$
$= \min\{(\bar{\eta}_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\varepsilon), (\bar{\eta}_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\iota)\}$.

Hence $(\bar{\eta}_{\mathrm{A}})_{\beta}^{\mathrm{M}}$ is a F-SA of ζ.
Conversely, assume that $(\xi_{\mathrm{A}})_{\beta}^{\mathrm{M}}$ and $(\bar{\eta}_{\mathrm{A}})_{\beta}^{\mathrm{M}}$ are two $MI$of F-SAs of ζ.
For every $\varepsilon, \iota \in \zeta$,$(\bar{\eta}_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\varepsilon \diamond \iota) \geq \min\{(\bar{\eta}_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\varepsilon), (\bar{\eta}_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\iota)\}$ and

$1 - (\eta_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\varepsilon \diamond \iota) = (\bar{\eta}_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\varepsilon \diamond \iota)$
$\geq \min\{(\bar{\eta}_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\varepsilon), (\bar{\eta}_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\iota)$
$= \min\{1 - (\nu_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\varepsilon), 1 - (\eta_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\iota)\}$
$= 1 - \max\{(\eta_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\varepsilon), (\eta_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\iota)\}$.

That is, $(\bar{\eta}_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\varepsilon \diamond \iota) \leq \max\{(\bar{\eta}_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\varepsilon), (\bar{\eta}_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\iota)\}$.
Hence $A_{\beta}^{M} = \{(\varepsilon, (\xi_A)_{\beta}^{M}, (\eta_A)_{\beta}^{M}) \mid \varepsilon \in \zeta\}$ be MI of F-SA of ζ. ■

Re. 3.15.
$I_{\xi_{\mathrm{A}}} = \{\varepsilon \in \zeta \mid (\xi_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\varepsilon) = (\xi_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\epsilon)\}$ is subset of ζ and the set $I_{\eta_{\mathrm{A}}} = \{\varepsilon \in \zeta \mid (\eta_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\varepsilon) = (\eta_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\epsilon)\}$ is subset of ζ.

Theorem 3.16.
Let $A_{\beta}^{M} = \{(\varepsilon, (\xi_A)_{\beta}^{M}, (\eta_A)_{\beta}^{M}) \mid \varepsilon \in \zeta\}$ be MI of F-SA of ζ, then the sets $I_{\xi_{\mathrm{A}}}$ and $I_{\eta_{\mathrm{A}}}$ are SAs of ζ.

Proof.
Let $\varepsilon, \iota \in I_{\xi_{\mathrm{A}}}$, then
$(\xi_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\varepsilon) = \beta.\xi_{\mathrm{A}}(\varepsilon) = \beta.\xi_{\mathrm{A}}(\epsilon) = (\xi_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\epsilon) = \beta.\xi_{A}(\iota) = (\xi_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\iota)$ and so,
$(\xi_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\varepsilon \diamond \iota) = \beta.\xi_{\mathrm{A}}(\varepsilon \diamond \iota) \geq \min\{\beta.\xi_{\mathrm{A}}(\varepsilon), \beta.\xi_{\mathrm{A}}(\iota)\} = \beta.\xi_{\mathrm{A}}(\epsilon)$, by Prop. (3.6), we know that $(\xi_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\varepsilon \diamond \iota) = (\xi_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\epsilon)$ or equivalently $\varepsilon \diamond \iota \in I_{\xi_{\mathrm{A}}}$.
Again, let $\varepsilon, \iota \in \mathrm{I}_{\eta_{\mathrm{A}}}$, then
$(\eta_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\varepsilon) = \beta.\eta_{\mathrm{A}}(\varepsilon) = \beta.\eta_{\mathrm{A}}(\epsilon) = (\eta_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\epsilon) = \beta.\eta_{\mathrm{A}}(\iota) = (\eta_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\iota)$ and so,$(\eta_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\varepsilon \diamond \iota) = \beta.\eta_{\mathrm{A}}(\varepsilon \diamond \iota) \leq \max\{\beta.\eta_{\mathrm{A}}(\varepsilon), \beta.\eta_{\mathrm{A}}(\iota)\} = \beta.\eta_{\mathrm{A}}(\epsilon)$.
Again by Prop. (3.5), we know that $(\eta_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\varepsilon \diamond \iota) = (\eta_{\mathrm{A}})_{\beta}^{\mathrm{M}}(\epsilon)$ or equivalently $\varepsilon \diamond \iota \in \mathrm{I}_{\eta_{\mathrm{A}}}$. Hence, $I_{\xi_{\mathrm{A}}}$ and $I_{\eta_{\mathrm{A}}}$ are SA in ζ. ■

4- Homomorphism of *M*ultiplication intuitionistic of BD-algebra.

Def. 4.1.[11].
Let $(\zeta;\diamond,\epsilon)$ & $(\zeta';\diamond',\epsilon')$ be two non-empty sets. A map. $f: \zeta \to \zeta'$ is **homomorphism** (**homo.**)if $f(\varepsilon \diamond \iota) = f(\varepsilon) *' f(\iota)$, $\forall \varepsilon, \iota \in \zeta$ &$f(\epsilon) = \epsilon'$.

Def. 4.2.

Let $f : (\zeta;\diamond,\epsilon) \to (\zeta';\diamond',\epsilon')$ be a homo. of BD -As for any $\mathrm{A}^{\mathrm{M}}_{\beta} = \{(\iota,(\xi_{\mathrm{A}})^{\mathrm{M}}_{\beta}(\iota),(\eta_{\mathrm{A}})^{\mathrm{M}}_{\beta}(\iota)) \mid \iota \in \zeta'\}$ in ζ' and $\beta \in (0,1)$ we define $(\mathrm{A}^{\mathrm{M}}_{\beta})^f = \{(\varepsilon,((\xi_{\mathrm{A}})^{\mathrm{M}}_{\beta})^f(\varepsilon),((\eta_{\mathrm{A}})^{\mathrm{M}}_{\beta})^f(\varepsilon)) \mid \varepsilon \in \zeta\}$ in ζ by

$((\xi_{\mathrm{A}})^{\mathrm{M}}_{\beta})^f(\varepsilon) = (\xi_{\mathrm{A}})^{\mathrm{M}}_{\beta}(f(\varepsilon))$ and $((\eta_{\mathrm{A}})^{\mathrm{M}}_{\beta})^f(\varepsilon) = (\eta_{\mathrm{A}})^{\mathrm{M}}_{\beta}(f(\varepsilon))$, $\forall\, \varepsilon \in \zeta$.

Theorem 4.3. Let $f : (\zeta;\diamond,\epsilon) \to (\zeta';\diamond',\epsilon')$ be a homo. of BD -A ζ into AB-A ζ'. If $A^M_\beta = \{(\varepsilon,(\xi_A)^M_\beta,(\eta_A)^M_\beta) \mid \varepsilon \in \zeta\}$ is MI of F-SA in ζ, $\Rightarrow$ the image of $\mathrm{A}^{\mathrm{M}}_{\beta}$ is MI of F-SA in ζ'.

Proof.

The image of $\mathrm{A}^{\mathrm{M}}_{\beta}$ is MI of F-SA in ζ', since $\mathrm{A}^{\mathrm{M}}_{\beta}$ is MI of F-SA in ζ and for any $\mathrm{a},\mathrm{b} \in \zeta$ $\exists \varepsilon,\iota \in \zeta'$ $\ni$ $f(\mathrm{a}) = \varepsilon, f(\mathrm{b}) = \iota \Rightarrow$

$$\begin{aligned}(\xi_{\mathrm{A}})^{\mathrm{M}}_{\beta}(\varepsilon \diamond \iota) &= (\xi_{\mathrm{A}})^{\mathrm{M}}_{\beta}\big(f(\mathrm{a}) \diamond' f(\mathrm{b})\big)\\ &\geq \min\{(\xi_{\mathrm{A}})^{\mathrm{M}}_{\beta}\big(f(\mathrm{a})\big),(\xi_{\mathrm{A}})^{\mathrm{M}}_{\beta}\big(f(\mathrm{b})\big)\}\\ &= \min\{(\xi_{\mathrm{A}})^{\mathrm{M}}_{\beta}(\varepsilon),(\xi_{\mathrm{A}})^{\mathrm{M}}_{\beta}(\iota)\}, \text{ and}\end{aligned}$$

$$\begin{aligned}(\eta_{\mathrm{A}})^{\mathrm{M}}_{\beta}(\varepsilon \diamond' \iota) &= (\eta_{\mathrm{A}})^{\mathrm{M}}_{\beta}\big(f(\mathrm{a}) \diamond' f(\mathrm{b})\big)\\ &\leq \max\{(\nu_{\mathrm{A}})^{\mathrm{M}}_{\beta}(f(\mathrm{a})),(\eta_{\mathrm{A}})^{\mathrm{M}}_{\beta}(f(\mathrm{b}))\}\\ &= \max\{(\eta_{\mathrm{A}})^{\mathrm{M}}_{\beta}(\varepsilon),(\eta_{\mathrm{A}})^{\mathrm{M}}_{\beta}(\iota)\}.\end{aligned}$$

$\Rightarrow$ the image of $\mathrm{A}^{\mathrm{M}}_{\beta}$ is F-SA in ζ'. ■

Theorem 4.4. Let $f : (\zeta;\diamond,\epsilon) \to (\zeta';\diamond',\epsilon')$ be a homo. of BD -A ζ into BD -A ζ'. If $A^M_\beta = \{(\varepsilon,(\xi_A)^M_\beta,(\eta_A)^M_\beta) \mid \varepsilon \in \zeta\}$ is MI of F-SA in ζ', $\Rightarrow$ the pre-image of $\mathrm{A}^{\mathrm{M}}_{\beta}$ is MI of F-SA in ζ.

Proof.

Let $(\mathrm{A}^{\mathrm{M}}_{\beta})^f$ is MI of F-SA in ζ, since $\mathrm{A}^{\mathrm{M}}_{\beta}$ is MI of F-SA in ζ' and let $\varepsilon,\iota \in \zeta$,

$$\begin{aligned}((\xi_{\mathrm{A}})^{\mathrm{M}}_{\beta})^f(\varepsilon \diamond \iota) &= (\xi_{\mathrm{A}})^{\mathrm{M}}_{\beta}\big(f(\varepsilon \diamond \iota)\big)\\ &= (\xi_{\mathrm{A}})^{\mathrm{M}}_{\beta}\big(f(\varepsilon) \diamond' f(\iota)\big)\\ &\geq \min\{(\xi_{\mathrm{A}})^{\mathrm{M}}_{\beta}\big(f(\varepsilon)\big),(\xi_{\mathrm{A}})^{\mathrm{M}}_{\beta}\big(f(\iota)\big)\}\\ &= \min\{((\xi_{\mathrm{A}})^{\mathrm{M}}_{\beta})^f(\varepsilon),((\xi_{\mathrm{A}})^{\mathrm{M}}_{\beta})^f(\iota)\}, \text{ and}\end{aligned}$$

$$\begin{aligned}((\eta_{\mathrm{A}})^{\mathrm{M}}_{\beta})^f(\varepsilon \diamond \iota) &= (\eta_{\mathrm{A}})^{\mathrm{M}}_{\beta}\big(f(\varepsilon \diamond \iota)\big)\\ &= (\eta_{\mathrm{A}})^{\mathrm{M}}_{\beta}\big(f(\varepsilon) \diamond' f(\iota)\big)\\ &\leq \{(\eta_{\mathrm{A}})^{\mathrm{M}}_{\beta}\big(f(\varepsilon)\big),(\eta_{\mathrm{A}})^{\mathrm{M}}_{\beta}\big(f(\iota)\big)\}\\ &= \{((\eta_{\mathrm{A}})^{\mathrm{M}}_{\beta})^f(\mathrm{x}),((\eta_{\mathrm{A}})^{\mathrm{M}}_{\beta})^f(\iota)\}.\end{aligned}$$

Hence $(\mathrm{A}^{\mathrm{M}}_{\beta})^f$ is F-SA in ζ. ■

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